STUDIES ON THE EARTHQUAKE-RESISTANT DESIGN OF SUSPENSION BRIDGE TOWER AND PIER SYSTEM

by

Yoshikazu YAMADA,[I] Yozo GOTO,[II] Hirokazu TAKEMIYA,[III]

SYNOPSIS

Following the previous studies,[1),2),3)] this paper reports the dynamic characteristics of the tower and pier system of suspension bridges obtained by complex mode analysis and discusses its earthquake-resistant design. Particular interests are given on the proper damping estimation in connection with the use of classical normal mode method and on the system synthesis based on the minimum cost.

INTRODUCTION

One of the authors has reported at the past three Conferences (WCEE) on earthquake response analyses and earthquake-resistant design of long span suspension bridges with particular emphasis on the tower and pier system.[1),2),3)] Since this system comprises the very flexible tower shaft and the rigid body pier, its dynamics are complicated coupled with the foundation stiffness.

Usually, for the response analysis of multi-degree-of-freedom systems the particular proportional damping matrix which enables the classical normal mode analysis is assumed. The previous paper[3)] investigated the tower and pier system dynamics by assuming equal damping effect for each normal mode and discussed on the modal correlation. But it is not proper to do so for this system since the tower is expected to have a small amount of damping while the pier is great.

This paper assumes a kind of non-proportional damping matrix by taking different damping effects between tower and pier. Such a damping matrix violates the normal mode approach, inducing the normal modes coupling. The following PART 1 concerns the dynamic analysis of this system by applying the state vector approach which results in a complex eigenvalue problem. To see the modal coupling effect clearly the response analysis is made in a stationary stochastic process due to a white noise excitation. Also the direct-integration of the system is carried out by using strong earthquake records as input motions.

[I] Professor of Civil Engineering, Kyoto University, Kyoto, Japan

[II] Research Associate Fellow of Civil Engineering, Kyoto University (Member of Engineering Research Laboratory, Ohbayashi-Gumi Ltd.)

[III] Assistant Research Fellow of Civil Engineering, Kyoto University

However, for the practical earthquake-resistant design of structures the simple method of response evaluation is desired such as the use of the response spectrum. For this purpose response comparison is made from different approximations.

In designing structures, the primary concern is to get the optimum solution which satisfies several imposed constraint conditions during its dynamic behavior when subjected to earthquake excitations. PART II attempts the optimization for the tower and pier system by choosing the minimum cost as an objective function. Since this process includes the iterative scheme of trial and error, an adequate substitutive approximate response analysis is used by referring PART I, instead of the time consuming complex mode analysis.

PART I ANALYSIS

SYSTEM DESCRIPTION

The system to be analysed in this paper is shown in Fig.1. The construction of the governing equation of undamped system was given in the previous paper.[2] Consider now the following special damping matrix to incorporate the different tower's and the pier's damping effect.

$$[c] = [c]_m^{1/2}([c]_m^{-1/2}[c]_s[c]_m^{-1/2})^{1/2}[c]_m^{1/2} \tag{1.1}$$

where

$$[c]_m = 2\beta_T\omega_1\,[m] + 2(\beta_P - \beta_T)\omega_1\begin{bmatrix}[m]_P & [0]\\ [0] & [0]\end{bmatrix}$$

$$[c]_s = \frac{2\beta_T}{\omega_1}[k] + \frac{2(\beta_P - \beta_T)}{\omega_1}\begin{bmatrix}[k]_P & [0]\\ [0] & [0]\end{bmatrix}$$

and ω_1 is the undamped fundamental natural frequency of the system, and β_T and β_P denote, respectively, the presumed damping factors of the tower and pier part. The matrices $[m]$ and $[k]$ are the mass and the stiffness ones and $[m]_P$ and $[k]_P$ represent, respectively, the matrices composed of the elements only related to the pier's motion from $[m]$ and $[k]$. Then the damped governing equation becomes

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = -[\tilde{m}]\{\ddot{z}_0\} \tag{1.2}$$

The application of the classical normal mode method yields

$$[I]\{\ddot{q}\} + [\tilde{c}]\{\dot{q}\} + [\omega_i^2]\{q\} = \{p\} \tag{1.3}$$

where $[I]$ is a identity matrix, $[\omega_i^2]$ is a diagonal matrix of square of undamped modal frequencies and $[\tilde{c}]$ is a damping matrix still having the off-diagonal elements which induce the coupling motion between classical normal modes. If $\beta_P = \beta_T$ were assumed in the above damping matrix, Eq.(1.1), the equal modal damping factors β_T are derived.

COMPLEX NORMAL MODE ANALYSIS

This method starts with reducing a set of differential equations of the 2nd order, Eq.(1.2) into those of the 1st order by introducing the new co-ordinates of state vectors[4)]

$$\{u(t)\} = \begin{Bmatrix} \{\dot{x}(t)\} \\ \{x(t)\} \end{Bmatrix} \qquad \{v(t)\} = \begin{Bmatrix} \{0\} \\ -[\tilde{m}]\{\ddot{z}_0\} \end{Bmatrix} \tag{1.4}$$

This leads Eq.(1.2) into the form of

$$[A]\{\dot{u}\} + [B]\{u\} = \{v\} \tag{1.5}$$

where

$$[A] = \begin{bmatrix} [0] & [m] \\ [m] & [c] \end{bmatrix} \qquad [B] = \begin{bmatrix} -[m] & [0] \\ [0] & [k] \end{bmatrix}$$

First the modal characteristics are investigated from the free damped vibration. To solve the associated eigenvalue problem the double QR method was efficiently used.

Modal Frequencies Fig.2 gives the modal frequencies comparison obtained from undamped and damped systems, where one can see a slight discrepancy between them at the modes proximity regions, i.e. at A and C region in the figure. The closely adjacent undamped modal frequencies are enforced to get closer (β_P= 0.10 case) or further to coincide each other (β_P= 0.20 case) with the increase of the pier's damping effect. In other situation (at the well separated modes region) the damped modal frequencies remain at their respective undamped modal frequencies.

Modal Damping Factors The quotients of the real part of the complex eigenvalues divided by the corresponding imaginary part are defined as approximate modal damping factors and are shown in Fig.3 and Table 1. Note that at the well separated modes region the damping amount imposed on the pier part appears at a specific mode, which is different by the different foundation region. For instance, at foundation B this is the 2nd mode. The damping factors of other modes remain almost at the tower's damping amount. At the modes proximity region, on the other hand, when β_P= 0.10, the concerned modes shear equally the pier's damping effect and the damping factors of other modes are near the tower's damping amount. The interchange of such modal damping factors is done gradually. However, when the pier's damping becomes so great as to yield the coincidence of the modal frequencies (β_p= 0.20 case), the above modes shearing of the pier's damping does not occur. Rather, it results in the abrupt change of that transferring. More specifically, in the foundation region up to the first modes proximity the 1st mode inherits the pier's damping; in the foundation region between the first and the second modes proximity, the 2nd mode does; and beyond the second modes proximity the 3rd mode does for the foundation region considered herein.

Mode Shapes The effects of the off-diagonal elements $\tilde{c}_{ij}(i \neq j)$ also appear at the vibration mode shapes. Fig.4 illustrates the complex modes as vectors on the phase plane. In this figure the rotation angle of the pier is scaled up for explanation. Note here that the phase differences by points, instead of all the points having the same angle or 180^0 out of phase as in the case of no damping or the proportional damping cases. This phenomenon is completely related to the pier's rocking motion and is strong as the value of β_p. At the first modes proximity region A the phase differences are seen at the 1st and the 2nd mode but not at the 3rd mode. At the well separated modes region B they appear only at the 2nd mode, and at the second modes proximity region C they do only at the 3rd mode. The above phase lag occurs along the height.

When the time variation of the damped complex modes are dipicted, the phase differences by points are recognized as the movement of nodes of the classical normal modes, as in Fig.5, instead of stationary ones for the undamped or the proportional damping cases.

Response Characteristics When the classical normal mode coupling is investigated by the response power spectral density, it may clearly be seen in the frequency domain. Fig.6 shows the representative system response power spectral densities due to white noise excitation. Note that the frequency contents appearance of the system in modes proximity foundation region is quite different between around the pier top and the above part. The former part still has the small peaks at the respective undamped modal frequencies such that they can form a spread-out envelope peak, while the latter part produces a new sharp peak which falls between the concerned undamped peak frequencies just like one mode peak. As the result, at the first modes proximity region the response power spectral density is significantly increased. This means a greater response in this region than in the well separated modes region. Furthermore, recalling the frequency contents of usual earthquake motions, the system in the second modes proximity region is also anticipated to increase the response considerably. For the well separated modes system, on the other hand, each modal peaks appear clearly. But pier's big damping subdues the 2nd modal contrubution on response significantly as is seen in Fig.6.

APPROXIMATE RESPONSE EVALUATION

In the response analysis the off-diagonal elements of the damping matrix $[\tilde{c}]$ in Eq.(1.3) may be neglected for some instances due to their smallness in magnitude. Table 1 gives the comparison of the damping factors such obtained with those from the complex eigenvalues. Note that for the β_p= 0.10 case the modal damping factors from both methods are almost same at any foundation region considered. But for the β_p=0.20 case the same thing holds only at the well separated modes region. At the modes proximity region the damping factors from the classical normal modes still shear the pier's damping amount almost equally between the concerned modes. This is different from the modal damping factors from complex eigenvalues aforementioned.

In Fig.7 are compared the rms responses due to white noise excitations computed exactly, by classical normal modes neglecting the modal coupling effect (approximate solution I), by the same method but taking the modal damping factors from the complex eigenvalues (approximate solution III) and by these R.M.S. responses. Note that at the first modes proximity region the approximate solution III is preferable for the upper half part of tower and its R.M.S. response (approximate solution IV) or the R.M.S. response of approximate solution I (approximate solution II) for the lower part and the pier. At the well separated modes region any approximate solutions considered herein are acceptable. At the second modes proximity region the approximate solution I is best to match the exact solution. However, in the use of the response spectrum for dynamic analysis there is no way to incorporate the modal correlation attributable to the phase difference between classical modes. In this case the approximate solution II may be adequately used not only at the well separated modes region but also at the modes proximity region in the overestimated sense.

PART II SYNTHESIS

Optimization technique is applied to the tower and pier system in consideration of the previous system dynamic characteristics. Herein, this process is defined as the nonlinear programming which choose a set of design variables {X} so as to minimize a specific objective function under some constraints.

FORMULATION

1. Assume a uniform stiffness distribution along the tower shaft and a rigid body pier for the system considered.
2. Select the design variables such that

$$\begin{aligned} X_1 &= B \quad \text{pier width along the bridge axis} \\ X_2 &= I \quad \text{moment of inertia of the tower} \end{aligned} \tag{2.1}$$

3. Assume the following relationships for stress computation

$$\begin{aligned} Z &= a\ I^{3/4} \quad \text{for section modulus} \\ A &= b\ I^{1/2} \quad \text{for cross sectional area of the tower} \end{aligned} \tag{2.2}$$

   where a and b are constants,and a=0.78 and b=0.80 were used in this study,
4. Consider the following constraints: (1) Tower shaft is safe against buckling. (2) Compressive stress at the tower base does not exceed the allowable stress. (3) Compressive stress of the tower shaft does not

exceeds its allowable stress due to bending. (4) Maximum displacement of the pier top remain within a reasonable range. (5) Compressive stress of the foundation does not exceed its allowable stress. (6) Pier is safe against overturning.

5. Choose the minimum cost as an objective function, i.e.

$$W = W_T + k\, W_P \tag{2.3}$$

under the constraints in 4. In this expression W_T represents the weight of the tower and W_P does that of the pier and k is the ratio of unit cost of the pier to that of the tower.

NUMERICAL EXAMPLE

For a numerical example, consider the tower and pier system of Fig.8. As an technique to solve the nonlinear programming problem the alternative step method was used. The response analysis was made by the R.M.S. method with use of the response spectrum of Fig.9.

The computation results are summerized as follows, referring Table 2.

1. The width of the pier B is determined only by the maximum displacement of the pier top when it is limitted as 10 cm. If this limit is increased up to 15 cm, then B is to be determined by the constraints on the tower.
2. The moment of inertia of tower, I is determined by the constraints of buckling or normal compressive collapse at the tower base when the elastic modulus of the foundation Eg is greater than 25×10^4 t/m^2. Otherwise, I is to be determined by the bending at the tower base.
3. The proper partial widening of the tower section is very effective to reduce the objective function.

CONCLUSION

The proper damping estimation of individual classical normal modes was indicated for the tower and pier system of suspension bridges. By using these quantities a way of its rational earthquake-resistant design was shown based on the response analyses and the minimum cost optimization.

BIBLIOGRAPHY

(1) Konishi,I.,and Yamada,Y.,"Earthquake Response of a Long Span Suspension Bridge", Proc. of 2nd WCEE, Vol.II, pp.863-878, 1960

(2) Konishi,I.,and Yamada,Y.,"Earthquake Response and Earthquake Resistant Design of Long Span Suspension Bridges", Proc. of 3rd WCEE, Vol.III, IV-312, 1965

(3) Konishi,I.,and Yamada,Y.,"Studies on the Earthquake Resistant Design of Suspension Bridge Tower and Pier Systems", Proc. of 4th WCEE, Vol.I B-4, pp.107-118, 1969

(4) Hurty,W.C.,and Rubinstein,M.F., Dynamics of Structures, Prentice-Hall, pp.313-337, 1964

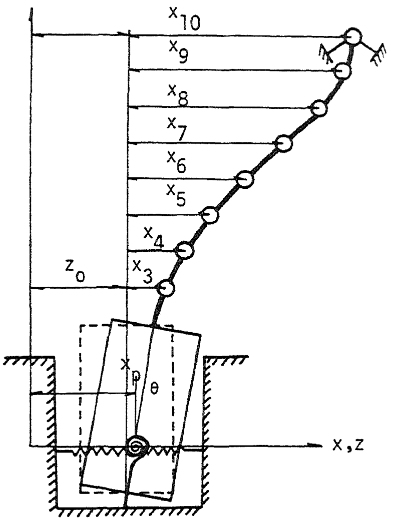


Fig.1 TOWER AND PIER SYSTEM MODEL

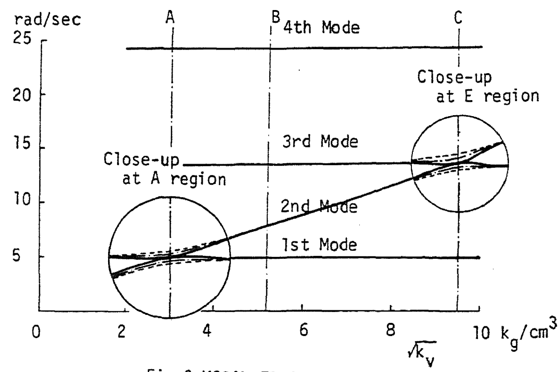


Fig.2 MODAL FREQUENCIES

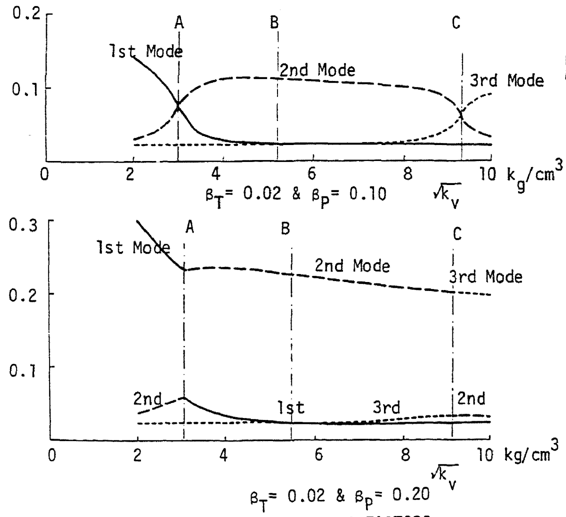


Fig. 3 MODAL DAMPING FACTORS

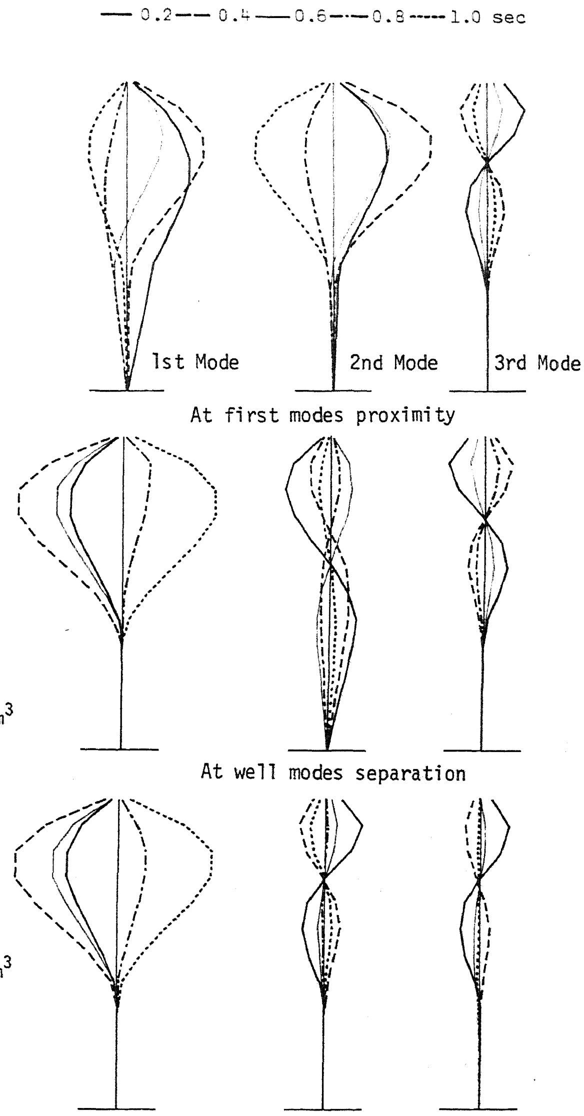


Fig.5 TIME VARIATION OF COMPLEX MODES
β_T= 0.02 & β_p= 0.20

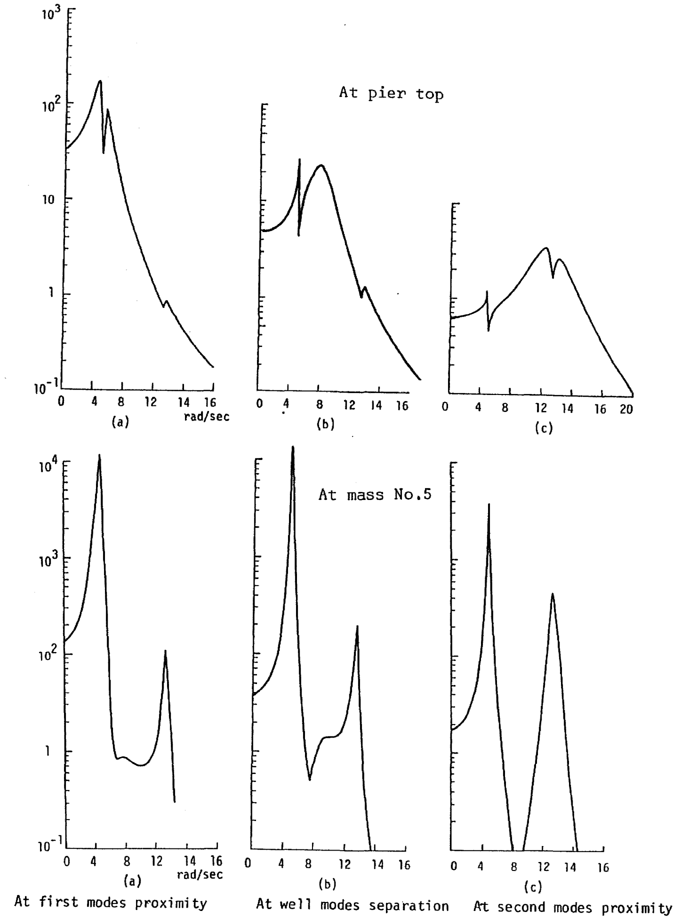


Fig.6 RESPONSE POWER SPECTRAL DENSITY
$\beta_T = 0.02$ & $\beta_P = 0{,}20$

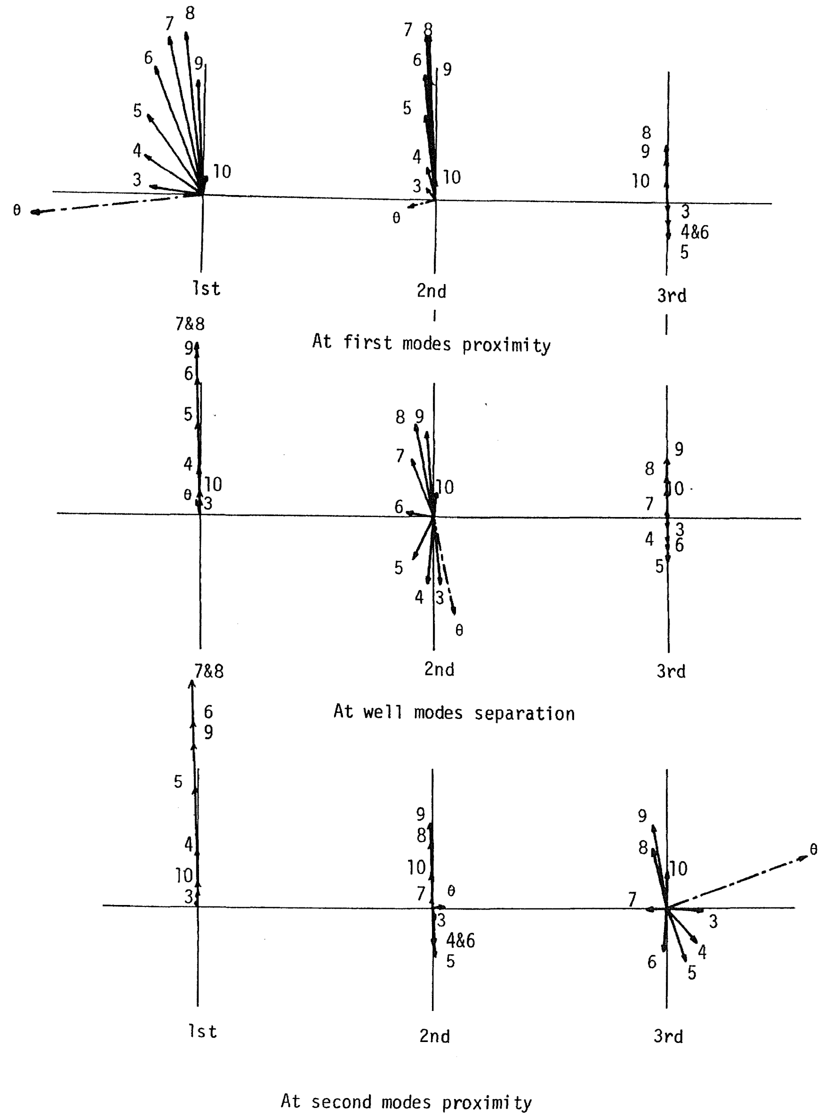
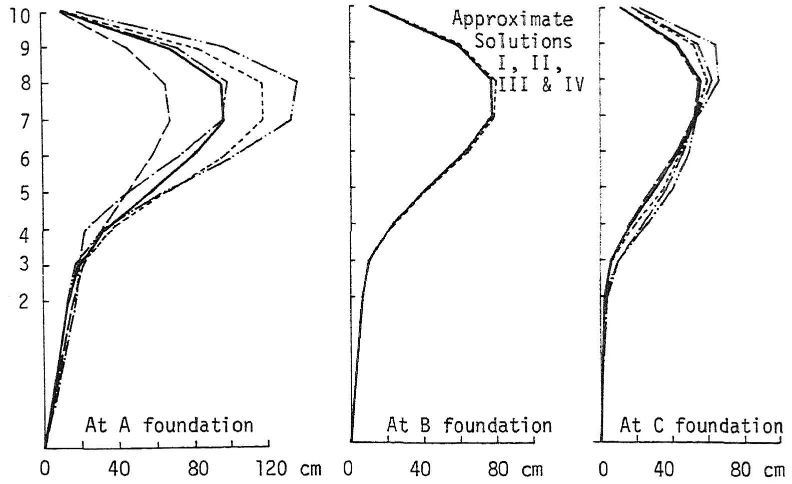


Fig.4 COMPLEX MODES ($\beta_T = 0.02$ & $\beta_P = 0.20$)



Exact Solution
Use complex modal analysis
Approximate Solutions
I : Use classical modal analysis neglecting modal coupling effect
II : Use R.M.S. evaluation in I
III: Use classical modal analysis taking damping factors from complex eigenvalues
IV : Use R.M.S. evaluation in III

Fig.7 rms RESPONSES
$\beta_T = 0.02$ & $\beta_P = 0.20$

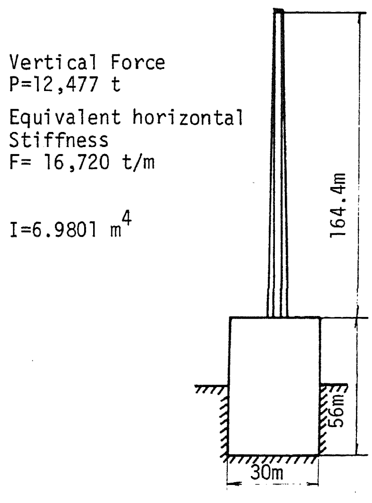


Fig.8 SYSTEM CONSIDERED

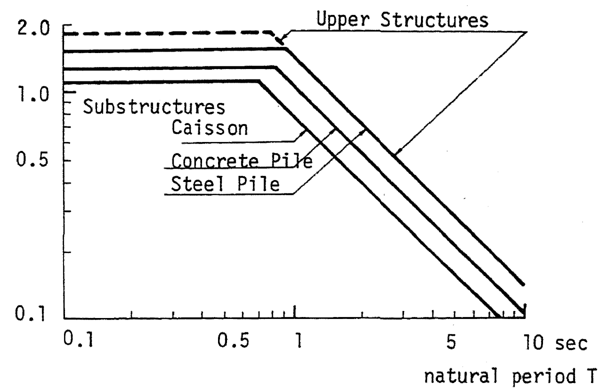


Fig.9 RESPONSE SPECTRUM

Table 1 MODAL DAMPING FACTORS

$\beta_T = 0.02$ & $\beta_P = 0.10$

	First proximity		Well separation		Second proximity	
Modes	$\tilde{c}_{ii}/2\omega_i$	μ_i/ν_i	$\tilde{c}_{ii}/2\omega_i$	μ_i/ν_i	$\tilde{c}_{ii}/2\omega_i$	μ_i/ν_i
1	0.0741	0.0743	0.0214	0.0214	0.0201	0.0201
2	0.0752	0.0757	0.1088	0.1095	0.0526	0.0452
3	0.0205	0.0205	0.0211	0.0211	0.0709	0.0787
4	0.0202	0.0202	0.0201	0.0201	0.0203	0.0203

$\beta_T = 0.02$ & $\beta_P = 0.20$

	First proximity		Well separation		Second proximity	
Modes	$\tilde{c}_{ii}/2\omega_i$	μ_i/ν_i	$\tilde{c}_{ii}/2\omega_i$	μ_i/ν_i	$\tilde{c}_{ii}/2\omega_i$	μ_i/ν_i
1	0.1409	0.2364	0.0232	0.0228	0.0204	0.0204
2	0.1439	0.0560	0.2195	0.2259	0.0934	0.0311
3	0.0211	0.0210	0.0225	0.0222	0.1343	0.2008
4	0.0205	0.0204	0.0204	0.0203	0.0208	0.0208

Table 2

(1)	(2) Mass No. 1 2 3 4 5 6 7 8	(3)	(4)	(5)	Eg x $10^4 t/m^2$	(6)	I (m^4)	B (m)	W
	x	x			20	10	1.829	21.84	4495
		x		x	25	10	1.558	18.55	3920
			x	x	30	10	1.558	17.17	3725
	x	x			20	10	1.508	22.75	4495
	x	x			25	10	1.599	18.47	3926
	x				20	15	1.662	22.04	4458
	x				25	15	1.634	18.02	3877
				x	30	15	1.556	17.25	3736
	x				20	15	1.636	22.11	4457
	x				25	15	1.650	17.98	3877
x			x		30	15	1.507	17.30	3721

(1): Compressive stress at tower base (2):Compressive stress due to bending
(3): Displacement of the pier top (4): Overturning (5): Buckling
(6): Limit of the pier top displacement