LIQUEFACTION ANALYSIS OF SATURATED GRANULAR SOILS

by

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SYNOPSIS

A method of analysis is presented which is suitable for the evaluation of the liquefaction potentiality of saturated soil deposits and earth-structures. The saturated soil is modelled as a fluid saturated porous granular solid. The system is discretized by finite element method and the matrix equations of motion of this coupted fluid-solid system is solved by step-by-step integration method. The results of the analysis yields the motions of the constituent materials and time histories of the pore pressures and the intergranular stresses. The onset of the liquefaction during the analysis is determined by a liquefaction criterion. Finally the analysis of the horizontal layers of soil subjected to three components of base motion is discussed.

INTRODUCTION

The phenomenon of the loss of strength of saturated granular soils during earthquakes is generally referred to as liquefaction. The process of liquefaction transforms an element of soil from a state of saturated granular solid to a state of viscous fluid. As a result of this change of material state the soil in a liquefied zone no longer has a supporting capability and therefore can under go excessive movements, eventually causing extensive damage to earth and earth-supported structures. Many cases of wide spread damage directly related to soil liquefaction have been reported in the literature [1], [2], [3].

A systematic approach to the development of capabilities for the evaluation of the liquefaction potentiality of the soil deposits and earth structures necessitates an understanding of the mechanism of liquefaction, development of analytical tools and experimental studies to determine the necessary material parameters. As to the mechanism of the development of local liquefaction, there appears to be a general agreement among the researchers [4], [5], [6] that the transfer of stresses from the granular solid skeleton of the soil to the pore water is the main cause of the diminishing of the intergranular contact and therefore the liquefaction of an element of soil. This process of development of liquefaction indicates that the pore water pressure determination under dynamic loading conditions is an important step in the analysis of the liquefaction potentiality of soil structures.

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A method for determination of the pore pressures and intergranular stresses has been proposed in author's previous works [7], [8], [9], in which the soil is treated as a saturated porous granular solid. This coupled two-phase modelling of the saturated soil offers a realistic means of accounting for the physical make-up of soils and leads to the determination of motions and stress histories of the constituent materials. The purpose of this paper is to present a method for the analysis of liquefaction of the saturated granular soil using a coupled fluid-solid mixture model. The formulation of the finite element spatial discretization and the relevent solution techniques are presented and various aspects of the material nonlinear models for soils are discussed with specific reference to the liquefaction problem. Finally, a special formulation is presented for the analysis of horizontal soil layer subjected to three components of the base motion.

The method of analyzing liquefaction potential which is proposed in this work can be considered a intergranular stress approach and it differs basically from the total stress approach advocated by seed and his co-workers, Reference [10].

PRELIMINARY CONCEPTS

In the two-phase representation of the saturated soils the granular solid skeleton and the fluid are treated as independent materials with individual material properties. The coupling between the volume changes of fluid and solid skeleton is taken into account through an additional material parameter. The flow of fluid with respect to the solid is assumed to be governed by a generalized form of the Darcy's flow law, for which the material parameter is the coefficient of permeability. The bulk modulus of the fluid, the coupling material parameter and the coefficient of permeability can be reasonably assumed to remain constant in the present dynamic analysis. The solid granular skeleton, in contrast, is a highly nonlinear material. A realistic constitutive relation for the solid skeleton of saturated granular soils must be capable of simulating the important nonlinear features such as dilatancy, shear failure and load reversal effects. Stress dilatancy, being a major contributing factor to the pore pressure built up is of special importance in liquefaction analysis.

The onset of liquefaction in an element of saturated soil is to be determined by a "liquefaction criterion" which can be conveniently defined in terms of intergranular stresses. Here, the liquefaction criterion is simply defined as vanishing of the mean intergranular pressure. The initiation of liquefaction in any analysis, as determined by satisfying the liquefaction criterion, marks the boundary, between two distinct states for an element of soil.

<u>Pre-liquefaction State</u> - At this state the soil is treated as a two-phase fluid saturated porous solid. The important characteristics of a potentially liquefying soil at this stage is the increase of the pore pressures accompanied by the decrease of the mean intergranular pressure.

Post-liquefaction State - After the initiation of liquefaction the nature of an element of soil changes from a fluid saturated granular medium to a viscous fluid.

The transformation from the pre-liquefaction state to a post-liquefaction state is to be considered an irreversible process--once an element of soil has liquefied, it is likely to remain liquefied for the remaining duration of the earthquake.

The analysis in the pre-liquefaction stage will lead to determination of the potentiality of liquefaction. If the extent of the development of the liquefaction and the associated stress and pore pressure distribution are of interest, then the analysis should be carried into the post-liquefaction stage. Doing so, requires accounting for the change of phase from fluid saturated granular solid to a viscous fluid in an element of soil which has satisfied the liquefaction criterion. This can be easily accomplished by the modification of the material properties of the granular solid consistent with such phase change.

THEORY

Finite Element Formulation

The independent state variables of coupled fluid-solid system are the state variable of the fluid and solid. The displacement vector for discretized system can be symbolically shown as

$$\underline{U}^T = < \underline{u}, \underline{w} > \qquad (1)$$

where $\underline{u}$ is the vector of the displacements of the solid and $\underline{w}$ is the vector of the displacements of the fluid with respect to solid. In a finite element discretization the displacements within each element are related to the nodal point displacements through the interpolation functions:

$$\underline{u}^n = \underline{\Phi}_u^n \underline{u}$$
$$\underline{w}^n = \underline{\Phi}_w^n \underline{w} \qquad (2)$$

where $\underline{\Phi}_u^n$ and $\underline{\Phi}_w^n$ are the finite element interpolation functions for the displacements of the solid and the displacements of the fluid with respect to solid. The superscript n denotes the nth element in the system.

The strain-displacement relations for the solid and the fluid constituents in tensor notation are:

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$
$$\zeta = w_{i,i} \tag{3}$$

where e_{ij} are the components of the solid strain and is the fluid volume change. Using the Equations (2) and Equation (3) the strain-displacement relations for each element can be determined.

$$\begin{Bmatrix} \underline{\epsilon}^n \\ \zeta^n \end{Bmatrix} = \begin{bmatrix} \Phi_e^n & 0 \\ o & \Phi_\zeta^n \end{bmatrix} \begin{Bmatrix} \underline{u} \\ \underline{w} \end{Bmatrix} \tag{4}$$

$\underline{\Phi}_e^n$ and $\underline{\Phi}_\zeta^n$ are obtained from the interpolation functions of Equation (2) by appropriate differentiation. Equation (4) can be symbolically written as

$$\underline{e}^n = \underline{B}^n \underline{U} \tag{5}$$

The stress-strain relation for the coupled system is as follows:

$$\begin{Bmatrix} \underline{\tau} \\ \pi \end{Bmatrix} = \begin{bmatrix} \underline{C}_{ss} & \underline{C}_{sf} \\ \underline{C}_{sf}^T & \underline{C}_{ff} \end{bmatrix} \begin{Bmatrix} \underline{\epsilon} \\ \zeta \end{Bmatrix} \tag{6}$$

τ_{ij} are the components of the total stress tensor and π is the pore pressure. The intergranular stresses are obtained from the following relation

$$\sigma_{ij} = \tau_{ij} - \delta_{ij}\,\pi \tag{7}$$

$\underline{C}_{ss}$ is in general a 6 x 6 matrix of material coefficients for the granular solid when no motion of the fluid with respect to solid is allowed (undrained condition). $\underline{C}_{sf}$ is a 6 x 1 matrix of coupling material coefficients between fluid and solid $\underline{C}_{ff}$ is the bulk modulus of the fluid. $\underline{C}_{ff}$ and $\underline{C}_{sf}$ are assumed to be constants but $\underline{C}_{ss}$ is a nonlinear function of state and will be discussed in the next section.

Equation (6) can be symbolically written as

$$\underline{\sigma}^n = \underline{C}^n \underline{e}^n \tag{8}$$

The stiffness matrix of the coupled system is given by the following equation

$$\underline{K} = \sum_{n=1}^{N} \int_{v^n} \underline{B}^{n^T} \underline{C}^n \underline{B}^n \, dv^n \tag{9}$$

The integration in the above equation is carried out over the volume of each element. N is the total number of the element.

The resistance to the flow of fluid with respect to the solid is represented by the dissipation matrix $\underline{H}$. Matrix $\underline{H}$ is given by the following equation

$$\underline{H} = \sum_{n=1}^{N} \int_{v^n} \underline{\Phi}_w^{n^T} (\underline{k}^n)^{-1} \underline{\Phi}_w^n \, dv^n \tag{10}$$

where $\underline{k}^n$ is the matrix of coefficients of permeability for the n-th element. The details of the derivation of the above matrices are given in previous works of author's [8], [9].

Dynamic Equations of Motion

The matrix equations of motion for a discrete system are

$$\begin{bmatrix} \underline{M}_s & \underline{O} \\ \underline{O} & \underline{M}_f \end{bmatrix} \begin{Bmatrix} \underline{\ddot{u}} \\ \underline{\ddot{w}} \end{Bmatrix} + \begin{bmatrix} \underline{D}_s & \underline{O} \\ \underline{O} & \underline{H} \end{bmatrix} \begin{Bmatrix} \underline{\dot{u}} \\ \underline{\dot{w}} \end{Bmatrix} + \begin{bmatrix} \underline{K}_{ss} & \underline{K}_{sf} \\ \underline{K}_{sf}^T & \underline{K}_{ff} \end{bmatrix} \begin{Bmatrix} \underline{u} \\ \underline{w} \end{Bmatrix} = \begin{Bmatrix} \underline{P}_s \\ \underline{P}_f \end{Bmatrix} \tag{11}$$

The first equation is the equation of motion of the bulk of the fluid solid system whereas the second equation is the equation of motion of the fluid with respect to solid. In the above equation of motion $\underline{M}_s$ and $\underline{M}_f$ are the lumped mass matrices of the bulk and the fluid respectively; $\underline{D}_s$ is a structural damping matrix representing the loss of energy by means other than the dissipation of the fluid; $\underline{H}$ is dissipation matrix, Equation (10); matrices $\underline{K}_{ss}$, $\underline{K}_{sf}$ and $\underline{K}_{ff}$ are the appropriate submatrices of the global stiffness matrix given by Equation (9). The load vectors $\underline{P}_s$ and $\underline{P}_f$ can either be a general time dependent dynamic forces or for the earthquake base motion they have the following form:

$$\begin{aligned} \underline{P}_s &= \underline{M}_s \underline{I} \, \ddot{u}_g \\ \underline{P}_f &= \underline{O} \end{aligned} \tag{12}$$

Equation (11) can be symbolically written as

$$\underline{M} \, \underline{\ddot{U}}_t + \underline{D} \, \underline{\dot{U}}_t + \underline{K}_t \underline{U}_t = \underline{P}_t \tag{13}$$

The subscript (t) is used to signify the value of the functions at time (t). It is important to note that the stiffness matrix of the system is a function of the displacement history therefore in a general form it is time dependent.

Time Integration of the Equations Motion

Due to the nonlinear nature of Equation (13) incremental solution techniques should be employed. However, the simple incremental solution techniques often lead to accumulation of the step-wise errors resulting in significant loss of accuracy and unstability. An incremental solution method with equilibrium check has been proposed by Wilson et al [11] which prevents the accumulation of the step-wise errors. This method is best suited for the purpose of the present study.

The matrix equation of motion is rewritten in the following form:

$$\underline{M}\,\ddot{\underline{U}}_{t+\tau} + \underline{D}\,\dot{\underline{U}}_{t+\tau} + \underline{K}_t\Delta\underline{U}_{t+\tau} = P_{t+\tau} - F_t \tag{14}$$

where $\Delta\underline{U}_{t+\tau} = \underline{U}_{t+\tau} - \underline{U}_t$ and $\underline{F}_t$ is the internal resisting force vector given by the following equation

$$\underline{F}_t = \sum_{n=1}^{N} \int_{v^n} \underline{B}^{n^T} \underline{\sigma}_t^n dv^n \tag{15}$$

In Equation (14) $\tau = \theta\Delta t$ where Δt is the time step and θ is a constant, $\theta \geq 1.4$. The accelerations are assumed to vary linearly within the time interval of τ. The following relations result from the linear acceleration assumption

$$\begin{aligned} \Delta\underline{U}_{t+\tau} &= \tau\dot{\underline{U}}_t + \frac{\tau^2}{3}\ddot{\underline{U}}_t + \frac{\tau^2}{6}\ddot{\underline{U}}_{t+\tau} \\ \dot{\underline{U}}_{t+\tau} &= \dot{\underline{U}}_t + \frac{\tau}{2}\ddot{\underline{U}}_t + \frac{\tau}{2}\ddot{\underline{U}}_{t+\tau} \end{aligned} \tag{16}$$

The substitution of $\ddot{\underline{U}}_{t+\tau}$ and $\dot{\underline{U}}_{t+\tau}$ from Equations (16) into Equation (14) results in the following matrix equation which can be solved for $\Delta\underline{U}_{t+\tau}$

$$\underline{K}_t^*\,\Delta\underline{U}_{t+\tau} = \underline{P}_{t+\tau}^* \tag{17}$$

where

$$\underline{K}^*_t = \underline{K}_t + \frac{6}{\tau^2}\underline{M} + \frac{3}{\tau}\underline{D}$$

$$\underline{P}^*_{t+\tau} = \underline{P}_{t+\tau} - \underline{F}_t + \frac{6}{\tau^2}\underline{M}\,\underline{a}_t + \frac{3}{\tau}\underline{D}\,\underline{b}_t$$

$$\underline{a}_t = \tau\dot{\underline{U}}_t + \frac{\tau^2}{3}\ddot{\underline{U}}_t$$

$$\underline{b}_t = \frac{2\tau}{3}\dot{\underline{U}}_t + \frac{\tau^2}{6}\ddot{U}_t$$

Finally the displacements, velocities and accelerations at time $t + \Delta t$ can be computed from the following relations

$$\underline{U}_{t+\Delta t} = \frac{1}{\theta^3}\,\Delta\underline{U}_{t+\tau} + \underline{U}_t + \frac{\tau}{\theta}\,(1-\frac{1}{\theta^2})\,\dot{\underline{U}}_t + \frac{\tau^2}{\theta^2}(1-\frac{1}{\theta})\,\ddot{\underline{U}}_t$$

$$\dot{\underline{U}}_{t+\Delta t} = \frac{3}{\theta^2}\,\Delta\underline{U}_{t+\tau} + (1-\frac{3}{\theta^2})\,\dot{\underline{U}}_t + \frac{\tau}{2\theta}(2-\frac{3}{\theta})\,\ddot{\underline{U}}_t \qquad (18)$$

$$\underline{U}_{t+\Delta t} = \frac{6}{\theta\tau^2}\,\Delta\underline{U}_{t+\tau} - \frac{6}{\theta\tau}\,\dot{\underline{U}}_t + (1-\frac{3}{\theta})\,\ddot{U}_t$$

MATERIAL MODEL FOR GRANULAR SOILS

The key to the success of the liquefaction analysis of the type proposed in this work lies in the appropriate mathematical modelling of the important features of the constitutive response of the granular solid skeleton of the soil. It has long been recognized that the loose sands are more susceptible to liquefaction under seismic loading conditions than the dense sands due to the fact that the loose sands tend to compact under the shear deformation whereas dense sands expand when subjected to shear deformation. This reduction of the volume in loose sands under the shear deformation contributes significantly to the pore pressure built-up and consequently to the reduction of the mean intergranular pressure, leading to liquefaction. In fact the earlier approaches to liquefaction analysis were based on the concept of "critical void ratio" [12]--the sands with such void ratio exhibit no dilation or compaction under shear deformation. The critical void ratio in effect establishes a boundary between the loose and dense sand with regards to the stress dilatancy under shear deformation. The appropriate representation of the stress dilatancy properties of granular soils requires special attention in liquefaction analysis, especially the analysis of liquefaction of horizontal layers of soil where the shear stresses are the dominant components of the earthquake induced stresses. Of numerous nonlinear material models for soils, the critical state model [13] and the plasticity model with strain hardening cap [14] appear to be good candidates for the type of analysis proposed here. However, additional analytical and experimental testing and correlation of such models are necessary before practical applications.

LIQUEFACTION ANALYSIS OF HORIZONTAL LAYERS OF SOIL

In seismic analysis of the horizontal layers of soil it is generally assumed that the seismic excitation propagates from the bedrock upward. The base excitation in general has three components. However, in most studies the vertical component of the base motion is neglected and the response of the system--modeled as a one dimensional shear beam--to a single horizontal component of the base is determined [10]. Such schemes may be justified in amplification studies of the horizontal layers of soil, specially when linearly elastic material properties are used. However, the application of the simple shear beam analysis to liquefaction potentiality studies is questionable. The lack of liquefaction in a horizontal layer of soil, determined from a one-dimensional shear beam analysis is not necessarily an indication that the same layer of soil will not liquify when subjected to all the three components of the base motion simultaneously.

A realistic analysis of the liquefaction potentiality of a horizontal soil layer must include all three components of the base motion for the following reasons:

1. The volume change of the granular solid skeleton, which is coupled with the fluid volumetric strain, has three contributing sources; vertical propagation of the pressure wave and the vertical propagation of the two components of the shear wave which produce the stress dilatancy.

2. Due to the nonlinear nature of the soils, the tangent shear moduli are dependent on the mean intergranular pressure which in turn is coupled with the pore pressures.

With simple shear beam models it is not possible to subject the layer of soil to all three components of the base motion simultaneously. Here a special finite element model is introduced for the purpose of dynamic analysis of the horizontal layer of soil subjected to three components of base acceleration.

A vertical column of soil is discretized as an assemblage of special one dimensional elements with four degrees of freedom at each: three translational displacements of solid u_x, u_y, u_z and vertical displacement of fluid with respect to solid w_z. The special form of the system imposes a restriction on the variation of the displacements, namely that the displacements are only a function of the vertical coordinate z.

$$u_x = u_x(z); u_y = u_y(z); u_z = u_z(z); w_z = w_z(z) \tag{19}$$

Substitution of the Equation (19) into Equation (3) yields the following form of the strain/displacement relations:

$$\epsilon_{xx} = 0; \epsilon_{yy} = 0; \epsilon_{zz} = \frac{\partial u_z}{\partial z}; \epsilon_{xy} = 0; \epsilon_{yz} = \frac{\partial u_y}{\partial z}; \epsilon_{xz} = \frac{\partial u_x}{\partial z}; \zeta = \frac{\partial w_z}{\partial z} \tag{20}$$

The finite element matrices for the system can be obtained by using Equations (4-10) and Equation (20) along with linear interpolation functions.

CONCLUSIONS

The nonlinear dynamic analysis technique for the saturated granular soils which was outlined in the previous section introduces the capability of evaluation of the liquefaction potentiality of saturated sandy soils by making use of the time histories of the intergranular stresses and the pore pressures. The convenience of defining the material properties for the granular soil and the criterion for the onset of liquefaction in terms of the intergranular stresses makes the proposed method more attractive.

The following steps are necessary in further development of the proposed method before the successful application to practical problems.

1. Parametric studies using the proposed method in order to determine the qualitative effect of various parameters such as depth, water table, relative density, earthquake intensity.

2. Analysis of actual cases of liquefaction and co-relation with field data.

3. Experimental studies of the material parameters and co-relation with test data.

Work on parametric studies is already underway.

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