THE SEISMIC RESPONSE OF ELEVATED WATER TANKS SUPPORTED ON CROSS BRACED TOWERS

by

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SYNOPSIS

A method of predicting the seismic response of cross braced elevated water tower structures is presented. The convective action of the liquid is allowed for by a two mass representation of the storage tank and its contents whereas the braced supporting framework is idealised as a series of one dimensional elements with mass concentrated at the joints. The use of an electronic digital computer to integrate the equations of motion of the mathematical models of the system, including allowance for preselected multiphase lateral stiffness characteristics of the supporting structure, is described. The method is applied to the calculation of the displacement responses, to digitised seismic ground accelerations, of a typical elevated water tower having ductile and non-ductile braces, both pretensioned and non-pretensioned.

INTRODUCTION

That liquid storage tower structures differ substantially, in their response to earthquakes, from the more normal form of multistorey building structure is implied in earthquake design codes(1). The specified seismic design coefficients for water towers are of the order of twice those listed for buildings but this reflects uncertainty rather than the results of analysis by a rational design procedure. The code provisions are somewhat arbitrary and an improved design approach requires an understanding of both the dynamic interaction of the elements comprising a water tower and the overall behaviour of the structural system under earthquake loading.

Some thirty years ago considerable effort was expended in investigating the seismic behaviour of water towers. Jacobsen(2) used a large shaking table for experiments on model towers and similar work at M.I.T.(3) emphasized the necessity for a dynamic design approach to be adopted.

Westergaard's work(4) on the changes of water pressure arising on dams due to earthquake motions was incorporated with the theory developed from model considerations by Hoskins and Jacobsen(5) who found satisfactory agreement with theoretical results when the effective water mass acting in an impulsively loaded rectangular tank was measured. Carder(6) tested steel water tanks by a series of pull back tests on the supporting towers and thereby studied the influence of foundation conditions, bracing properties and loading characteristics on the manner in which steel water towers vibrated, with the object of determining their likely behaviour in an earthquake. An appreciation of the effect of initial tension in cross bracing rods was shown in the work of McLean and Moore(7). They pointed out that in a laterally loaded three panel tower the upper tie rods buckled first, then

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the second panel rods and finally the diagonals in the lowest panel. A tower may pass through all stages in half a cycle of motion and as a result the period may be a composite of the four computed periods. Williams[8] described investigations of the elastic behaviour of a cross braced elevated water tank and showed that for a single mass structure in near resonant conditions critical displacements may be built up before the type of ground motion changes.

Extensive model studies were undertaken by Ruge[9] with the object of deriving data useful in the designing of seismically loaded water tanks. The concept of sacrificial cross bracing was discussed and he proposed the incorporation of specially designed spring elements in the structure; however only qualitative reference to damping and inelastic action was made. Ruge drew attention to the advantages gained by progressive rather than simultaneous failure of the structural elements. Murphy[10] carried out tests on an 84 ft cross braced steel water tower at the Woburn Railway workshops as a result of which the natural period of vibration of the structure was determined. He suggested that the damage caused to the tower in the 1942 Wellington earthquake was consistent with that predictable using a Californian type response curve. Ulrich and Carder[11] discussed the increase in stiffness and decrease in damping observed in tests of a cross braced water tower when loose tie rods had been tightened. They submitted that their observations supported the idea that if a structure becomes damaged by a strong earthquake the period will probably increase because of the loss of rigidity and as the elastic resilience is partly destroyed the internal damping will be somewhat higher and this serve to protect the structure, partly offsetting its damaged condition. The behaviour of typical elevated water tanks in the 1952 Californian earthquake was described by Steinbrugge and Moran[12]. They reported stretching of anchor bolts and bracing rods and the complete collapse of some tanks. They deduced the modes of failure and inferred that torsional motion had occurred in some cases. Steinbrugge and Moran observed that wind braced tanks had not behaved satisfactorily whereas those designed for seismic loading had behaved better. They concluded that the currently effective design coefficients were not too large and that more study of the seismic response of cross braced elevated water towers was needed. After analysing the behaviour of water tank structures in the 1952 Californian earthquake Moran and Cheney[13] reported satisfactory correlation between the behaviour predicted using the response spectrum approach and the actual damage sustained.

Steinbrugge and Flores[14] described the stretching of cross bracing in elevated water tank towers resulting from the 1960 Chilean earthquake and Clough and Jenschke[15] carried out computer analyses of cross braced frames based on the steel frame buildings at the University of Concepcion in which diagonal bracing was incorporated. After this was broken the unbraced structure resisted the earthquakes occurring on the following day with no further structural damage thus giving rise to the question of whether the bracing was either necessary or beneficial. A digital computer study was made of the strengthening effect of the diagonal bracing when the buildings were subjected to ground motions of three different earthquakes. Results indicated that for some earthquakes the bracing may be beneficial whereas in other cases the bracing induced forces in the frame which exceed the strength of the braced structure. Thus the bracing may actually result in the structure being less earthquake resistant than if it was unbraced. Housner[16] has proposed a method of limit design based on the energy input by recorded strong ground motions. The velocity response spectra concept was used to define the energy fed into a structure and part of this energy was assumed to be dissipated by yielding of the structure. Housner showed that a cross braced tower designed to resist, at the limit of its elastic range, an equivalent static load of 0.125 g will be safe in a disturbance which would produce 0.40 g in an elastic structure, providing that the bracing rods may be permitted to stretch.

A method of determining the hydrodynamic pressures developed in a fluid container subjected to horizontal accelerations has also been proposed by Housner[17]. The simplifications of earlier work including that of Jacobsen[18] and Graham and Rodriguez[19] involves some approximation but the technique is considered sufficiently accurate for seismic design use[20]. Both Cloud[21] and Blume[22] have reported reasonable correlation between the observed behaviour of water tanks and the vibrational characteristics predicted using Housner's method. In addition satisfactory agreement between the behaviour of petroleum storage tanks in the 1964 Alaskan earthquake and the trends expected from theoretical considerations based on Housner's work has been demonstrated[23].

Chandrasekaran and Krishna[24] have reported on experimental and analytical investigations of the seismic behaviour of reinforced concrete water towers. They recommended incorporation of diagonal steel braces in concrete frame towers to improve the strong motion response and suggested that a reinforced concrete elevated water tank may be analysed satisfactorily as a single degree of freedom system. This simplification was also used by Ramiah and Gupta[25] whereas Sonobe and Nishikawa[26], Infrim and Bratu[27], Garcia[28] and Shepherd[29] have advocated a two degree of freedom representation.

The computer simulation techniques which have been applied widely to building seismic response analysis in the last decade have received scant attention by those concerned with the earthquake behaviour of elevated liquid storage tanks. However Hanson and Fan[30] have used a digital computer to examine the effect of minimum cross bracing on the inelastic response of multistorey buildings and have shown that significant reductions in the ductility and energy absorption capacity requirements of the main frame members may be achieved by the provision of suitable diagonal members.

The principle of supplying load resistance by direct tension or compression members rather than by elements acting primarily in bending is not in itself new[31]. The problems which may arise due to incorrect design of a braced framework are well recognised[32,33]. Also the concept that a braced frame may ride out a strong motion earthquake by progressive weakening of the structure has been suggested previously[34]. Nevertheless an examination of the seismic behaviour of a range of cross braced elevated water tower structures using an electronic digital computer simulation technique does not appear to have been undertaken. The feasibility of predicting the response of elevated water tower structures which incorporate expendable cross bracing components appeared worthy of study and this consideration prompted the investigations described in this paper.

BASIS OF RESPONSE DETERMINATION

The direct numerical integration of the equations of motion of a multi-degree of freedom system has been applied by several investigators[35,36,37] to seismic response determination, particularly to inelastic response calculations where the system is assumed to behave in a linear elastic manner during each time increment used in the stepwise numerical integration process and the non-linear behaviour is determined as the response of a sequence of successively differing systems.

For the purpose of determining the effect of earthquake loading on an elevated water tank supported on a braced framework the tank can be modelled as a two mass system[29] (the mass m_1 being the convective mass, m_0 the impulsive mass and m_S the mass of the tank structure) carried on an idealised frame of members whose own mass is lumped at the beam positions (M_1, M_2, M_3 in figure 1). The system shown, in which the elevation of the tower has three braced panels, then becomes one of four discrete masses, M_1, M_2, $M_3 + m_S + m_0$ and m_1 connected by links which are essentially elastic but which, in the case of the diagonal members, have stiffnesses which vary with the lateral displacement as the bracing members slacken, yield or fracture.

A special purpose computer program has been developed[38] to enable equations of the form

$$[M].\{\Delta\ddot{x}\} + [C].\{\Delta\dot{x}\} + [K]_{LAT}.\{\Delta x\} = -[M].\Delta\ddot{x}_g \quad \ldots\ldots (1)$$

in which $[M]$ is the mass matrix
$\{\Delta\ddot{x}\}$ is the vector of incremental accelerations
$[C]$ is the damping matrix
$\{\Delta\dot{x}\}$ is the vector of incremental velocities
$[K]_{LAT}$ is the lateral stiffness matrix of the system
$\{\Delta x\}$ is the vector of incremental displacements
and $\Delta\ddot{x}_g$ is the incremental ground acceleration

to be set up and solved by numerical integration techniques, using digitised ground accelerations as the excitation, to determine the displacement response of an elevated water tower in a selected direction of translation.

As is shown in summary form in the flow chart presented in figure 2, the program requires details of the mass and stiffness of the unbraced tower to be provided as input, together with damping coefficients which effectively determine the proportion of equivalent viscous damping applied to the induced vibrations. The initial stiffness characteristics of the cross bracing are determined in a loop in which the bracing details are read, their load-deflection properties evaluated and the positions of the frame corresponding to any prestressed member just becoming slack calculated. The total stiffness matrix of the braced frame is formed from the initially read bare frame stiffness by addition of the contributions of the cross bracing members. A subroutine is used to produce a set of exciting accelerations at selected times by suitable modification of a digitised accelerogram which forms part of the input to the program. The stepwise numerical integration of equation 1 is undertaken, using a linear variation in acceleration technique based on the work of Newmark[39] to determine the successive displacements of the system. Allowance is made for diversion past unnecessary steps in the solution process where these are avoidable, specifically by checking whether the stiffness has changed since the previous cycle and acting accordingly. Provision is made for output in the form both of lists of the displacement response of each mass at successive times and of plots of displacement-time histories. The validity of using the mathematical model incorporated in this program to represent the essentially multi-linear lateral load-deflection characteristics of practical cross braced frames was established by a series of laboratory tests(40) which confirmed that, within the range of sway likely to be encountered in practice, the representation was satisfactory(38).

APPLICATION OF PROPOSED ANALYSIS METHOD

To illustrate the analysis procedure, Moran and Cheney's 100,000 gallon water tank(13) supported on a three storey, single bay, cross braced frame, 105 ft high, has been used as the basis for the series of seismic response predictions with the sole modification that the original tank, which had a domed top and bottom, is replaced by a cylindrical one. Differences in the response were determined between the situation in which entirely elastic response is assumed, and those in which multiphase response of the bracing elements is taken into account. Also the effects were examined of varying the levels of prestress of the bracing system.

In sequence, the lateral stiffness matrix for the unbraced frame, the post yield axial stiffness characteristics(40) of the bracing rods, and the normal mode frequencies of the unbraced frame supporting an empty tank were calculated in order to provide input data for the main response analysis program. Since it is generally accepted that elevated water tank structures exhibit relatively low equivalent viscous damping characteristics, 2% critical was selected for each of the first two modes of vibration of the structural system and the appropriate damping coefficients(41) were determined. The digitised North/South accelerations of the 1940 El Centro earthquake were also provided as input to the main program and a series of displacement-time responses were obtained for the particular cases of tank capacity and bracing properties described below.

The response of the Moran and Cheney frame with no prestress in the bracing, but assuming unlimited elastic capacity in all members, supporting a full tank of water free to act convectively, is shown in figure 3. For the purpose of comparison, the analysis was repeated with a full tank but with no allowance for the convective action of the water, i.e. treating it as a solid mass. The maximum displacement values are:-

	Water acting convectively	No convective action of water
Mass 1	3.78 in. at 17.91 sec.	2.05 in. at 14.57 sec.
Mass 2	6.46 in. at 17.91 sec.	3.50 in. at 14.57 sec.
Mass 3	8.56 in. at 17.91 sec.	4.64 in. at 14.58 sec.

In figure 4 the 2% damped convective response of the system in which the full water tank is supported on the Moran and Cheney braced tower with no prestress in the bracing, but with the post yield axial stiffness set at the appropriate predetermined value, is presented. The analysis was repeated with the rods prestressed to one quarter of their yield level but with an otherwise unaltered situation. The maximum displacements and bottom bay total yield distortions are:

	No prestress in bracing	Prestress in bracing
Mass 1 :	6.60 in. at 14.52 sec.	5.04 in. at 4.42 sec.
Mass 2 :	8.23 in. at 14.52 sec.	6.55 in. at 4.41 sec.
Mass 3 :	9.43 in. at 14.53 sec.	7.73 in. at 4.40 sec.
Bracing yield :	4.0 and 1.4 in.	3.1 and 2.7 in.

The 2% damped convective response of a full tank system in which the structural bracing was selected to exhibit multiphase characteristics, including fracturing of the high tensile steel members, is shown in figure 5. The bracing was similar to that of the Moran and Cheney frame but prestress forces equal to one quarter of the yield loads were incorporated. Moreover, the bottom bay bracing was considered to be composed of the equivalent of 2 in^2 high tensile steel and 2.82 in^2 bracing. As the high tensile steel fractured at about three seconds after the commencement of the accelerogram, the effect was to produce a flexible lower bay for the remainder of the response. When half the Moran and Cheney mild steel bracing was prestressed to one quarter of the yield level and combined, in parallel, with two 0.5 in. diameter high tensile steel rods in each bay, prestressed to one quarter of their ultimate strength, the response shown in figure 6 was obtained. Again two of the high tensile steel rods fractured during the response. The maximum displacements and bottom bay bracing yield distortions are

	Figure 5	Figure 6
Mass 1 :	7.44 in. at 22.89 sec.	3.65 in. at 8.66 sec.
Mass 2 :	7.75 in. at 22.88 sec.	5.02 in. at 6.34 sec.
Mass 3 :	7.95 in. at 20.29 sec.	5.10 in. at 6.34 sec.
Bracing yield :	4.48 and 5.50 in.	1.41 and 1.71 in.

COMMENT ON ANALYSES

The comparison between the extended elastic displacements determined in the first case, including the convective action of the water, and in the second neglecting it, emphasises the possible amplification of the response which the sloshing of the liquid may cause and is consistent with the requirement(1) of substantially increased seismic design coefficients for elevated fluid containers relative to the basic values specified for normal building structures.

Since the hypothetical extended elastic bracing invoked in the above consideration can not, in fact, be achieved by typical mild steel braces of the dimensions specified, the actual displacement-time history of the Moran and Cheney frame is of the form shown in figure 4. The system responds through a phase of acting as if there is no effective bracing in the bottom bay after about thirteen seconds, followed subsequently by a response consistent with only one brace being active intermittently in the bottom bay. The breaking up of the regular form of oscillation results in a substantial reduction in the vibrational amplitude of the later part of the plot. The introduction of bracing prestress was made in recognition of the fact that many tank supporting structures do have the cross bracing pretensioned to ensure that it is effective in contributing to the lateral stiffness, actually doubling this at low amplitudes of vibration until the prestressing is cancelled out by the effect of the lateral displacement. The sensitivity of the system to the presence of pretension is clearly indicated by the maximum displacement results presented above. In the prestressed case the initially stiffer system takes less time to build up its response to the particular applied excitation, the maximum amplitude occurring earlier than in the non-prestressed configuration. The pattern of an effectively unbraced, changing to partially braced, bottom bay is repeated with consequent reduction in the vibratory response.

When a deliberate attempt is made to promote the development of multiphase behaviour, plots of the form shown in figures 5 and 6 are typical of the responses of the system in which early yielding of the mild steel and fracturing of the high tensile steel occurs. In the case of the figure 6 system, the middle bay bracing member as well as those in the lower bay suffer loads exceeding the elastic limit. However the lateral displacements and requirements of ductility (defined as the ratio of total displacement to the displacement at yield) are lower than in any of the other convective full tank systems. In the typical cases where the bracing ductility is of the order of 6, this can be readily achieved by mild steel components in tension and since, after the fracturing or slackening of the bracing, the residual bottom bay members will remain elastic up to a sway of approximately 10 ins., the Moran and Cheney frame would be fully capable of surviving the remainder of the earthquake excitation.

CONCLUSIONS

The use of the proposed method of predicting the seismic response of cross braced elevated water tower structures will enable designers to achieve earthquake resistance in this type of structure with greater confidence than hitherto. However it is evident that the operating conditions and the level of seismic resistance sought will need to be specified more accurately than has been the case in the past. For instance, if a tank is to be used to provide a constant head supply it is most unlikely that it will be in any state other than either full or completely empty when excited by an earthquake. Hence it would not be necessary to check the response of a series of partially full conditions, as would be the case for a supply tank which could well be at any stage of draw-down when an earthquake occurs. Where the possibility exists of the tank contents being frozen the situation in which the non-convective response could prove to be critical must be examined. The statistical probability of ground shaking of a particular magnitude being experienced at a chosen site should be taken into account in selecting a suitable exciting accelerogram and, wherever possible, the characteristics of this record should reflect the site properties. Although selected multiphase lateral stiffness characteristics can be achieved with satisfactory reliability[41], an assessment of the acceptability of sacrificial bracing elements must be made and, when such response is anticipated, suitable remedial action must be possible in order to restore a structure after a major earthquake has affected it. It is acknowledged that the possibility of torsional response is almost impossible to prevent in elevated water tower structures but extension of the analysis method may be made to incorporate provision for torsion. Essentially this would involve suitable modification of the coupled stiffness matrix[42] as the bracing rods slacken, yield or fracture. In summary it seems that a more sophisticated approach to the design of elevated water tower structures than that used previously is both possible and desirable. The use of a torsion spanner to set prestress levels may appear to be a relatively novel technique to many civil engineers but the example of those who have already shown their willingness to adopt elaborate detailing and to include ingenious devices in seismic resistant structures(43) could well be followed by others with advantage.

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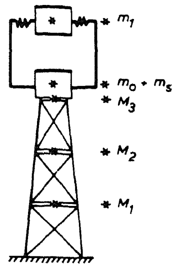
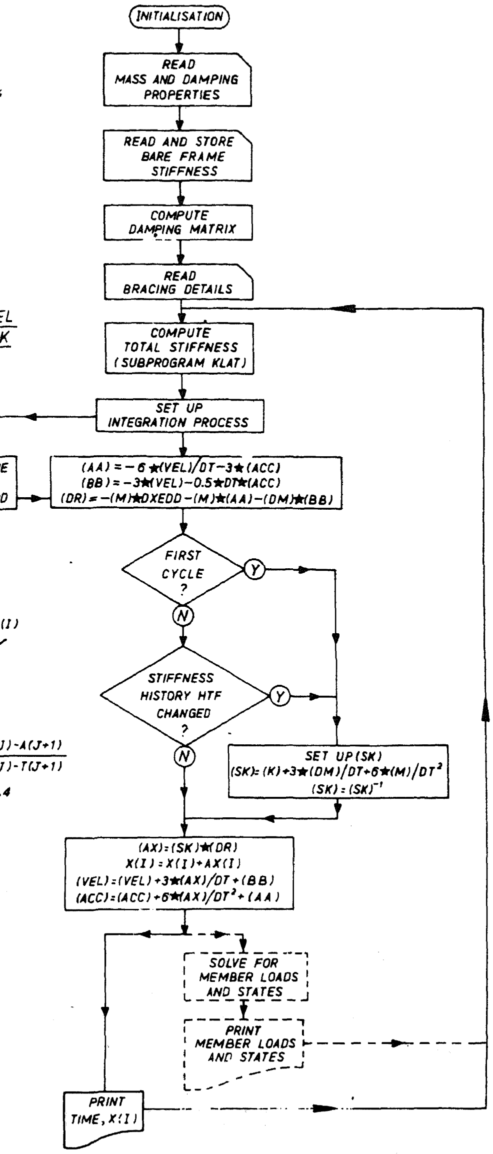
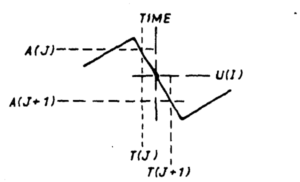


Fig. 1 : IDEALISED THREE PANEL ELEVATED WATER TANK





$$U(I) = A(J) + (TIME - T(J)) \star \frac{A(J) - A(J+1)}{T(J) - T(J+1)}$$

$$DXEDD = (U(I) - U(I-1)) \star 386.4$$

Fig 2 : FLOW CHART FOR COMPUTER PROGRAM

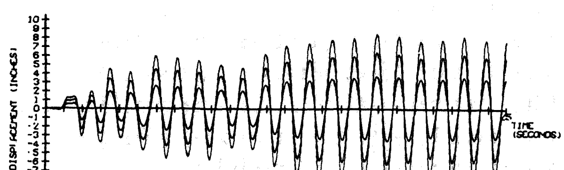


Fig. 3.

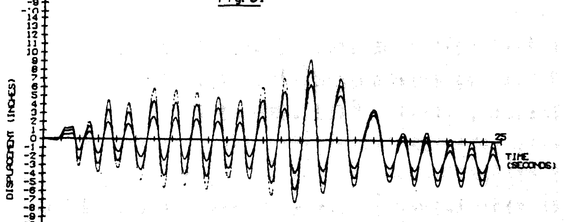


Fig. 4.

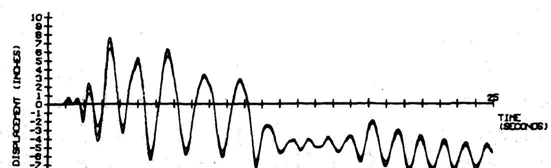


Fig. 5.

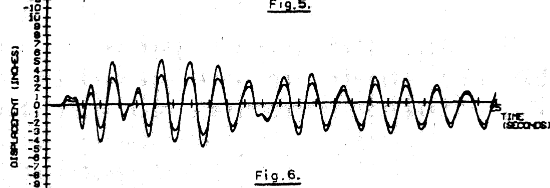


Fig. 6.

Fig. 3, 4, 5 & 6 Displacement - Time Responses