TORSION IN SYMMETRICAL BUILDINGS

by

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SYNOPSIS

A rational basis is developed for determining torsional earthquake effects in symmetrical buildings arising from earthquake wave motions. It is concluded that the ratio of the accidental eccentricty e to the long plan dimension a varies almost directly with the fundamental frequency of vibration of the building and with the transit time of the earthquake wave motion. It is estimated that eccentricities e/a of 5 percent may be exceeded for frequencies higher than 1 Hertz, and eccentricities of 10 percent may be exceeded for frequencies higher than 2 Hertz. Corner columns and end shear walls should be designed with great conservatism.

GLOSSARY OF TERMS

a = long plan dimension of structure

b = short plan dimension of structure

c = wave velocity for ground surface motions

d_d = maximum value of transient ground displacement, u.

d_v = maximum value of transient ground velocity, $\dot{u}$

d_a = maximum value of transient ground acceleration, $\ddot{u}$

$d_{\dot{a}}$ = maximum value of time derivative of ground acceleration, $\dddot{u}$

D = relative linear displacement spectral value

D_d = spectral relative displacement bound for linear motion

D_v = spectral pseudo relative velocity bound for linear motion

D_a = spectral pseudo acceleration bound for linear motion

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e_y = effective eccentricity of seismic force in x-direction

e = e_y

f_x = natural frequency of flexural vibration of structure in x-direction

f_T = natural frequency of torsional vibration of structure

F = added torsional response displacement spectrum value

F_d = spectral added torsional displacement bound

F_v = spectral added pseudo relative torsional velocity bound

F_a = spectral added pseudo torsional acceleration bound

I_p = polar moment of inertia of mass of structure

k_x = distributed shear stiffness in x-direction

k_y = distributed shear stiffness in y-direction

M = mass of structure

t = time

u_x = transient displacement of ground in x-direction

u_y = transient displacement of ground in y-direction

$\dot{u}$ = derivative of u with respect to t

x = horizontal coordinate, in direction of design force considered

y = horizontal coordinate at right angles to x

φ = ground rotation or torsion

φ_d = maximum value of φ

φ_v = maximum value of $\dot{\varphi}$

φ_a = maximum value of $\ddot{\varphi}$

ρ_o = polar radius of gyration of mass M

τ = a/c

ω_x = circular natural frequency of flexural vibration in x-direction

ω_T = circular natural frequency of torsional vibration

INTRODUCTION

The need for providing resistance to torsion in buildings subjected to earthquakes has been recognized(1) for only two decades by specific provisions in design codes. However, these provisions are based almost entirely on judgment and have no sound theoretical basis except in the instances where the centers of resistance and mass do not coincide. For such cases methods are available for computing approximate values of torsional input. It is the purpose of this paper to develop a rational basis for determining the torsional earthquake effects in symmetrical buildings, and to arrive at design recommendations similar to those now used in some codes, but with a more appropriate consideration of the effects of building size, period of vibration, and type of framing on the necessary design eccentricities for seismic forces.

A design eccentricity of 5 percent of the maximum building dimension at each level is used in the Uniform Building Code(2) and the Recommended Lateral Force Requirements of the Structural Engineers Association of California.(3) In the latter reference, the statement is made that "this topic of torsion is acknowledged as one which is subject to further study."

The approach used herein is to develop an estimate of torsional ground motions from a consideration of measured strong ground motions assumed to propagate as a wave. From these motions an estimate is made of a torsional response spectrum. The combination of torsional response and flexural or ordinary response is then determined. The relative responses in torsion and flexure of several typical building configurations are computed, taking into account the differences in frequencies in these modes of response. Finally, values of eccentricity are determined to account for the computed responses, in terms of the building width and the wave propagation velocity.

A study of these results leads to certain conclusions and recommendations indicating that the design eccentricity should vary with the natural frequency of the building, or inversely as the period of vibration, and varies also as the transit time of the wave motion across the base of the building. It is also concluded that for the same torsional frequency the eccentricity is 1.0 to 1.7 times as high for a building with shear walls or with a small number of columns as for a building with relatively uniformly distributed shearing resistances over its plan area, with the larger ratio being applicable to lower torsional frequencies. Finally, it is indicated that the ratio of design eccentricity to longest plan dimension is a maximum for a building with a square plan, and drops to about half as much for a long narrow building.

TORSIONAL GROUND MOTION

We consider a horizontal x, y coordinate system at the ground surface, with transient displacements u_x and u_y in the coordinate directions. Then a wave motion for u_x with a wave velocity c in the direction of +y is designated by the relation

$$u_x = f(y - ct). \tag{1}$$

This relation corresponds to a wave with no change in shape. A similar relation is assumed for u_y:

$$u_y = f(x - ct). \tag{2}$$

From the fundamental relations of the Theory of Elasticity, the rotation φ of the ground surface about a vertical axis is:

$$\varphi = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) = \sim - \frac{\partial u_x}{\partial y} \tag{3}$$

We assume that both terms in (3) can be of about the same value, and consider the rotation to be equal to only one of them rather than the average of the two in the remainder of the study.

From Eq. (1) we find:

$$\varphi = - \frac{\partial u_x}{\partial y} = - f'(y - ct) \tag{4}$$

Also we determine the ground velocity $\dot{u}$ as:

$$\dot{u} = - cf'(y - ct) \tag{5}$$

Hence, by comparison of Eqs. (4) and (5) we find:

$$\varphi = \dot{u}/c \tag{6}$$

By a similar development with higher derivatives with respect to time, we find values of maximum torsional displacement φ_d, torsional velocity, φ_v, and torsional acceleration φ_a, related to maximum values of maximum ground velocity d_v, maximum ground acceleration d_a, and maximum time derivative of ground acceleration $d_{\dot{a}}$, as follows:

$$\left.\begin{aligned} \varphi_d &= d_v/c \\ \varphi_v &= d_a/c \\ \varphi_a &= d_{\dot{a}}/c \end{aligned}\right\} \tag{7}$$

We may now use these maximum values of ground rotation to estimate the response spectrum bounds for torsion in about the same way we use the maximum values of ground displacement, velocity and acceleration to estimate the response spectrum bounds for linear displacement. We can also use the actual time variations if we desire.

TORSIONAL STRUCTURAL RESPONSE

Methods of approximating the response spectrum for linear displacements are available.[(4),(5)] For about 7 percent critical damping the relative displacement bound D_d, the pseudo relative velocity bound D_v, and the pseudo acceleration bound D_a are related to the maximum ground motions approximately as follows:

$$\left.\begin{aligned} D_d &= \sim 1.2\ d_d \\ D_v &= \sim 1.6\ d_v \\ D_a &= \sim 2.0\ d_a \end{aligned}\right\} \quad (8)$$

We may expect slightly higher amplification factors, however, in the corresponding torsional response spectra because of the greater number of oscillations in each corresponding component. Before estimating the torsional response spectrum bounds, however, it is desirable to transform the torsional components so that they are directly comparable with the linear components of motion.

In Fig. 1 there is shown a rectangular building of dimensions a by b, with its maximum linear ground displacement d_d and its maximum torsional displacement φ_d indicated. It is of interest to compare the maximum torsional response displacement of the building with the maximum linear response displacement. To determine these we first note that multiplication of both sides of Eq. (7) by a transforms the terms into ground motions that pertain to twice the increased motion in the x-direction of the ground due to torsion:

$$\left.\begin{aligned} \varphi_d a &= d_v\ a/c = d_v\ \tau \\ \varphi_v a &= d_a\ a/c = d_a\ \tau \\ \varphi_a a &= d_a\ a/c = d_a\ \tau \end{aligned}\right\} \quad (9)$$

where $\tau = a/c$ = the transit time of the wave motion to pass over the long dimension of the building.

The right hand sides of Eqs. (9) have the same units as the right hand sides of Eqs. (8). We now define corresponding response spectrum bounds for the added torsional displacement, in terms of one-half the quantities in Eqs. (9), in order that we can make a direct comparison. We use slightly larger amplifications as explained above to obtain the bounds for the spectral torsional added displacement F_d, the spectral pseudo relative torsional added velocity F_v, and the spectral torsional added pseudo acceleration F_a, as follows:

$$\left.\begin{aligned} F_d' &= 1.33\ \varphi_d\ a/2 = 0.67\ d_v\ \tau \\ F_v &= 1.67\ \varphi_v\ a/2 = 0.83\ d_a\ \tau \\ F_a &= 2.4\ \varphi_a\ a/2 = 1.2\ d_{\dot a}\ \tau \end{aligned}\right\} \qquad (10)$$

Before we can proceed to draw the response spectrum for added torsional displacement we must first make an estimate of $d_{\dot a}$. This can be done in several ways; for example, one can estimate the time derivative of the recorded accelerations from strong-motion records, or one can use a relation governing the general shape of the trapezoid of maximum ground motions formed on a tripartite logarithmic plot of relative displacement, pseudo relative velocity and pseudo acceleration. The basic relations, with slightly different notation, have been published.(6) With the present notation we find:

$$d_d\ d_a > d_v^2 \qquad (11)$$

For most earthquakes, it has been determined(6) that

$$d_d\ d_a = \sim (5 \text{ to } 15)\ d_v^2 \qquad (12)$$

It can also be shown that a relation similar to Eq. (11) can be derived, as follows

$$d_v\ d_a > d_{\dot a}^2 \qquad (13)$$

and by analogy to Eq. (12)

$$d_v\ d_{\dot a} = \sim (5 \text{ to } 15)\ d_a^2 \qquad (14)$$

The El Centro earthquake of 1940 gives a value of slightly over 5 in Eq. (12) and the best estimate of $d_{\dot a}$ from the accelerogram leads to a value of about 6 in Eq. (14). Hence we now can determine a reasonably consistent set of values for our further computations.

A set of values of maximum ground motion components corresponding approximately to the N-S component of the El Centro earthquake of

May 18, 1940 is given in Table 1. The table also lists the linear displacement spectrum bounds computed from Eqs. (8), twice the maximum added torsional ground displacement components, computed from Eqs. (9), and the spectral torsional added displacement bounds computed from Eqs. (10). The torsional displacements and response spectrum bounds are given in terms of τ, the transit time. Particular values of the spectral bounds are given in Table 1 for $\tau = 0.1$ and $\tau = 0.05$, which can be considered to correspond to a building having a maximum plan dimension of 100 ft. and a ground having shear wave velocities of 1000 ft/sec. and 2000 ft/sec., respectively.

The response spectra tabulated in Table 1 for linear displacement and for added torsional displacement for $\tau = 0.1$ and 0.05 are plotted on a standard tripartite logarithmic plot in Fig. 2, for comparison.

TORSIONAL AND FLEXURAL FREQUENCIES

In order to compare the response displacements in torsion with the linear, or flexural, displacements we must compare the natural frequencies of vibration. General formulas are given here for several types of framing in rectangular buildings, and specific relative values are given in Table 2. The buildings considered are shown in Fig. 3.

In general, consider a structure having a total mass M, a mass polar moment of inertia I_p, and stiffness k_x and k_y in the x and y directions where the elements of stiffness are distributed over areas, along lines (i.e., walls), or concentrated at columns.

Then the circular frequency of vibration in flexural oscillation in the x direction is

$$\omega_x^2 = \Sigma\, k_x/M \tag{15}$$

and the circular frequency of vibration in torsional oscillation is

$$\omega_T^2 = \Sigma\, (k_x y^2 + k_y x^2)/I_p \tag{16}$$

The ratio of these is:

$$\frac{\omega_T^2}{\omega_x^2} = \frac{\Sigma\, (k_x y^2 + k_y x^2)}{\rho_o^2\, \Sigma\, k_x} \tag{17}$$

where $\rho_o^2 = I_p/M$.

For a rectangular structure, with a mass M uniformly distributed over the rectangular plan area,

$$\rho_o^2 = (a^2 + b^2)/12 \qquad (18)$$

When k_x and k_y are uniformly distributed over a rectangle, as in Fig. 3(a):

$$\Sigma\, k_x = k_x\, ab \qquad (19)$$

$$\Sigma\, (k_x y^2 + k_y x^2) = ab(k_x a^2 + k_y b^2)/12 \qquad (20)$$

Hence, for Fig. 3(a) the relative frequency is obtained from:

$$\frac{\omega_T^2}{\omega_x^2} = \frac{k_x a^2 + k_y b^2}{k_x(a^2 + b^2)} = \frac{1 + k_y b^2/k_x a^2}{1 + b^2/a^2} \qquad (21)$$

In this case, for either $k_y = k_x$, or for $b = 0$, it follows that $\omega_T = \omega_x = 1$.

For a rectangular structure with the shearing resistance concentrated along the outer boundary, as in Fig. 3(b):

$$\Sigma\, k_x = 2b\, k_x \qquad (22)$$

$$\Sigma\, (k_x y^2 + k_y x^2) = ab(k_x a + k_y b)/2 \qquad (23)$$

and therefore

$$\frac{\omega_T^2}{\omega_x^2} = \frac{3a(k_x a + k_y b)}{k_x(a^2 + b^2)} = 3\,\frac{1 + bk_y/ak_x}{1 + b^2/a^2} \qquad (24)$$

In this case, for $k_y = k_x$ and $b = a$, or for $b = 0$, it follows that

$$\omega_T/\omega_x = \sqrt{3}.$$

Values for other cases, such as a plan with 9 equal symmetrical columns uniformly spaced, or 4 equal symmetrical columns at the corners, as in Figs. 3(c) and (d), are readily computed from Eq. (17).

Values of ω_T/ω_x for particular aspect ratios of b/a for the special cases shown in Fig. 3 are tabulated in Table 2.

EFFECTIVE ECCENTRICITY

With the value of ω_T/ω_x known for a particular structure, one can enter the response spectra of Fig. 2, and for a particular value of τ one can determine the relative value of the added torsional response displacement F to the relative linear spectral displacement D. The values of F and D are always to be read from the displacement scales in Fig. 2, no matter which spectral bound is used. Specific values so determined for several frequencies f_x, framing types, and aspect ratios are given in Table 3 for f_T and F/D.

For a particular structure, when F/D is known, one can determine the eccentricity e_y, later used as e without subscript, of the lateral seismic force P_x, in terms of the long plan dimension a, that accounts for the added torsional displacement. To do this requires the following relations.

Let P_x designate the seismic force that accounts for the linear maximum deflection D:

$$D = P_x/\Sigma\, k_x \tag{25}$$

Let $P_x e$ be the torsional moment that accounts for the added torsional displacement + F on one end and - F at the other end of the long plan dimension:

$$F = P_x e\, a/2\, \Sigma\, (k_x y^2 + k_y x^2) \tag{26}$$

Then, from Eqs. (25) and (26), one derives:

$$\frac{F}{D} = \frac{e\, a\, \Sigma\, k_x}{2\, \Sigma\, (k_x y^2 + k_y x^2)} \tag{27}$$

For the special case of Fig. 3(a), with the values from Eqs. (19) and (20) substituted in Eq. (27), one obtains:

$$\frac{F}{D} = \frac{6\, e/a}{1 + b^2 k_y/a^2 k_x} \tag{28}$$

When b or $k_y = 0$, F/D = 6 e/a; and when b = a and $k_y = k_x$, F/D = 3 e/a.

For the special case of Fig. 3(b), with the values from Eqs. (22) and (23) substituted in Eq. (27), one obtains:

$$\frac{F}{D} = \frac{2\ e/a}{1 + bk_y/ak_x} \tag{29}$$

When b or $k_y = 0$, $F/D = 2\ e/a$; and when $b = a$ and $k_y = k_x$, $F/D = e/a$.

Values of aF/eD for several special cases are given in Table 2.

RESULTS OF CALCULATIONS

For the special cases of Figs. 3(a), (b), (c) and (d) values are given in Table 2 for aspect ratios b/a = 1, 0.5, and 0, from which one can determine f_T/f_x, and the eccentricity e to account for computed values of F/D. With selected natural frequencies for f_x of 0.318, 0.5, 1.0, 1.59, 3 and 5 cycles per second, by use of Fig. 2 values were determined for f_T and are shown in Table 3. For each condition, the value of F/D is also tabulated, for a value of $\tau = 0.1$. The values are proportional to τ.

The corresponding values of e/a are also given in Table 3 for $\tau = 0.1$, and these are also proportional to τ.

EFFECT OF YIELDING

The calculations reported herein are for elastic behavior of essentially simple structures. The results are not applicable to cases where yielding occurs. However, it is clear that the maximum combined stresses due to torsion and linear displacement will occur at or near the corners or ends of the building. When yielding occurs because of local weakness, it almost certainly will occur at one extreme point. Then the symmetry of the structure is impaired and further eccentricities are caused which makes yielding develop further and faster in the already yielded member, since the center of resistance moves further away from the yielded member. This indicates that yielding in torsion may be much more serious than yielding in flexure or in linear displacement, and the design should provide greater assurance of resistance to torsional yielding than to other types of yielding.

For this reason, corner columns or end shear walls should be more conservatively designed than other members of the structure.

CONCLUSIONS AND RECOMMENDATIONS

Although the specific data reported herein are directly applicable only to simple structures, it is believed that the conclusions drawn are valid for complex structures as well, since the data are applicable to the relative responses of structures in linear deformation and torsion, and apply to the higher modes of deformation as well as to the fundamental mode, in the same way. The following specific conclusions and recommendations are made.

1. The design eccentricity e for torsion, relative to the long plan dimension a, for a building nearly square in plan with uniformly distributed shear resistance, is approximately 0.9 $f_T\ \tau$, where f_T is the fundamental torsional frequency of the building, and τ is the transit time for a wave of motion to traverse the long plan dimension at the shear wave velocity c. This relation holds for frequencies greater than 0.3 cycles per second up to 10 cycles per second.

2. The design eccentricity e for a square building with shear resistance concentrated at the perimeter is about 1.5 $f_T\ \tau$ for f_T less than 1.6 cycles per second, and drops to 0.9 $f_T\ \tau$ for higher frequencies.

3. The design eccentricity for a building with a small number of columns, most of them placed on the perimeter, is about the same as for a building with all the shear resistance concentrated at the boundaries

4. The design eccentricity drops to 1/2 its maximum value for a building with uniformly distributed resistance as the shorter plan dimension b of the building plan approaches zero relative to the longer plan dimension a. For an aspect ratio b/a, the eccentricity varies approximately as $(1 + b^2/a^2)/2$.

5. The design eccentricity for a building with resistance concentrated at the perimeter varies approximately as $(1 + b/a)/2$.

6. It appears that buildings with short periods or high frequencies should be designed for a larger value of eccentricity than buildings with long periods or low frequencies. The UBC, for example, may not provide sufficient torsional resistance for framed buildings with periods shorter than about 0.6 sec. or shear-wall buildings with periods shorter than about 1 sec. A value of accidental eccentricity of 10 percent of the longer plan dimension would be reasonable for shorter periods, possibly even increasing to 15 percent at a period of 0.2 sec.

7. Corner columns and end shear-walls should be designed even more conservatively than other elements of resistance of a building, in order to permit the building to resist accidental torsions without serious risk of damage.

BIBLIOGRAPHY

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(2) Uniform Building Code, 1967 Edition, International Conference of Building Officials, Pasadena, 1967. See Sect. 2314(g), p. 124.

(3) Recommended Lateral Force Requirements and Commentary, Seismology Committee, Structural Engineers Association of California, San Francisco, 1967. See Sect. 2313(g), p. 15; p. 38; p. 68-69.

(4) Reference (1), p. 71.

(5) Nathan M. Newmark, "Current Trends in the Seismic Analysis and Design of High Rise Structures," Proc. Symposium on Earthquake Engineering, University British Columbia, Vancouver, 1965, p. VI-1 to VI-55. See Figs. 6 and 7.

(6) Nathan M. Newmark, "Problems in Wave Propagation in Soil and Rock," Symposium on Wave Propagation and Dynamic Properties of Earth Materials, University of New Mexico, Albuquerque, 1968. See Eqs. (10) and (11).

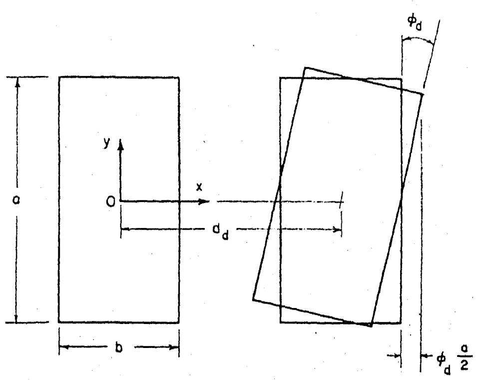

FIG. 1 TRANSLATION AND ROTATION OF BUILDING FOUNDATION

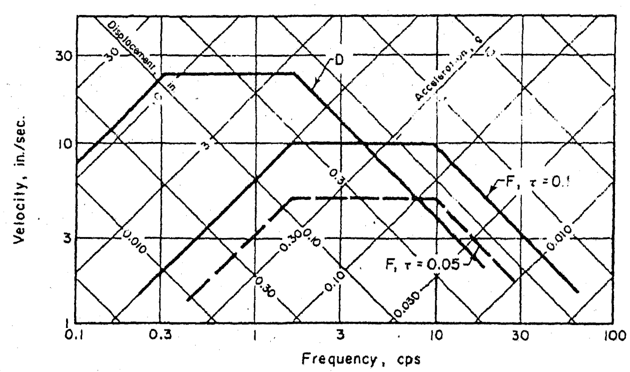

FIG. 2 RESPONSE SPECTRA FOR LINEAR MOTION AND ADDITIONAL TORSIONAL MOTION

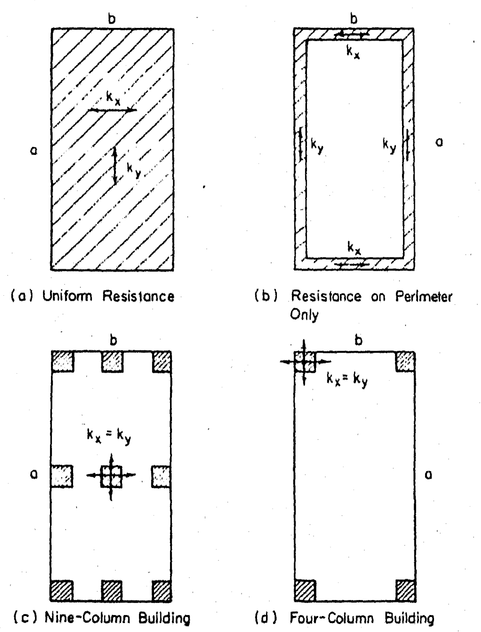

FIG. 3 TYPES OF BUILDINGS CONSIDERED

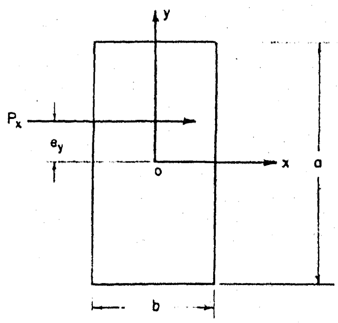

FIG. 4 ECCENTRIC SEISMIC FORCE PRODUCING TORSION AND DISPLACEMENT

TABLE 1. GROUND MOTION AND RESPONSE SPECTRUM VALUES

	Quantity	Displacement, in.	Velocity, in/sec.	Acceleration, in/sec^2
(a)	Linear grou nd motion	10	15	120
(b)	Response spe trum bounds for (a)	12	24	240
(c)	Additional torsional ground motion times factor of two	15τ	120τ	5000τ
(d)	Response spectrum bounds for additional torsional motion	10τ	100τ	6000τ
(e)	Values of (d) for $\tau = 0.1$	1.0	10.0	600 = 1.55g
(f)	Values of (d) for $\tau = 0.$	0.5	5.0	300 = 0.78g

TABLE 2. PARAMETERS FOR STRUCTURES CONSIDERED

Condition b/a	k_y/k_x	Quantity	Structure: Uniform Dist. Fig. 3(a)	Perimeter Fig. 3(b)	9 Column Fig.3(c)	4 Column Fig. 3(d)
1.0	1.0	$\omega_T/\omega_x = f_T/f_x$	1	1.732	1.414	1.732
		aF/eD	3	1	1.5	1
0.5	1.0	$\omega_T/\omega_x = f_T/f_x$	1	1.897	1.414	1.732
		aF/eD	4.8	1.333	2.4	1.6
0.0	All	$\omega_T/\omega_x = f_T/f_x$	1	1.732	1.414	1.732
		aF/eD	6	2	3	2

TABLE 3. RESULTS OF CALCULATIONS FOR TORSIONAL ECCENTRICITY
All values are for $\tau = 0.1$

Structure		Uniform, Fig.3(a)			Perimeter Fig. 3(b)			9-Column Fig.3(c)		4-Column, Fig. 3(d)	
b/a		1.0	0.5	0	1.0	0.5	0	1.0	0	1.0	0
f_x, Hertz	Item										
0.318	f_T	0.318	0.318	0.318	0.550	0.600	0.550	0.450	0.450	0.550	0.550
	F/D	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083
	e/a	0.028	0.017	0.014	0.083	0.062	0.041	0.056	0.028	0.083	0.042
0.5	f_T	0.5	0.5	0.5	0.87	0.95	0.87	0.71	0.71	0.87	0.87
	F/D	0.131	0.131	0.131	0.131	0.131	0.131	0.131	0.131	0.131	0.131
	e/a	0.044	0.027	0.022	0.131	0.098	0.065	0.088	0.044	0.131	0.065
1.0	f_T	1.00	1.00	1.00	1.73	1.90	1.73	1.41	1.41	1.73	1.73
	F/D	0.262	0.262	0.262	0.240	0.219	0.240	0.262	0.262	0.240	0.240
	e/a	0.087	0.055	0.043	0.240	0.180	0.120	0.174	0.087	0.240	0.120
1.59	f_T	1.59	1.59	1.59	2.75	3.0	2.75	2.25	2.25	2.75	2.75
	F/D	0.416	0.416	0.416	0.241	0.220	0.241	0.294	0.294	0.241	0.241
	e/a	0.139	0.087	0.069	0.24	0.18	0.12	0.196	0.098	0.241	0.120
3	f_T	3.0	3.0	3.0	5.2	5.7	5.2	4.2	4.2	5.2	5.2
	F/D	0.79	0.79	0.79	0.46	0.42	0.46	0.56	0.56	0.46	0.46
	e/a	0.26	0.16	0.13	0.46	0.35	0.23	0.38	0.19	0.46	0.23
5	f_T	5.0	5.0	5.0	8.7	9.5	8.7	7.1	7.1	8.7	8.7
	F/D	1.31	1.31	1.31	0.75	0.69	0.75	0.92	0.92	0.75	0.75
	e/a	0.44	0.27	0.22	0.75	0.56	0.37	0.62	0.31	0.75	0.37