

Abstract.

Applying non-linear Maxwell-Kelvin body for dynamic model of clay from relation between stress and strain obtained by dynamic triaxial test, non-linear visco-elastic constants and poisson's ratio were clarified. Using these physical constants, dynamical analysis of earth dam was made as two dimensional initial value problem based on extentional vibration by finite difference method. For the starting point of this analysis, statical analysis of distribution of stress in earth dam was made as two dimensional boundary value problem by finite difference method. In this case also, non-linear elastic modulus and poisson's ratio obtained by statical triaxial test were used. From these methods of analysis, it has become possible to discuss the stability of earth dam concretely and precisely based on actual properties of soil.

1. Introduction.

A-seismic problem of structures made of soil or structures whose foundations are built in soil is extremely important, however it is difficult to make rational analysis in the present status. That is mainly because, physical constants of soil which are fundamental elements for the analysis, have not been accurately known. In the previous paper¹⁾, authors gave instances of non-linear physical constants which were obtained by dynamical tri-axial tests on respective one kind of clay, sand and crushed stone. In the present paper, seismic analysis of the earth dam is tried to be made using physical constants of clay.

For the time being, a-seismic design of the earth dam is confined to slip circle method, but in regard to the research of dynamical analysis, Dr. Matsumura and Takada²⁾ formerly carried on the study as shear vibration. However it is doubtful to treat the vibration of the earth dam as shear vibration. On the other hand, Prof. Ishizaki and Hatakeyama³⁾ presented the method to analyse equation of equilibrium expressed by displacement basing on finite difference method treating the vibration of the earth dam as extentional. The displacement response of soil on dynamic load is undoubtedly extentional, and the observation results on the prototype earth dam seems to show that it is better to be treated as extentional vibration. But in this analysis, perfect elasticity was assumed as the property of soil and damping of vibration was neglected. After that, Prof. Clough and others⁴⁾ obtained extentional solution of the earth dam basing on the finite element method on the assumption that soil material has constant elasticity and the fraction of critical damping is 20% in each mode. However their analysis has no direct connection with the actual physical properties of soil.

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| 1) Hatano, Watanabe. | Technical Report C 68002, 1968 |
| 2) Matsumura, Takada. | Public Works Research Institute Report 28, 1934 |
| 3) Ishizaki, Hatakeyama. | Disaster Prevention Research Inst. Bul. 52. 1962 |
| 4) Clough, Chopra. | P.A.S.C.E. Apr. 1966 |

As already mentioned by authors¹⁾, it is reasonable that clay is expressed as Maxwell-Kelvin body which is non-linear in nature and has frequency characteristics. So that vibration behaviors and distributions of stresses and displacements of the earth dam are tried to be analysed by the finite difference method using the non-linear constants obtained by the actual tests on the specimen. As the starting point of dynamical analysis, it is necessary to know the stress distribution in the dam body caused by dead weight and water load, Therefore statical analysis of the earth dam is also conducted. Furthermore the variation of pore water pressure in the dam body during earthquakes is analysed, because of its importance in the stability of the earth dam.

2. Expression of physical constants of clay.

The nature of clay is so diverse in kind that there is much difference in the property according to the kinds. But the dynamic model of cohesive soil does not change so much in its form and it is enough to change only the numerical values of constants. For this reason, the attempt was made to use the physical constants of clay as numerical values which authors have already shown in the previous paper.

In the following, the outline of the physical properties is shown. As materials of clay, Bentonite on sale (composition: SiO₂ 77.3%, Al₂O₃ 13.5%, Specific gravity: 2.58, Grain size: 99% less than 0.053 m/m) was chosen, because materials of uniform quality can be obtained easily. As a specimen, a cylinder of 7.5 cm in diameter, 15 cm in height, was made compacting the Bentonite with water content 27-29% in four layers. And the density was 1.76-1.80 gr/cm³. Axial compression σ_1 and confined pressure $\sigma_{2,3}$ were measured with pressure meters of S.R. gauge type, and vertical and tranversal displacement were measured with π gauges of S.R. gauge type.

(1) Statical physical constants. As for the relation between axial pressure and axial deformation, characteristics of soft spring appear according to the degree of confined pressure, and Fig. 1 shows statical secant modulus of elasticity E_{os} corresponding to the each stage of confined pressure. And the following expression was obtained.

$$E_{os} = \exp \left\{ 2.31 \frac{14 + 1.1\sigma_{2,3}^2 - \sigma_1}{4.8 + 0.45\sigma_{2,3}^{1.6}} \right\} : E_{os} < 490 - 27\sigma_1 \quad (1)$$

$$E_{os} = 490 - 27\sigma_1 : \exp \left\{ 2.31 \frac{14 + 1.1\sigma_{2,3}^2 - \sigma_1}{4.8 + 0.45\sigma_{2,3}^{1.6}} \right\} > 490 - 27\sigma_1$$

(Unit kg, cm)

The statical poisson's ratio ν_s is given in Fig. 2 and the influence of σ_1 is so much that ν_s is expressed as in the following.

$$\nu_s = 0.245 + 0.0164 \sigma_1 \quad (\text{Unit kg, cm}) \quad (2)$$

(2) Dynamical physical constants. Fig. 3 shows an example of the process of the relation between axial stress $\sigma_1 = \sigma_{2,3} + a + b \sin \omega t$ and axial strain ϵ_1 in case of having added confined pressure $\sigma_{2,3}$ and furthermore dynamical axial stress $a + b \sin \omega t$. The gradient of a straight line connecting the point $(\sigma_{2,3} + a - b, \epsilon_1)$ and the point $(\sigma_{2,3} + a + b, \epsilon_1)$ in each load cycle, that is the

dynamical secant modulus of elasticity E_0 , indicates frequency characteristics of cohesive soil which considerably increases in proportion to the frequency of load. An example of this property is shown in Fig. 4. And it is also shown in Fig. 3 that the permanent strain accumulates as load history runs. Judging from these characteristics, Maxwell-Kilvin body is adopted as dynamic model. From change of value of E_0 due to frequency $f = \omega/2\pi$ and permanent strain, each element E, η, E_1, η_1 of Maxwell-Kelvin body can be calculated in each stage of $\sigma_{2,3}, a, K = a/b$. As a result, it was ascertained that E, η, E_1 , and η_1 had the following characteristics.

(i) Keeping $\sigma_{2,3}$ and a constant, if K is decreased, that is, b is increased, all physical constants decrease. In this case, the ratio of the decrease of the physical constant to the decrease of K , becomes small when the value a becomes large.

(ii) If $\sigma_{2,3}$ is increased keeping a and K constant, the physical constants increase.

(iii) If a is increased keeping $\sigma_{2,3}$ and K constant, the physical constants decrease.

According to the above characteristics, the optimum value of coefficient C was obtained from all experimental data, representing each physical constant as $E, \eta, E_1, \eta_1 = C_1 - C_2 a + (C_3 - C_4 a) K + C_5 \sigma_{2,3}$. The result is as follows:

$$\begin{aligned} E &= 6.76 \times 10^2 - 1.11 \times 10^2 a + (3.84 \times 10^2 - 3.85 \times 10 a) K + 1.52 \times 10^2 \sigma_{2,3} \\ \eta &= 1.37 \times 10^5 - 2.5 \times 10^3 a + (5.17 \times 10^4 - 8.5 \times 10^3 a) K + 1.17 \times 10^4 \sigma_{2,3} \\ E_1 &= 2.44 \times 10^3 - 3.44 \times 10^2 a + (1.28 \times 10^3 - 1.3 \times 10^2 a) K + 6.6 \times 10^2 \sigma_{2,3} \\ \eta_1 &= 8.72 \times 10^4 - 1.31 \times 10 a + (4.76 \times 10^4 - 5.0 a) K + 6.0 \times 10^4 \sigma_{2,3} \end{aligned} \quad (3)$$

$0.5 \leq f \leq 12, 0.8 \leq a \leq 4.5, 0 \leq \sigma_{2,3} \leq 44, 1.1 \leq K \leq 7.5$ (Unit Kg, cm, sec)

With regard to dynamical poisson's ratio, definite relation between its value and frequency or amplitude of dynamic load b , was not seen. Therefore, as shown in Fig. 5 it is expressed by the following equation as the function of a mean level of dynamic load a .

$$\nu = 0.21 + 5.5 \times 10^{-2} a \quad 0.5 \leq f \leq 12, 0.8 \leq a \leq 4.5 \quad (4)$$

3. Analysis of vibration of the earth dam.

(Unit Kg, cm, sec)

(1) Fundamental equations and the outline of analysis. Regarding the earth dam as a system of multiple particles, establishing equation of motion expressed by displacement, an attempt was made to solve by finite difference method. Here follows two-dimensional solution, however, it can be expanded immediately to a three-dimensional problem without any theoretical difficulties. If equation of motion is established assuming that absolute displacements in directions of x and y are u and v , and considering that physical properties vary with the place, the following equation can be obtained.

$$\rho(x,y) \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} + (\lambda + \mu) \frac{\partial^2 v}{\partial x \partial y} + \left(\frac{\partial \lambda}{\partial x} + 2 \frac{\partial \mu}{\partial x} \right) \frac{\partial u}{\partial x} + \frac{\partial \mu}{\partial y} \frac{\partial u}{\partial y} + \frac{\partial \mu}{\partial y} \frac{\partial v}{\partial x} + \frac{\partial \lambda}{\partial x} \frac{\partial v}{\partial y} \quad (5)$$

$$\rho(x,y) \frac{\partial^2 v}{\partial t^2} = \mu \frac{\partial^2 v}{\partial x^2} + (\lambda + 2\mu) \frac{\partial^2 v}{\partial y^2} + (\lambda + \mu) \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial \mu}{\partial x} \frac{\partial v}{\partial x} + \left(\frac{\partial \lambda}{\partial y} + 2 \frac{\partial \mu}{\partial y} \right) \frac{\partial v}{\partial y} + \frac{\partial \lambda}{\partial y} \frac{\partial u}{\partial x} + \frac{\partial \mu}{\partial x} \frac{\partial v}{\partial y} \quad (6)$$

here

$$\mu = \frac{1}{2(1+\nu)} E_0, \quad \lambda = \frac{\nu}{(1+\nu)(1-2\nu)} E_0$$

$\rho(x,y)$: density of clay ν : dynamical poisson's ratio
 E_0 : dynamic modulus of elasticity

Boundary conditions for the solution of the above are treated as in the following.

(i) up-stream face: shear stress in the parallel direction and stress in the vertical direction to the up-stream face equal zero.

(ii) crest of dam: "

(iii) down-stream face: "

(iv) base of dam: $u = f(x,t)$ displacement wave of earthquakes. (7)

(Remarks)

(In case of dynamic analysis in condition of full reservoir, internal stresses caused by dead weight and statical water pressure are calculated and physical constants at each point of the dam body corresponding to these stresses must be adopted as the initial values of the physical constants. As for hydro-dynamic pressure, it is possible to add the influence on vibration considering virtual mass of water, however, in case of the earth dam, compared with mass of dam body it is so little that there is almost no influence, therefore in here it is neglected. Accordingly, in case of dynamical analysis, condition of (i) is used on the up-stream face even in the condition of full reservoir, and internal stress is examined adding the stress by dead weight and statical water pressure to dynamical stress obtained.)

In analysing (5) and (6) under the condition of (7), displacement and stress are calculated as an initial value problem by making the finite difference equation of time interval Δt and mesh intervals h, k . In accordance with the one obtained in the previous paragraph, the displacements of every point at the time $t + \Delta t$ are calculated using the values of λ and μ determined by the stress at the time t . Thus, approximate solution of non-linear vibration is obtained by repeating this calculation.

(2) Treatment as Maxwell-Kelvin body. The ratio E_0 of stress to strain is expressed in the following using the well-known relation in Maxwell-Kelvin body.

$$E_0 = E \frac{p^2 + pE_1/\eta_1}{p^2 + p(E_1/\eta_1 + E_2/\eta_2 + E_3/\eta_3) + EE_1/\eta_1} = E - E \left(\frac{\gamma}{p+\alpha} + \frac{\delta}{p+\beta} \right) \quad (8)$$

here $p = \frac{d}{dt}$

The following relation can be obtained from the above equation.

$$\alpha + \beta = E_1/\eta_1 + E_2/\eta_2 + E_3/\eta_3, \quad \alpha\beta = EE_1/\eta_1 = \beta\delta + \alpha\delta, \quad \gamma + \delta = E_1/\eta_1 + E_2/\eta_2 \quad (9)$$

Using operator E_0 for the displacement u and v , the following equation is established.

$$E_0 u = E[u - \gamma \bar{u} - \delta \bar{\bar{u}}], \quad E_0 v = E[v - \gamma \bar{v} - \delta \bar{\bar{v}}] \quad (10)$$

Internal points x, y , time t and displacements are expressed as follows.

$$x = ih, y = jk, t = nat \quad (11) \quad u_{ij}^n = u(x, y, t), \quad v_{ij}^n = v(x, y, t) \quad (12)$$

The following equation can be obtained from (8), (10), (11) and (12)

$$\begin{aligned} \bar{u}_{ij}^n &= \frac{1}{p+\alpha_{ij}} u_{ij}^n = \int_0^{nat} e^{-\alpha_{ij}(nat-\tau)} u(x_{ij}, \tau) d\tau = \int_{(n-1)\Delta t}^{nat} e^{-\alpha_{ij}(nat-\tau)} u(x_{ij}, \tau) d\tau + \int_0^{(n-1)\Delta t} e^{-\alpha_{ij}(nat-\tau)} u(x_{ij}, \tau) d\tau \\ &= u_{ij}^n \int_{(n-1)\Delta t}^{nat} e^{-\alpha_{ij}(nat-\tau)} d\tau + e^{-\alpha_{ij}nat} \int_0^{(n-1)\Delta t} e^{-\alpha_{ij}(\tau)} u(x_{ij}, \tau) d\tau = u_{ij}^n \frac{1}{\alpha_{ij}} (1 - e^{-\alpha_{ij}nat}) + e^{-\alpha_{ij}nat} \bar{u}_{ij}^{n-1} \end{aligned}$$

In the quite same way

$$\begin{aligned} \bar{u}_{ij}^n &= u_{ij}^n \frac{1}{\beta_{ij}} (1 - e^{-\beta_{ij}at}) + e^{-\beta_{ij}at} \bar{u}_{ij}^{n-1} \\ \bar{v}_{ij}^n &= v_{ij}^n \frac{1}{\alpha_{ij}} (1 - e^{-\alpha_{ij}at}) + e^{-\alpha_{ij}at} \bar{v}_{ij}^{n-1}, \quad \bar{v}_{ij}^n = v_{ij}^n \frac{1}{\beta_{ij}} (1 - e^{-\beta_{ij}at}) + e^{-\beta_{ij}at} \bar{v}_{ij}^{n-1} \end{aligned} \quad (13)$$

In this way, elastic constants, λ and μ in the equations of motion, (5) and (6), can be replaced by visco-elastic constants of Maxwell-Kelvin body and poisson's ratio. And, the treatment of non-linear is that after calculating dynamical principal stress σ_1^n and σ_2^n at the time of t , λ and μ which are determined with these stresses are adopted as the constants at the time of $t+\Delta t$.

In this case, the following assumptions are made

- (i) $\sigma_{1static} - \sigma_{2static} = a$
- (ii) $\sigma_{2static} = \sigma_{2,3}$ provided that $\sigma_{1static} > \sigma_{2static}$
- (iii) either σ_1^n or σ_2^n of which acting direction is near to the direction of $\sigma_{1static}$, equal b .

(3) Equation of motion in finite difference. In making finite difference equation, the following equation can be obtained using the relation among (10), (11), (12) and (13).

$$\begin{aligned} \rho_{ij} \frac{u_{ij}^{n+1} - 2u_{ij}^n + u_{ij}^{n-1}}{\Delta t^2} &= (\lambda_{ij}^0 + 2\mu_{ij}^0) \frac{u_{i+1,j}^n - 2u_{ij}^n + u_{i-1,j}^n}{h^2} + \mu_{ij}^0 \frac{u_{i,j+1}^n - 2u_{ij}^n + u_{i,j-1}^n}{h^2} + (\lambda_{ij}^0 + \mu_{ij}^0) \frac{v_{i,j+1}^n - v_{i,j-1}^n}{4h} - \frac{v_{i+1,j}^n - v_{i-1,j}^n}{4h} \\ &+ \left(\frac{\lambda_{i,j+1}^0 - \lambda_{i,j-1}^0}{2h} + 2 \frac{\mu_{i,j+1}^0 - \mu_{i,j-1}^0}{2h} \right) \frac{u_{i,j+1}^n - u_{i,j-1}^n}{2h} + \left(\frac{\mu_{i,j+1}^0 - \mu_{i,j-1}^0}{2h} \right) \frac{u_{i,j+1}^n - u_{i,j-1}^n}{2h} \\ &+ \left(\frac{\mu_{i,j+1}^0 - \mu_{i,j-1}^0}{2h} \right) \frac{v_{i,j+1}^n - v_{i,j-1}^n}{2h} + \left(\frac{\lambda_{i,j+1}^0 - \lambda_{i,j-1}^0}{2h} \right) \frac{v_{i,j+1}^n - v_{i,j-1}^n}{2h} \end{aligned} \quad (14)$$

$$\begin{aligned} \rho_{ij} \frac{v_{ij}^{n+1} - 2v_{ij}^n + v_{ij}^{n-1}}{\Delta t^2} &= \mu_{ij}^0 \frac{v_{i,j+1}^n - 2v_{ij}^n + v_{i,j-1}^n}{h^2} + (\lambda_{ij}^0 + 2\mu_{ij}^0) \frac{v_{i,j+1}^n - v_{i,j-1}^n}{2h} + (\lambda_{ij}^0 + \mu_{ij}^0) \frac{u_{i,j+1}^n - u_{i,j-1}^n}{4h} - \frac{u_{i+1,j}^n - u_{i-1,j}^n}{4h} \\ &+ \left(\frac{\lambda_{i,j+1}^0 - \lambda_{i,j-1}^0}{2h} \right) \frac{v_{i,j+1}^n - v_{i,j-1}^n}{2h} + \left(\frac{\lambda_{i,j+1}^0 - \lambda_{i,j-1}^0}{2h} + 2 \frac{\mu_{i,j+1}^0 - \mu_{i,j-1}^0}{2h} \right) \frac{v_{i,j+1}^n - v_{i,j-1}^n}{2h} \\ &+ \left(\frac{\mu_{i,j+1}^0 - \mu_{i,j-1}^0}{2h} \right) \frac{u_{i,j+1}^n - u_{i,j-1}^n}{2h} \end{aligned} \quad (15)$$

$$\text{here } \bar{u}_{ij}^n = u_{ij}^n - \delta_{ij} \bar{u}_{ij}^n - \delta_{ij} \bar{u}_{ij}^n, \quad \bar{v}_{ij}^n = v_{ij}^n - \delta_{ij} \bar{v}_{ij}^n - \delta_{ij} \bar{v}_{ij}^n \quad (16)$$

$$\lambda_{ij}^0 = \frac{D_{ij}}{(1+\nu_{ij})(1-2\nu_{ij})} E_{ij}, \quad \mu_{ij}^0 = \frac{1}{2(1+\nu_{ij})} E_{ij} \quad (17)$$

From the above equations, stresses are calculated as in the following.

$$\sigma_{x,ij}^n = \lambda_{ij}^n \left(\frac{U_{i,j}^n - U_{i,j-1}^n}{2h} + \frac{V_{i,j}^n - V_{i,j-1}^n}{2k} \right) + 2\mu_{ij}^n \left(\frac{U_{i,j}^n - U_{i,j-1}^n}{2h} \right), \quad \sigma_{y,ij}^n = \lambda_{ij}^n \left(\frac{U_{i,j}^n - U_{i,j-1}^n}{2h} + \frac{V_{i,j}^n - V_{i,j-1}^n}{2k} \right) + 2\mu_{ij}^n \left(\frac{V_{i,j}^n - V_{i,j-1}^n}{2k} \right)$$

$$\tau_{xy,ij}^n = \mu_{ij}^n \left(\frac{U_{i,j}^n - U_{i,j-1}^n}{2h} + \frac{V_{i,j}^n - V_{i,j-1}^n}{2k} \right) \quad (18)$$

$$\sigma_{1,ij}^n, \sigma_{2,ij}^n = \frac{\sigma_{x,ij}^n + \sigma_{y,ij}^n}{2} \pm \sqrt{\frac{(\sigma_{x,ij}^n - \sigma_{y,ij}^n)^2}{4} + \tau_{xy,ij}^n{}^2}, \quad \theta_{ij}^n = \frac{1}{2} \tan^{-1} \frac{2\tau_{xy,ij}^n}{\sigma_{x,ij}^n - \sigma_{y,ij}^n} \quad (19)$$

(4) Expression of boundary condition. The (i) of boundary condition (7), that is, up-stream face, is expressed as in the following.

$$\sigma_x \cos^2 \theta_1 + \sigma_y \sin^2 \theta_1 + 2\tau_{xy} \sin \theta_1 \cos \theta_1 = 0$$

$$\tau_{xy} (\cos^2 \theta_1 - \sin^2 \theta_1) + (\sigma_y - \sigma_x) \sin \theta_1 \cos \theta_1 = 0 \quad (20)$$

In this case θ_1 is the angle between vertical and up-stream face, and σ_x, σ_y and τ_{xy} are generally expressed as in the following.

$$\sigma_x = \lambda \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) + 2\mu \frac{\partial U}{\partial x}$$

$$\sigma_y = \lambda \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) + 2\mu \frac{\partial V}{\partial y} \quad (21)$$

$$\tau_{xy} = \mu \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right)$$

Regarding the points, i and j , on the up-stream face, the following finite difference equations are used for the purpose of calculating the equation (21).

$$\frac{\partial U}{\partial x} = \frac{(4U_{i,j} - U_{i,j-2} - 3U_{i,j-1})}{2h}, \quad \frac{\partial U}{\partial y} = \frac{(3U_{i,j} + U_{i,j-2} - 4U_{i,j-1})}{2k}$$

$$\frac{\partial V}{\partial x} = \frac{(4V_{i,j} - V_{i,j-2} - 3V_{i,j-1})}{2h}, \quad \frac{\partial V}{\partial y} = \frac{(3V_{i,j} + V_{i,j-2} - 4V_{i,j-1})}{2k} \quad (22)$$

In conclusion, by inserting (22) to (21) and solving simultaneously (21) and (20), $\sigma_x, \sigma_y, \tau_{xy}, U_{i,j}$ and $V_{i,j}$ are obtained. As to the downstream face, the treatment is just the same, and the dam crest, almost the same.

The boundary condition at the dam base is given in the following general expression.

$$u_{i,j}^n = f(ih, n\Delta t) \quad (23)$$

(5) Condition of stability of solution and actual calculation. To examine the condition of stability of solution, the simplification of the fundamental equations, (5) and (6) was made, namely differentials of first-order of displacement were neglected and, physical constants λ and μ were assumed as fixed values. And the following condition of stability was attained.

$$\Delta t \leq \frac{1}{\sqrt{\frac{\lambda+3\mu}{\rho}} \sqrt{\frac{1}{h^2} + \frac{1}{k^2}}} \quad (24)$$

Mesh points to be applied the equations of finite difference (14) and (15) were taken as all the internal mesh points except the points on the boundary line. As shown in the equations (14) and (15), displacement at eight points surrounding the central points of i, j are necessary for the equation of motion. Accordingly, for the mesh points i, j which are just inside of the up and down stream faces, the values at fictitious point $i-1, j+1$ outside of the dam body, is necessary. That is because the term of $\frac{\partial^2 u}{\partial x \partial y}$ for the displacement u is generally expressed by the following equation of finite difference.

$$\frac{\partial^2 u_{i,j}}{\partial x \partial y} = \frac{1}{4hk} (u_{i,j+1} + u_{i-1,j-1} - u_{i-1,j+1} - u_{i+1,j-1}) \quad (25)$$

Now, if $\frac{\partial^2 u}{\partial x \partial y}$ is expressed by the equation (26)

$$\frac{\partial^2 u_{i,j}}{\partial x \partial y} = \frac{1}{4hk} (u_{i,j+1} + u_{i-1,j-1} - 2u_{i,j-1} + u_{i+1,j} - u_{i,j} + u_{i,j+1} - u_{i,j-1}) \quad (26)$$

not using the values of the fictitious point $i-1, j+1$ outside of the dam body, an equation of motion is expressed only by internal points and points on the boundary. In consequence, to set up the equation of motion for the points on the just inside of the boundary, the equation (26) is used.

4. Distribution of stress and displacement by dead weight and statical water pressure in the earth dam.

In the starting of calculation of vibrations, physical constants of each point should be determined, and calculation ought to be begun as dynamical initial value by using physical constants corresponding to the stress in the statical condition. In this sense, the distribution of stress and displacement of the earth dam by dead weight and statical water

pressure is to be analysed.

If the left side of the equation (5) is zero, and the left side of the equation (6) is $\rho(x,y)g$, fundamental equations can be obtained for the statical calculation. As boundary conditions the following are established.

- (i) In case of empty reservoir
up-stream face, crest of dam, down-stream face: same as (i), (ii) and (iii) of (7)

base of dam: $u = v = 0$

- (ii) In case of full reservoir
up-stream face: stress in the vertical direction to the up-stream face equal $-W_0(H_0 - y)$. stress in the parallel direction equal zero.

Crest of the dam, down-stream face: same as (ii) and (iii) of (7)

Base of the dam: $u = v = 0$

In here W_0 : weight of unit volume of water H_0 : depth of reservoir
water y : height from the dam base

The attempt is made to solve by changing the fundamental equation into an equation of finite difference basing on Successive Over Relaxation Method. If m order and $m+1$ order approximation are expressed by (m) and $(m+1)$, an equation of equilibrium of force is as follows:

$$2 \left\{ \frac{\lambda_{ij}^{(m)} + 2\mu_{ij}^{(m)}}{R^2} + \frac{\mu_{ij}^{(m)}}{R^2} \right\} u_{ij}^{(m)} = \frac{\lambda_{ij}^{(m)} + 2\mu_{ij}^{(m)}}{R^2} (u_{i+1,j}^{(m)} + u_{i-1,j}^{(m)}) + \frac{\mu_{ij}^{(m)}}{R^2} (u_{i,j+1}^{(m)} + u_{i,j-1}^{(m)}) + \frac{\lambda_{ij}^{(m)} + \mu_{ij}^{(m)}}{4R^2} (v_{i+1,j+1}^{(m)} - v_{i+1,j-1}^{(m)} - v_{i-1,j+1}^{(m)} + v_{i-1,j-1}^{(m)}) + \frac{1}{4R^2} (\lambda_{ij}^{(m)} - \lambda_{ij}^{(m+1)}) (u_{i+1,j}^{(m)} - u_{i-1,j}^{(m+1)}) + \frac{1}{2R^2} (\mu_{ij}^{(m)} - \mu_{ij}^{(m+1)}) (u_{i,j+1}^{(m)} - u_{i,j-1}^{(m+1)}) + \frac{1}{4R^2} (\lambda_{ij}^{(m)} - \lambda_{ij}^{(m+1)}) (v_{i,j+1}^{(m)} - v_{i,j-1}^{(m+1)}) + \frac{1}{4R^2} (\mu_{ij}^{(m)} - \mu_{ij}^{(m+1)}) (v_{i+1,j}^{(m)} - v_{i-1,j}^{(m+1)}) \quad (27)$$

$$2 \left\{ \frac{\lambda_{ij}^{(m)} + 2\mu_{ij}^{(m)}}{R^2} + \frac{\mu_{ij}^{(m)}}{R^2} \right\} v_{ij}^{(m)} = \frac{\lambda_{ij}^{(m)} + 2\mu_{ij}^{(m)}}{R^2} (v_{i,j+1}^{(m)} + v_{i,j-1}^{(m)}) + \frac{\mu_{ij}^{(m)}}{R^2} (v_{i+1,j}^{(m)} + v_{i-1,j}^{(m)}) + \frac{\lambda_{ij}^{(m)} + \mu_{ij}^{(m)}}{4R^2} (u_{i+1,j+1}^{(m)} - u_{i+1,j-1}^{(m)} - u_{i-1,j+1}^{(m)} + u_{i-1,j-1}^{(m)}) + \frac{1}{4R^2} (\lambda_{ij}^{(m)} - \lambda_{ij}^{(m+1)}) (v_{i,j+1}^{(m)} - v_{i,j-1}^{(m+1)}) + \frac{1}{2R^2} (\mu_{ij}^{(m)} - \mu_{ij}^{(m+1)}) (v_{i,j+1}^{(m)} - v_{i,j-1}^{(m+1)}) + \frac{1}{4R^2} (\lambda_{ij}^{(m)} - \lambda_{ij}^{(m+1)}) (u_{i+1,j}^{(m)} - u_{i-1,j}^{(m+1)}) + \frac{1}{4R^2} (\mu_{ij}^{(m)} - \mu_{ij}^{(m+1)}) (u_{i+1,j}^{(m)} - u_{i-1,j}^{(m+1)}) - S_{ij} g \quad (28)$$

5. Example of numerical calculation.

An earth dam being 50 meters high as shown in Fig. 6 was taken as an example of calculation. Mesh points were selected as shown in the Figure. Among them 99 points were provided for internal points, and 44 points for boundary points. So as to satisfy the condition of stability of solution, the time interval was set up as $\Delta t = 0.005 \text{ sec}$, taking the mesh intervals

of μ three varieties 5 meters, 12.5 meters, 15 meters, and μ 5 meters.

(1) In case of empty reservoir

(i) Statical calculation. The distribution of displacement and stress in case of total weight of a dam body acting at a time, was calculated by applying (1) and (2) to (27) and (28). The calculation is repeated and continued to the extent that the total of the difference between λ and μ of $m+1$ order and m order at each mesh point comes up to 1×10^{-3} of the total sum of the values of λ and μ of m order. The distribution of displacement and stress obtained by this calculation is given in Figure 7.

(ii) Dynamical calculation. By applying the equations (3) and (4) to the equations (14) and (15), the distributions of displacement and stress were calculated at every time interval. In this case, the value calculated from (3) and (4) by using the stress at each mesh point obtained basing on statical calculation of (i), was used for physical properties of dam body in initial time.

Figure 8 denote the transition of displacement of a dam crest in case of whole dam base being uniformly affected by simple harmonic vibrations of displacement and the vibration mode on the vertical center line of dam body. As simple harmonic vibrations of displacement, seven kinds of periods from the period 0.2 sec. to the period 1.5 sec., were selected, and amplitude of displacement was fixed in order that half amplitude of maximum acceleration may come up to 100 cm/sec^2 .

The calculation was being continued to the state affected by foundation motion of several waves. Figure 9 shows a resonance curve of the dam crest of the one which is assumed to be the fundamental mode of vibration during this period of time. Resonance period is 1.1 second and the acceleration on the dam crest in the resonance was 7.5 times of the acceleration of the foundation.

The examples of the distribution of the displacement of the dam body at a certain time during vibration and the distribution of the stress of the dam body corresponded to this (statical stress and dynamical stress are included) are given in Figure 10.

And then, the example of calculated developments of displacements of the earth dam due to the natural earthquakes which were observed on the soft foundation is shown.

The maximum acceleration that was obtained by differentiating the displacement of this earthquakes in Fig. 11 is 70 cm/sec^2 . as shown in the same figure.

The processes of the displacement of vibrations at the dam crest of the dam body are given in Figs. 11.

(2) In case of full reservoir.

(i) Statical calculation. Assuming that dead weight of a dam body acts at the same time as water pressure load does, the distribution of stress and displacement are calculated. The result is given in Figure 12.

(ii) Dynamical calculation. Seismic calculation was made on the displacement by natural earthquakes shown in Fig. 11 as earthquake input to the dam base, and the processes of displacements of vibrations at the dam crest and the middle of the dam body are shown in Figs. 13. In this case, the value of physical constants which was obtained by applying statical stress in the condition of full reservoir to (3) and (4) was used as an initial value of physical constants of the dam body. Compared with

mass of the dam body (13,000 tons per 1 meter in thickness) virtual mass of hydro-dynamic pressures (300 tons) is so small that the effect of hydro-dynamic pressures was neglected.

Fig. 14 denotes distributions of displacement and stress at one time point in process of this vibration.

In case of tremor when the reservoir water is at full level, the increment of internal pore-pressure in the earth dam is an important problem from viewpoint of stability. Taking the above-mentioned calculation as an example of application, this problem has been discussed. If Δp , the increment of pore-pressure caused by internal dynamical stress is modified so that the equation given by Skempton⁵⁾ may be applied to this instance, the following equation can be obtained.

$$\Delta p^n = \frac{1+\nu^n}{3} (\sigma_1^n + \sigma_2^n) \quad (29)$$

σ_1^n, σ_2^n : dynamical principal stress at the time t.

ν^n : dynamical poisson's ratio "

The conditions of the increment of pore-pressures at two different times were obtained using σ_1^n and σ_2^n in seismic calculation in the condition of full reservoir. The results are given in Figure 15.

6. Discussion and conclusion.

The method of analysis of non-linear vibration of the earth dam was made clear. Principal knowledge obtained from examples of calculation is as follows.

Vibrations in the dam body are not always stational even if vibrations of dam foundation are stational. That is because physical properties change every moment according to stress and also various modes vibrate superposing intricately.

As for the vibration mode of the lowest order in the central vertical section in seven kinds of stational vibrations of the foundation, a resonance curve was drawn. According to this curve, it was estimated the fraction of critical damping is 0.066 when a resonance period is 1.1 sec. This damping of vibrations is due to the internal viscosity of the dam body.

If the same calculation is made giving consideration to the thickness and physical properties of the foundation layer, it is possible to estimate the damping of vibration including dissipation of vibration energy of the dam body into the foundation.

It was made clear by this example of calculation that among vibration modes in the central vertical section given in Figs. 8 and 9, the fundamental mode is different from the mode in shearing vibrations, but similar to the mode observed on the prototype dam.

Although the distribution of internal stress and displacement caused by vibrations is complicated, according to the example of this calculation, in combination with statical stresses and dynamical stresses, tensile stresses in the parallel direction to the up and down-stream-faces appears on the up and down-stream faces in the upper part of the dam body. It can be said that the result of this calculation explains the actual cracks observed in the prototype earth dams which were affected by severe earth-

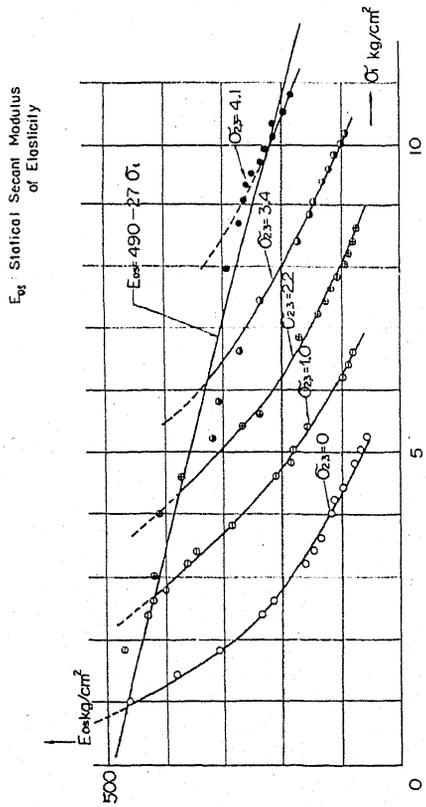
5) A. W. Skempton: The Pore-Pressure Coeff. And B. Geotechnique, 4 1954.

quakes.

Besides, during earthquakes in the condition of full reservoir, large increment of pore water pressure appears on the up-stream face as high as the central part of the dam body and its vicinity. This is one of factors which have a great influence on the stability of the earth dam.

From above mentioned methods of analysis, the concrete discussion on a-seismic problem of earth dams can be carried on.

Fig. 1 $E_{st} \sim \sigma_1$ curves for various levels of confined pressures (Static test)



(Static test)

Fig. 2 $\mu \sim \sigma_1$ relations for various levels of confined pressures σ_3

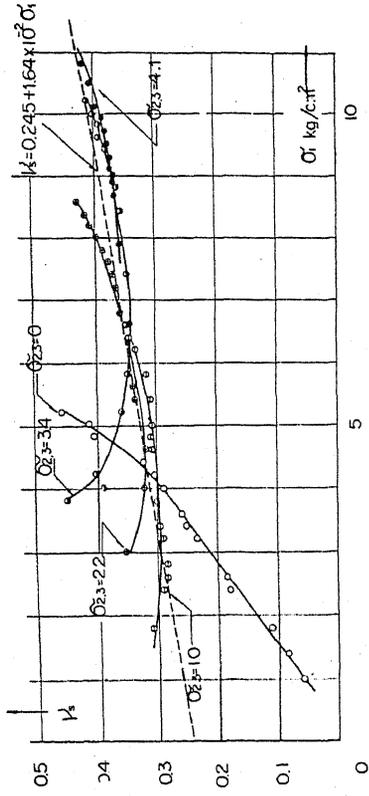


Fig. 3 Dynamic Load Intensity - Vertical Strain (Clay)

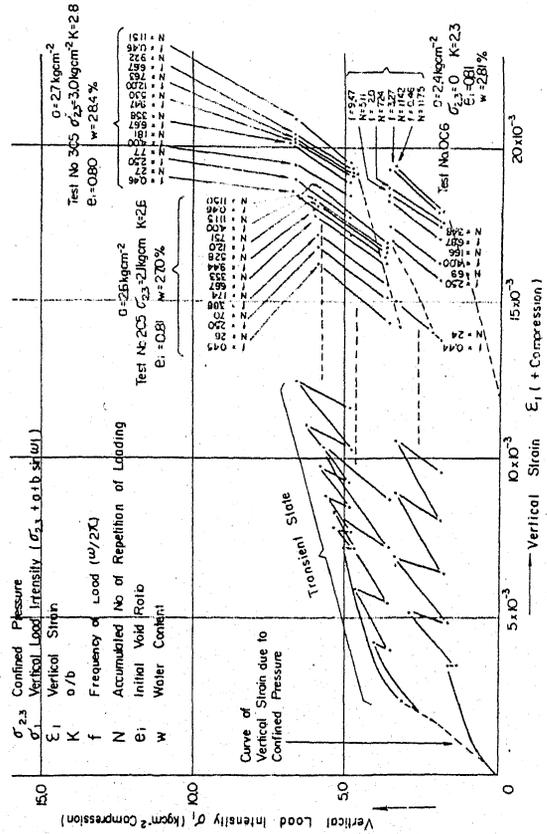


Fig. 4 Dynamic Modulus of Elasticity - Frequency of Load (Clay)

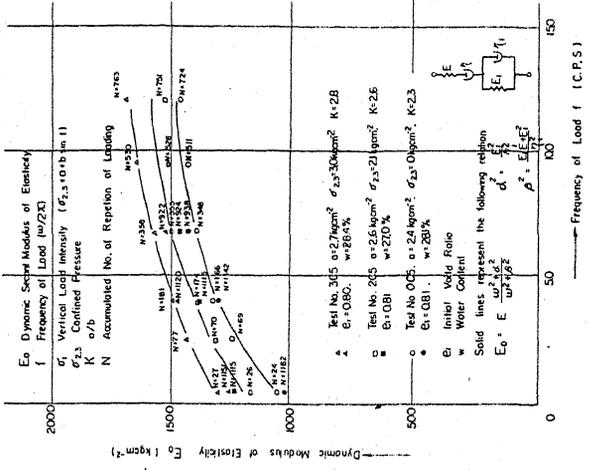


Fig. 5 Dynamic Poisson's Ratio — Mean Level of Dynamic Load σ (Clay)

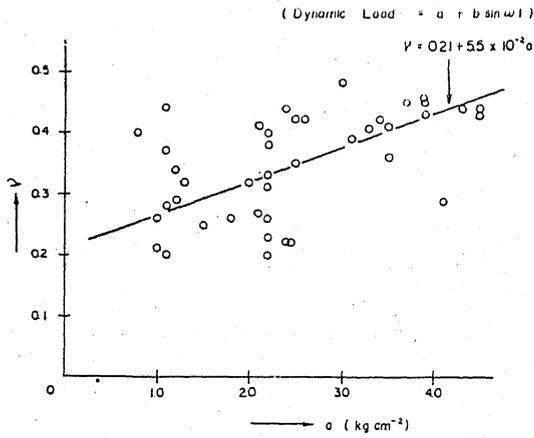


Fig. 6

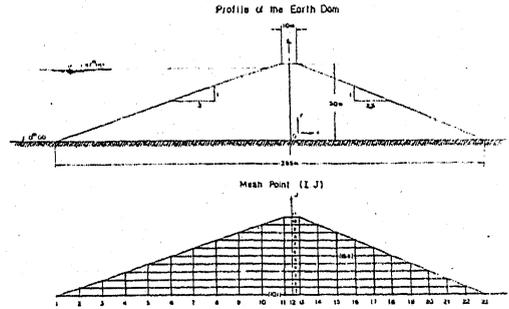


Fig. 7 Deformation and Principal Stresses of the Earth Dam Due to its Own Weight

(Reservoir empty)

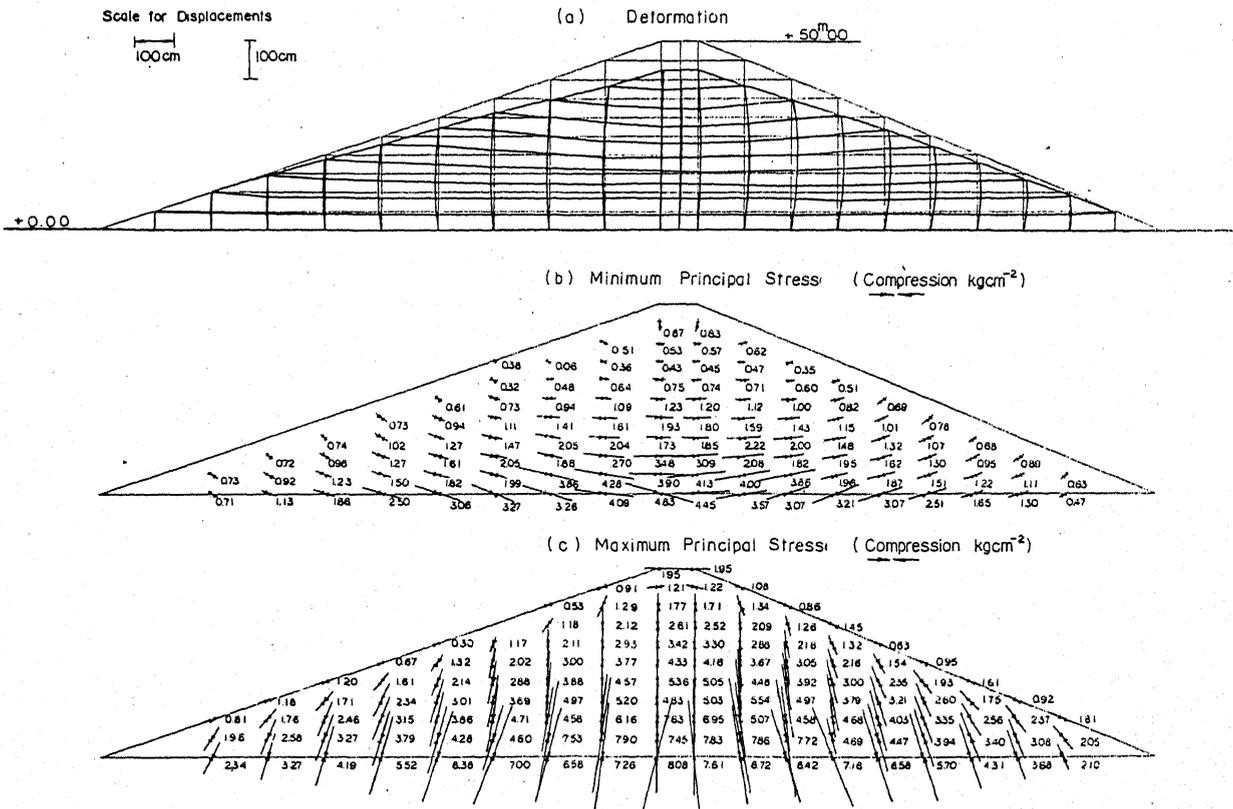


Fig. 8 Vibration of the Earth Dam Due to Sinusoidal Ground Motion (Reservoir empty)

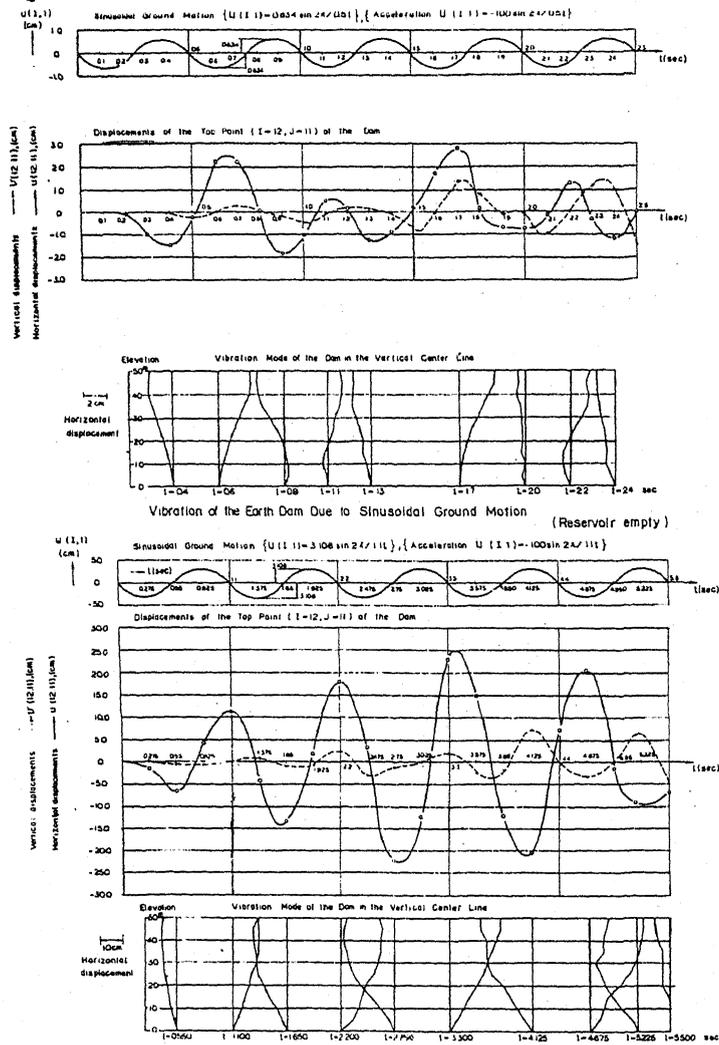


Fig. 9

Resonance Curve of the Top Point of the Earth Dam in the Fundamental Mode (Reservoir empty)

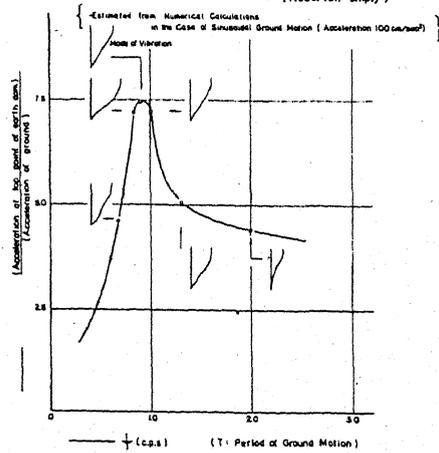


Fig. 10-1 Deformation and Principal Stresses due to Sinusoidal Ground Motion ($u(t,1) = 0.634 \sin \frac{2\pi}{0.5} t$) (Reservoir empty)

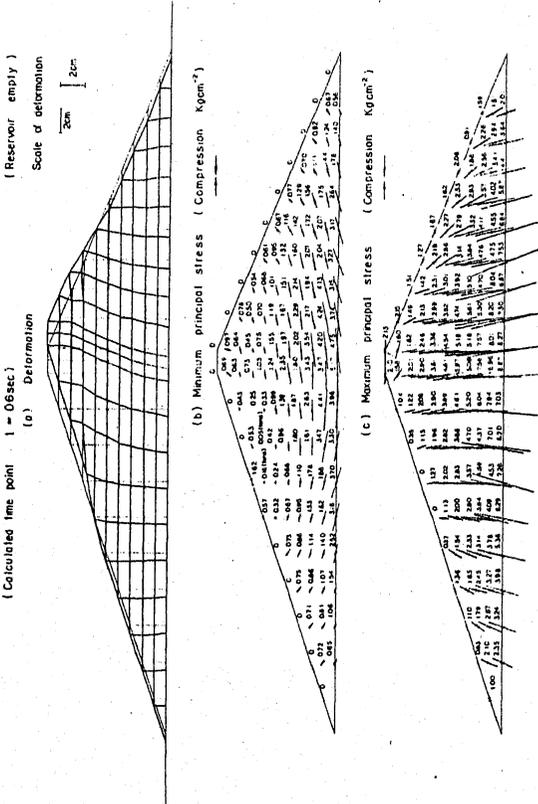


Fig. 10-2 Deformation and Principal Stresses due to Sinusoidal Ground Motion ($u(t,1) = 0.634 \sin \frac{2\pi}{0.5} t$) (Reservoir full)

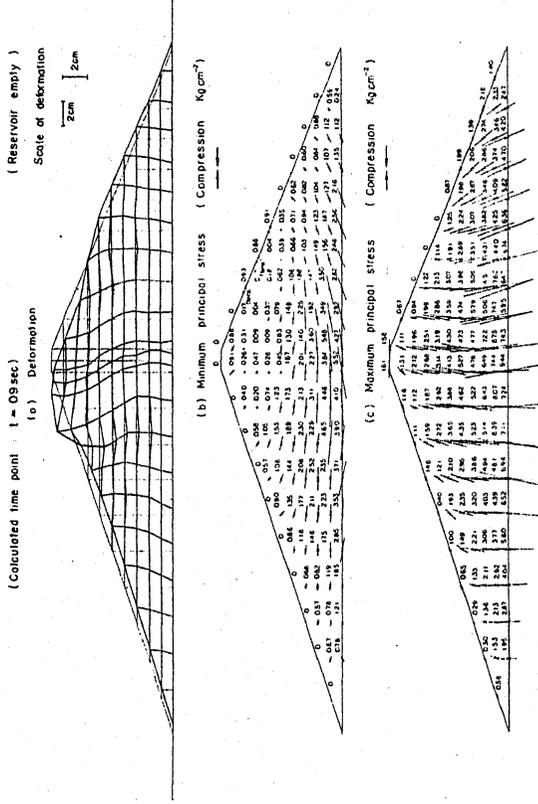


Fig. 11

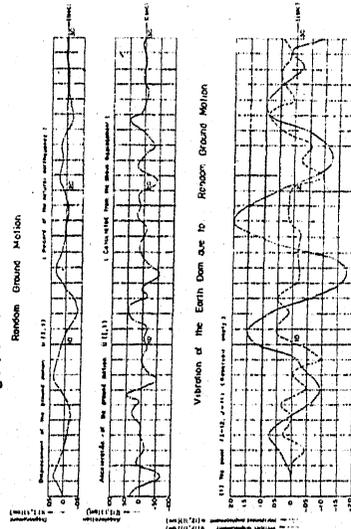


Fig. 12 Deformation and Principal Stresses of the Earth Dam Due to its Own Weight and Water Load. (Reservoir full)

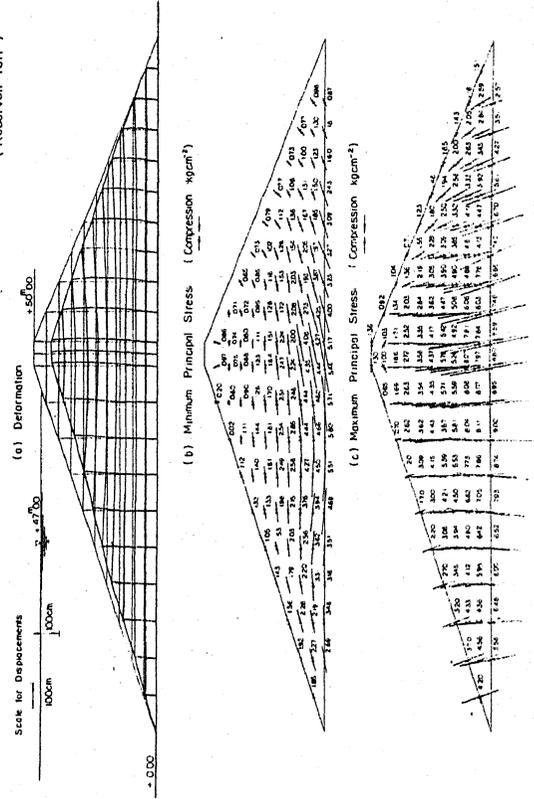


Fig.13

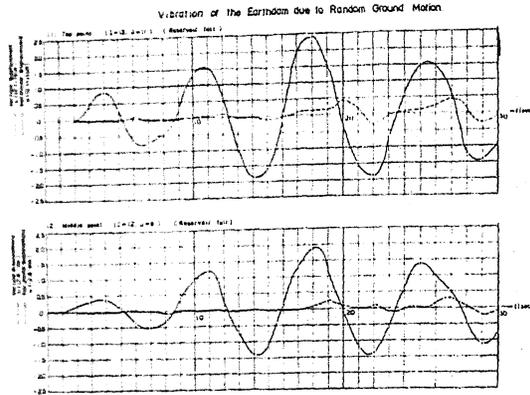


Fig.14 Deformation and Principal Stresses due to Random Ground Motion

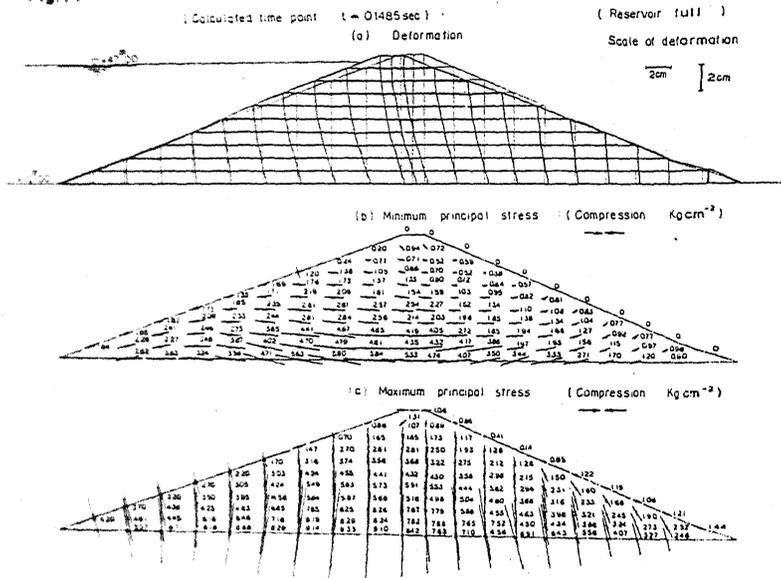


Fig.15 Increase of Pore Water Pressure due to Dynamic Stresses during Random Vibration (Reservoir full)

