RESPONSE ANALYSIS OF FRAMED STRUCTURES (II)

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SYNOPSIS

In the previous paper[2)], the writer treated the problems of how to construct the equation of motion for arbitrary shaped framed structures and developed the numerical integral method (direct method) for solving this equation. The problem rose in selecting the damping coefficient on the process of calculation for the response of actual structures. In this paper, the modal analysis for solving equation of motion is discussed, ahead of recent paper dealt with the formula of modal analysis for calculation, and the remained of the paper explained the results obtained from response analysis with respect to various conditions of suspension bridge.

INTRODUCTION OF FORMULA

The equation of motion for multi-mass-system is, by using matrices, expressed as follows

$$\mathbb{M}\ddot{x} + \mathbb{C}\dot{x} + \mathbb{K}x = -\mathbb{M}\mathbb{F}\ddot{x}_e \quad \text{--------} \quad (1)$$

where

$\mathbb{M}$ = mass matrix;

$\mathbb{C}$ = damping coefficient matrix;

$\mathbb{K}$ = stiffness matrix;

x = relative displacement vector of mass points;

x_e = earthquake movement;

$\mathbb{F}$ = the difference of absolute and relative displacement vector dividing by x_e, while the displacement is the same direction as seismic acceleration, the value of element in this vector is 1, otherwise equals zero;

The derivation of equation (1) and the numerical integral method (direct method) in solving equation (1) were explained by the previous paper[2)]. Responses of various types of framed structures obtained from direct method are in every time troubled by the problem of selecting the damping coefficient. In order to improve such fault, another formula based on modal analysis is derived from assuming that the damping ratio is proportion to the shape of mode. Now introducing a linear equation

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$$(\mathbb{M}\lambda^2 - \mathbb{K})\ X = 0$$

let υ_i be the root other than zero, and P_i^2 be the value of λ^2, that is to say, the eigenvector and eigenvalue. If $\mathbb{V}$ denotes the matrix arranging υ_i in a column, and $\mathbb{P}^2$ the matrix arranging P_i^2 diagonally, the relation between them is

$$\mathbb{V}^T \mathbb{K} \mathbb{V} = \mathbb{V}^T \mathbb{M} \mathbb{V} \mathbb{P}^2 \tag{2}$$

Each element of $\mathbb{P}$ is circular natural frequency, and each column of $\mathbb{V}$ shows proper mode of vibration. Further, changing the independent variables x_i of Eq.(1) to q_i by the relation

$$x = \mathbb{V} q \tag{3}$$

and multiplying $\mathbb{V}^T$ from the left side, Eq.(4.) is obtained.

$$\mathbb{V}^T \mathbb{M} \mathbb{V} \ddot{q} + \mathbb{V}^T \mathbb{C} \mathbb{V} \dot{q} + \mathbb{V}^T \mathbb{M} \mathbb{V} \mathbb{P}^2 q = -\mathbb{V}^T \mathbb{M} \mathbb{F} \ddot{x}_e \tag{4}$$

Because the critical damping coefficient matrix of motion (1) for a multi-mass-system is $2\mathbb{M}\mathbb{V}\mathbb{P}\mathbb{V}^{-1}$, Eq.(4) is transformed into

$$\ddot{q} + 2(h_0 \mathbb{P}^{-1} + h_1 \mathbb{U} + h_2 \mathbb{P})\mathbb{P}\dot{q} + \mathbb{P}^2 q = -(\mathbb{V}^T \mathbb{M} \mathbb{V})^{-1} \mathbb{V}^T \mathbb{M} \mathbb{F} \ddot{x}_e \tag{5}$$

When $P^{-1} S_{vi}(t)$ is the solution of the equation of motion for one mass system in which damping ratio h_i and circular natural frequency P_i of multi-mass-system are substituted, $\mathbb{S}_{vi}(t)$ being the matrix of diagonal arrangement of $S_{vi}(t)$, the solution of Eq.(5) is

$$q = \mathbb{P}^{-1} \mathbb{S}_v(t)\ (\mathbb{V}^T \mathbb{M} \mathbb{V})^{-1} \mathbb{V}^T \mathbb{M} \mathbb{F}$$

and the relative displacement is obtained by substituting in Eq.(3), as follows:

$$x = \mathbb{V} \mathbb{P}^{-1} \mathbb{S}_v(t)\ (\mathbb{V}^T \mathbb{M} \mathbb{V})^{-1} \mathbb{V}^T \mathbb{M} \mathbb{F} \tag{6}$$

Sectional forces would be then calculated from the displacement method of statics.

RESULTS OF THE CALCULATION

Three sets of models were made from the suspension bridge as shown in Fig. (1), and results of the calculation were compared with each other. Model No.1 includes 38 nodes, 48 members, model No.2 includes 54 nodes, 75 members, and model No.3, 90 nodes, 129 members respectively. After 20 cases of computation, we summarize the results as follows.

The natural period is divided by oscillated mode distinctly, for example, modes of yielding large movement against suspended structure, tower, and hanger, natural periods of them are lain in the range of above one second, below 1-2 second, and below 0.25 second respectively.

Especially, the long period is necessary so as to yield large movement for the suspended structure, in general, periods of about ∞, 50 sec., and 40 sec. are usually found and oscillated modes are shown in Fig.(2). Each oscillated mode in Fig. (2) is composed from stiffened truss of which a part is rigid, we can't obtain such results from usual method by solving the differential equation.

Almost all long periods of larger than 3 sec are very few changes by increasing the number of nodes as sequential as model No.1 → No.2 → No.3, but modes are increased, if their periods are about one sec for model No.2, No.3 and below one sec for model No.3.

Maximum displacements obtained from the response spectrum analysis at both of tower and suspended structure are larger than 10 cm, compared those values with the values of obtained from modal analysis, the displacements of tower is approximately larger than 10 cm, but that of suspended structure is excessed 1 m., maximum displacements yield nearly at the anchor of the side span for suspended structure and at the section of under 30-60 cm from the top for tower. If the structure is symmetric, the displacement of the center span will not occur, this means that almost all of energies are absorbed by side span.

The maximum tensile force in main cable is approximately 1000 ton., large values of the tensile force in hanger and of bending moment in stiffened structure are obtained by several cases, excluding the cases of such large values, we may summarize that the tensile force of about 17 ton in hanger and the bending moment of $10000^{t \cdot m}$ in stiffened truss are lasted.

From the static analysis, the tensile force of 10000 ton in cable and that of 380 ton in hanger, 36720 $^{t \cdot m}$ of bending moment in suspended structure due to live load are obtained, compared those values with the results from dynamic analysis, it is not the question.

On the one hand, maximum shearing force of 1500^{t} accompanying with max. bending moments of $42{,}000\text{-}60{,}000^{t \cdot m}$ acting at the pier base are obtained from dynamic response, where bending moments of $55{,}000\text{-}60{,}000^{t \cdot m}$ are determined from model No.1 and those of $42{,}000\text{-}46{,}000^{t \cdot m}$ from model No.2. In general, sectional forces in suspended structure are fairly different, from this fact, if we desire to obtain more accurate results, the model such as No.2 should be modelled, otherwise the satisfying results can not be obtained.

COMPARISON OF MODELS

The natural periods of model No.1 (38 nodes, 49 members) is included in that of model No.2 (54 nodes, 75 members), and that of model No.2 is also included in that of model No.3 (90 nodes, 129 members). The periods of larger than 3 sec for suspended structure and about one sec for tower appear in common with all models, periods of about one sec for models No.2 and No.3, 1.0-0.5 sec for model No.3 are more frequently appeared than those of model No.1 for suspended structure. In other words, three sets of models having long period produce similar modes, otherwise periods in the range of 0.5-1.5 sec, give different responses among them.

In comparison with max. displacements and sectional forces at the tower portion, it can't be found extremely difference among three sets of models. But there is quite difference in comparison with sectional forces against the suspended structure, depending on this fact, we may conclude that, if there is no difference in the range of long period among three models, the changes of the max. displacements also can't be yield. But in the range of short periods 0.5-1.5 sec for yielding large movement against suspended structure, modes have several difference, sectional forces also are affected by those differences. Tensile forces for cable and hanger in the side span of model No.1 are less than those in model No.2, but the inverse results are obtained in the center span.

As described in the above, if we would like to determine max. displacements only, proper number of nodes for suspended structure is enough, for the determination of sectional force, number of nodes should be increased at least to model No.2, otherwise we can not expect to obtain more accurate sectional force in suspended structure.

EFFECTS DUE TO DIFFERENCES OF SEISMIC WAVES

Seismic waves obtained from EL Centro and Taft are to be used for analysis of model No.1, and the results of modal analysis is shown in Fig.(3). Abscissa indicates the results determined from seismic waves of El Centro and longitudinal axis represents the results obtained from seismic waves of Taft. Both axes are scaled by logrithm. From this Fig., we find the results obtained from El Centro is larger than that obtained from Taft, while the results of the former is taken as a base for measurement, the results of the latter is lain in the range of 15-150%, almost all max. sectional forces (excluding the bending moments of center span) are lain in the range of 2/3-3/2 (66%-150%), and max. displacements (excepting the top of the tower) at tower portion are in the range of 55-70%, those values of suspended structure are in the range of 15-25%.

The reason of yielding such difference among those values are probably depended on the character of response spectrum, the more adequate reason is under investigated. The response velocity spectrum by using El Centro and Taft seismic waves converted into max. acceleration of 200 gals is indicated in Fig. (4).

COMPARISON BETWEEN THE RESPONSE SPECTRUM ANALYSIS AND MODAL ANALYSIS

The results determined from response spectrum analysis are plotted against abscissa, the results of modal analysis, using seismic waves of El Centro and Taft with damping coefficient $h = 0.1$, are plotted with respect to the longitudinal axis, and shown in Fig.(5) and Fig.(6). The results of using the response spectrum suggested by THE TECHNICAL RESEARCH COMMITTEE OF HONSHU SHIKOKU CONNECTING BRIDGE is shown in Fig.(4). The natural period of larger than one sec in Fig.(4), the results calculated from response spectrum analysis (plotted by —·— line) is rather higher than those obtained from El Centro and Taft with damping coefficient $h=0.1$ (shown by solid and dotted line).

The results in Fig.(5) and Fig.(6) have the same tendency such as in Fig.(4), we may conclude that the results obtained from the response spectrum analysis is larger than those from modal analysis, such phenomenon appears obviously in the center span of suspended structure.

INFLUENCE OF DAMPING COEFFICIENT

Damping coefficients of 0.1 and 0.5 are selected for response analysis with respect to the same structure, and the results is indicated in Fig.(7), simultaneously, the results in Fig.(8) is also obtained by using damping coefficients h=0.1 and 0.2. The results of using h=0.15 is lower than those of h=0.1 about 80% (Fig.(7)), in the case of h=0.2, the results of which is lower than those of h=0.1 about 60% (Fig. (8)). In general, there has a large tendency of descent for sectional force, but for the displacements, the case of h=0.15 is lower than that of h=0.1 about 90%, and the case of h=0.2 is lower than that of h=0.1 about 80%.

INFLUENCE OF THE UNSYMMETRICITY

For symmetric suspended bridge, the displacements and the sectional forces in center span become very small, if the character of symmetry is disturbed slightly, the displacements and the sectional forces will increase rapidly.

The results obtained from the response spectrum analysis and modal analysis with respect to the center span on change of own weight of the right tower are shown in Fig.9(a) and Fig.9(b), abscissa indicates the change ratio of own weight and longitudinal axes indicate, (i) displacements of span center (Fig.9(a)), (ii) tensile forces of main cable, hanger and bending moment of stiffened truss in center span (Fig.9(b)) respectively.

The results on change of sectional area (A) and moment of inertia (I) of the left tower are shown in Fig.(10). From those Figs., we know the fact that parameters of displacement, tensile force and bending moment in center span are changed steeply, when the suspension bridge has slight unsymmetry, hence in modelling suspension bridge, the notice of symmetry must be taken, otherwise the large error will be introduced into the results of center span by influence of the unsymmetry.

COMPARISON OF THE STRUCTURAL TYPES

Max. displacement is yielded at node 22 (Fig.(11)), the tensile forces of members 35 and 38 are very larger than those of hangers, in order to improve such fault, calculations are carried out by (i) increasing cross sections in member 35, 38, 39 and hanger, (ii) adding members of 50 and 51 on connecting with the tower (Fig. (11)). Unfortunately poor results are obtained by such improvement. The reason of causing such unsatisfactory result is that owing to varnish from the displacement, forces are concentrated on the strengthened member. As to improve large displacement, diagonal hangers to be used on suspension bridge are considerable.

The calculation also are carried out by connecting stiffened truss of side span with that of center span as a continuous stiffened truss, the large sectional forces are appeared at tower portion and members 38 and 39, we can't obtain satisfying results in those calculation.

CONCLUSION

From the above investigation, we may conclude as follows,

(1) For the aim to obtain the sectional force of tower or all nodes of displacements, simple model constructed from actual bridge provides satisfying results, if the aim to obtain the sectional force of suspended structure, the more accurate model constructed from actual bridge is necessary, otherwise we can't expect to obtain satisfying results.

(2) The response spectrum suggested by THE TECHNICAL RESEARCH COMMITTEE OF HONSHU-SHIKOKU CONNECTING BRIDGE for determined the displacements of tower (excluding the displacement at the top of tower) provides satisfying results, but the correction must be made on calculating the displacements of the suspended structure.

(3) The displacements and sectional forces in center span is affected very sensitively, if the suspension bridge is unsymmetric, hence the model constructed from unsymmetric actual bridge must be correctly.

(4) The members which suspend stiffened truss at both ends produce large sectional forces, in avoidance of large values in those members, the method of constrained displacement should not be applied.

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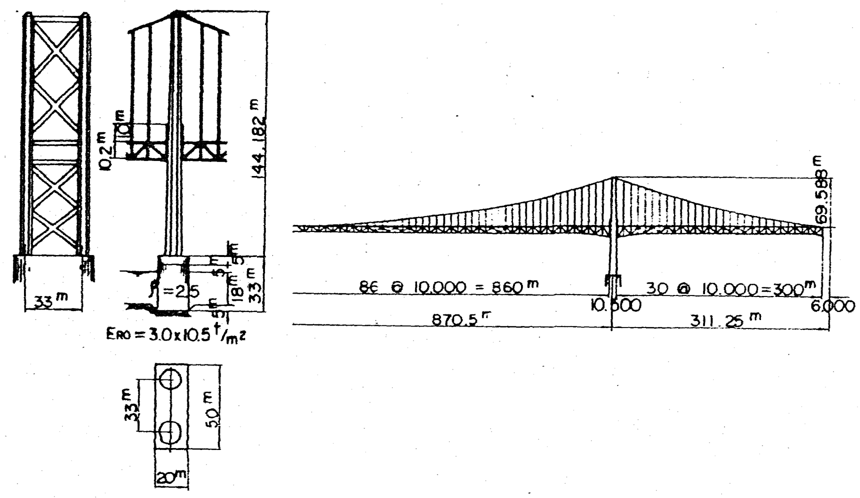

Fig. 1 Suspension Bridge to be used for calculation

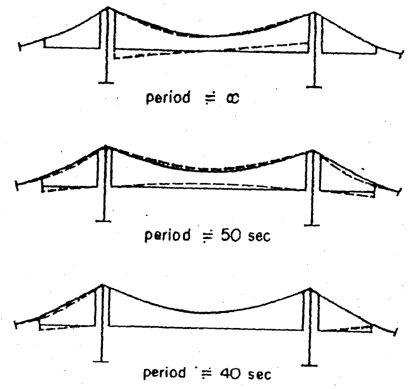

Fig. 2 Ocillated Modes

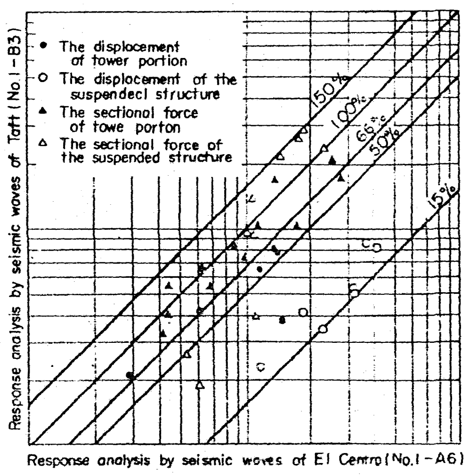

Fig. 3

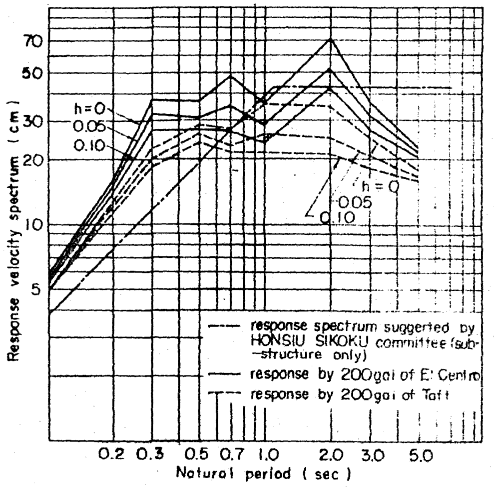

Fig. 4

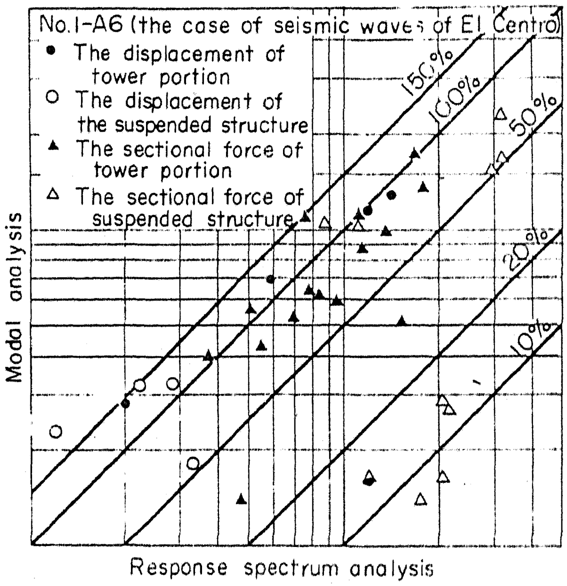

Fig. 5

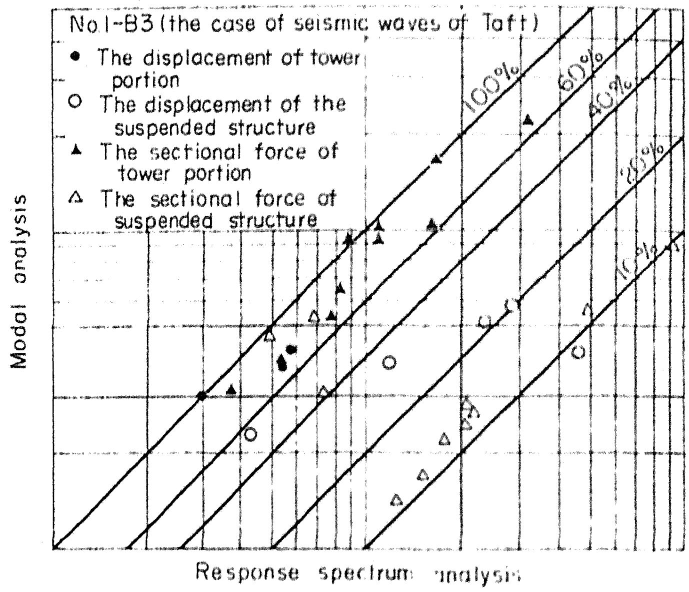

Fig. 6

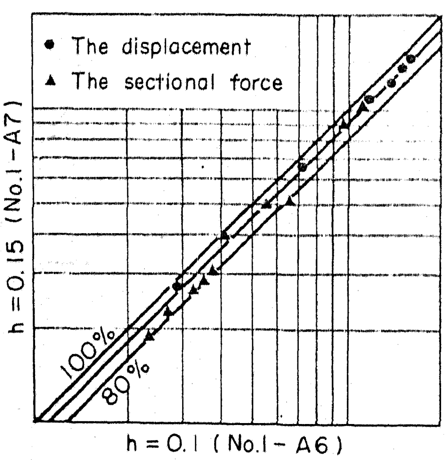

Fig. 7

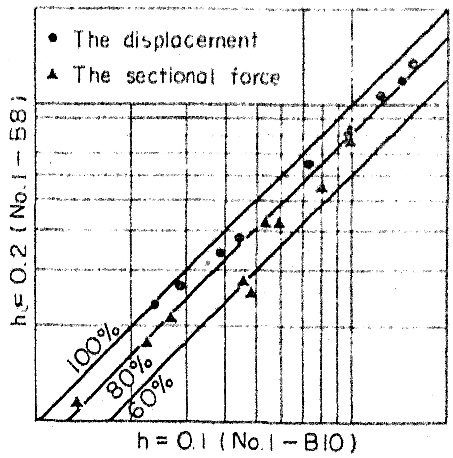

Fig. 8

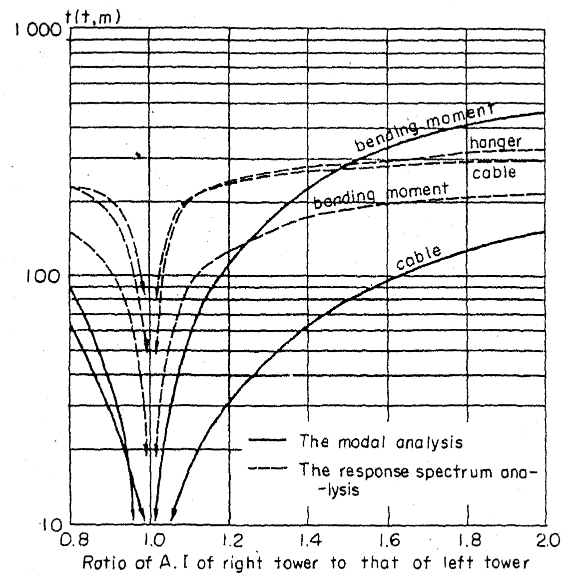

Fig. 10 (a)

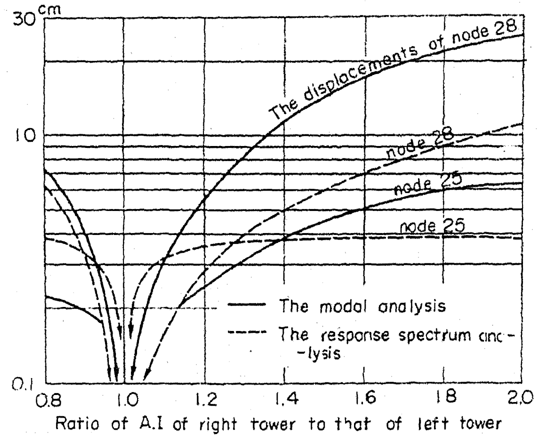

Fig. 10(b)

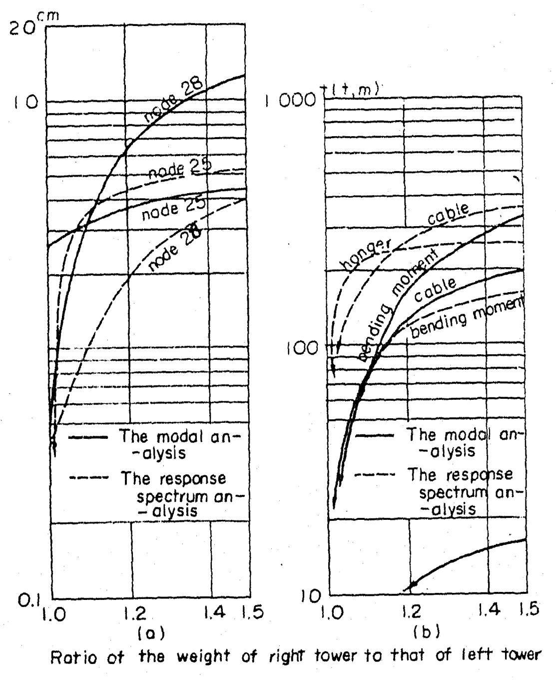

Fig 9

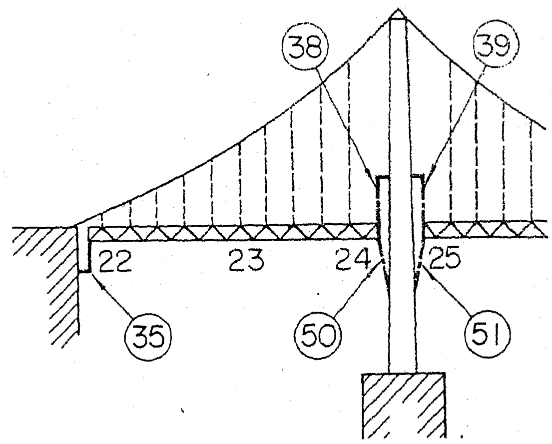

Fig. 11