

# EARTHQUAKE INTENSITY AND SMOOTHED EARTHQUAKE SPECTRA.

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## SYNOPSIS.

Definitions of earthquake intensity based on ground accelerations and velocities are proposed and discussed. A time-intensity function is defined and a proposal is made for its use as a time-shaping function for simulated accelerograms. A new definition of spectral intensity is proposed and a duality property of response spectra is established. Characteristic frequencies of ground motion are defined and used to classify accelerograms. A theoretical expression for smoothed Fourier amplitude spectra of ground acceleration is derived; this function may be used as a frequency-shaping function for artificial accelerograms. An expression for the smoothed pseudovelocity spectrum is given.

### 1. A Measure of Earthquake Intensity Based on Ground Accelerations.

In a previous paper<sup>(1)</sup>, the author has given a definition of earthquake intensity as a measure of seismic destructiveness based on a criterion of cumulative damage. Essentially, this definition equates intensity with the sum of the energies dissipated per unit of weight by a collection of simple structures whose frequencies are uniformly distributed in the interval  $(0, \infty)$ . Let the structures be allowed to oscillate parallel to the x-axis, and let  $E$  be the energy dissipated per unit of weight by a structure of circular frequency  $\omega_0$ , as a consequence of the motion induced on it by the earthquake. The intensity of the earthquake along the x-axis is then defined as

$$I_a = \int_0^{\infty} E d\omega_0 \quad (1.1)$$

If viscously damped simple linear oscillators, all with the same fraction of critical damping,  $\zeta$ , are chosen as mathematical models of the structures belonging to the collection, it can be shown<sup>(1)</sup> that

$$I_{a,xx}(\zeta) = \frac{\arccos \zeta}{g \sqrt{1-\zeta^2}} \int_0^{t_0} a_x^2(t) dt \quad (1.2)$$

Thus, with this choice, the intensity is expressible as the product of two factors. The first is a function of  $\zeta$  alone ;

$$f(\zeta) \equiv \frac{2 \arccos \zeta}{\pi \sqrt{1-\zeta^2}} \quad (1.3)$$

and may be called the structural factor. The second is the integral

$$I_{a,xx} \equiv \frac{\pi}{2g} \int_0^{t_0} a_x^2(t) dt = \frac{1}{2g} \int_0^{\infty} |F_{a_x}(\omega)|^2 d\omega \quad (1.4)$$

that depends only on the motion of the ground, and may, therefore, be given the name of seismological factor.

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Within the range of values of  $\beta$  valid for real structure, the structural factor is practically a constant<sup>(1)</sup>. Therefore the definition may be standardized choosing a value of  $\beta$  close to the values observed for real structures. The simplest choice is  $\beta=0$ . The standardized intensity along the x-axis, based on ground accelerations, is consequently defined as being equal to the seismological factor given in eq. (1.4). It can be shown that the standardized intensity is equal to the sum of the energies per unit of weight accumulated in the collection of oscillators at the moment the earthquake stops.

The mean square ground acceleration over the time  $t_0$

$$\overline{a_x^2} = \frac{1}{t_0} \int_0^{t_0} a_x^2(t) dt \quad (1.5)$$

has been used by some authors as a measure of intensity. It is obvious that

$$I_{a,xx} = \frac{\pi}{2g} \overline{a_x^2} t_0 \quad (1.6)$$

It appears that the mean square ground acceleration does not give an adequate importance to earthquake duration as a factor of earthquake destructiveness. Furthermore, the definition proposed by the author gives an structural-mechanical interpretation to  $\overline{a_x^2} \cdot t_0$  as a measure of intensity. Let us further remarks that the measure of intensity given in (1.2) or (1.4) is statistically additive in the following sense : let  $a_x(t)$  be the result of the superposition of two uncorrelated acceleration time series,  $a_x(t) = {}_1a_x(t) + {}_2a_x(t)$ ; then

$$\int_0^{t_0} a_x^2(t) dt = \int_0^{t_0} {}_1a_x^2(t) dt + \int_0^{t_0} {}_2a_x^2(t) dt \quad (1.7)$$

and therefore, the intensity of the resultant motion is equal to the arithmetical sum of the intensities of the component motions. The conclusion may be extended to include the case of ensemble averages if the resultant acceleration process is considered to be built up by orthogonal elementary processes.

In Ref. (1) it has been shown how an intensity tensor, defining earthquake intensity at a point as a tensorial magnitude, may be defined. To construct this tensor it suffices to have records of acceleration vs. time along three orthogonal directions through the point. The elements of the intensity tensor referred to the system of coordinate-axes  $Ox_1 \times Ox_2 \times Ox_3$  are defined by

$$I_{a,ij} = \frac{\pi}{2g} \int_0^{t_0} a_i(t) a_j(t) dt, \quad (i,j = 1,2,3) \quad (1.8)$$

The trace of the tensor is invariant for every rotation leaving the origin fixed. This scalar may be termed the scalar intensity at the origin; its value is given by

$$I_{a,11} + I_{a,22} + I_{a,33} = \frac{\pi}{2g} \int_0^{t_0} \overline{a^2} dt \quad (1.9)$$

If  $Ox_3$  is chosen to coincide with the vertical through 0, the sum

$$I_{a,h} \equiv I_{a,11} + I_{a,22} = \frac{\pi}{2g} \int_0^{t_0} (a_1^2 + a_2^2) dt \quad (1.10)$$

is invariant for rotations of the reference frame around  $Ox_3$ . This sum may be given the name of scalar intensity on the horizontal plane through 0. From the standpoint of earthquake damage, this may be the most significant scalar measure of intensity at 0.

An intensity ellipsoid can be defined by the equation

$$\{r\}^T [I_{\alpha, \beta}] \{r\} = 1 \quad (1.11)$$

It is easy to show that the radius vector  $|\vec{r}|$  is inversely proportional to the square root of the intensity along the line defined by the unit vector  $\{\hat{r}\}$ . The intensity ellipsoid may be used to study the radiation field at points near the epicenter. In this kind of study geometrical techniques like Mohr's circle may be employed, specially if the study is limited to motion on the horizontal plane. An exploratory study on the Parkfield earthquake has been done by LANGE<sup>(2)</sup>.

## 2. A Measure of Intensity Based on Ground Velocities.

In Ref. (1) the author mentioned the possibility of using a suggestion made by WESTERGAARD, in 1933<sup>(3)</sup>, to establish a measure of earthquake intensity based on the volume density of the kinetic energy of ground motion. To give a precise meaning to Westergaard's proposal, let  $F_{Vx}(\omega)$  be the Fourier transform of the component of ground velocity along the x-axis. The contribution to kinetic energy of the x-component of ground velocity in the frequency interval  $(\omega, \omega+d\omega)$  is proportional to  $|F_{Vx}(\omega)|^2 d\omega$ . Hence, the total contribution of the x-component of ground velocity is proportional to

$$\int_0^{\omega} |F_{Vx}(\omega)|^2 d\omega = \pi \int_0^{t_0} v_x^2(t) dt \quad (2.1)$$

This suggests the idea of taking the last integral or a quantity proportional to it, as a measure of earthquake intensity along the x-axis. It remains to be shown that this integral is in some way related to seismic destructiveness, this being the implicit intent of all intensity scales.

It may be shown that<sup>(4)</sup> if  $E$  is the energy dissipated per unit of weight by a simple linear oscillator with negligible viscous damping and natural period  $T_0$ , then

$$I_{v, xx} \equiv \int_0^{\infty} E dt_0 = \frac{\pi^2}{g} \int_0^{t_0} v_x^2(t) dt \quad (2.2)$$

This expression will be called the standardized intensity along the x-axis, based on ground velocities.

With some minor and obvious changes, the results obtained at the end of the preceding section may be adapted to define an intensity tensor based on ground velocity measurements. Similarly, new definitions for the scalar intensity at 0, and for the intensity on a horizontal plane through 0 may be given, based on ground velocities instead of ground accelerations. The measure of intensity defined in eq. (2.2) is also statistically additive, as may be readily shown.

It has obvious seismological implications and may be related directly to the flux of kinetic energy. The above result shows that it may also be given a structural-mechanical interpretation.

It must be remarked that  $\int_0^{\infty} E \, dT_0$  is not convergent if the damping is finite. Thus the definition of  $I_{v,xx}$  cannot be extended to include linear oscillators with finite damping as mathematical models of the structures belonging to the collection envisaged in definition (2.2). Thus  $I_{v,xx}$  is essentially a seismological definition of intensity; it cannot be interpreted as the product of a seismological factor and a structural factor, as was done for  $I_{a,xx}$ . This is one reason to prefer  $I_{a,xx}$  as compared with  $I_{v,xx}$  in earthquake engineering applications. A more important reason to prefer  $I_{a,xx}$  is that it can be determined directly from the accelerograph data, perhaps with an adequate base line correction, whereas the computation of  $I_{v,xx}$  involves the previous integration of the accelerogram to obtain ground velocities, and this is an operation that introduces errors the importance of which is difficult to assess.

Let us finally remark that the ratio  $I_{a,xx} : I_{v,xx}$  has the physical dimensions of the square of a frequency. As will be pointed out presently it bears a direct relation to one of the characteristic frequencies of the ground motion to be defined in Section 5. This ratio may be also interpreted as the average number of passages through zero from below for the velocity process.

### 3. Time-Intensity Function. Intensity Density.

HUSID and his co-workers<sup>(5)</sup> have shown that the integral

$$I_{a,xx}(t) \equiv \frac{\pi}{2g} \int_0^t a_x^2(t) dt \quad (3.1)$$

may be used as a rational criterion to select the "important" segment of a given real accelerogram from the standpoint of the computation of response spectra. Essentially the same function has been used by SARAGONI<sup>(6)</sup> to define a time-shaping function for the development of artificial accelerograms. The function of  $t$  defined in (3.1) will be called the time-intensity function. It measures the intensity of ground motion along the  $x$ -axis up to the instant  $t$ , as if the earthquake were to stop suddenly at that instant.

Let us introduce a new function  $i_{a,xx}(t)$  defined as

$$i_{a,xx}(t) \equiv \frac{1}{I_{a,xx}} \frac{d}{dt} I_{a,xx}(t) \quad (3.2)$$

It follows immediately that

$$i_{a,xx}(t) \geq 0 \quad \text{and} \quad \int_{-\infty}^{\infty} i_{a,xx}(t) dt = 1 \quad (3.3)$$

Furthermore,  $i_{a,xx}(t) = 0$ , for  $t < 0$ , (assuming the earthquake to start at  $t = 0$ ).

Therefore,  $i_{a,xx}(t)$  is a causal function having the properties of a density, in the sense given to the last term in probability theory. It will be called the intensity density at the instant  $t$ . This kind of

functions is extensively treated in the mathematical literature.

PAPOULIS<sup>(7)</sup> gives estimates of  $i_{a,xx}(t)$  for the cases in which  $a_x(t)$  is a causal function (i.e.,  $a_x(t)=0$ , for  $t<0$ ), and for the case in which  $a_x(t)$  is time-limited (i.e.,  $a_x(t)=0$  outside the interval  $0<t<t_0$ ). These estimates are based on the assumption that  $i_{a,xx}(t)$  is the density of the sum of a large member of independent random variables. Furthermore it is assumed that conditions for the validity of the causal form of the central limit-theorem of probability theory are satisfied. Under this assumptions the results are as follows :

1) When the acceleration is a causal function, the intensity density is approximately a  $\chi^2$  distribution :

$$i_{a,xx}(t) \cong \frac{t^\alpha e^{-t/\beta}}{\beta^{\alpha+1} \Gamma(\alpha+1)}, \text{ for } t>0; \quad i_{a,xx}(t)=0, \text{ otherwise} \quad (3.4)$$

2) When the acceleration is time-limited, the intensity density is approximately a beta distribution

$$i_{a,xx}(t) \cong M \left(\frac{t}{t_0}\right)^\alpha \left(1 - \frac{t}{t_0}\right)^\beta, \text{ for } 0 < t < t_0; \quad i_{a,xx}(t)=0, \text{ otherwise} \quad (3.5)$$

The above estimates may be corrected to obtain higher order approximations. In the first case, an expansion in terms of generalized Laguerre polynomials is obtained. In the second case, the expansion involves the use of Jacobi polynomials. The expressions of the parameters  $\alpha, \beta, M$  are given in Ref. (7).

The approximations given in eqs. (3.4) and (3.5) may be used as time-shaping functions in the development of artificial accelerograms of infinite duration, and of finite duration, respectively. A distribution essentially of the form (3.4) has been used with this purpose by SARAGONI<sup>(6)</sup>. Equation (3.5) should give better results for the simulation of accelerograms of short duration.

#### 4. Spectral Intensity. The Dual Property of Response Spectra.

Response spectra intensities have been defined by BENIOFF<sup>(8)</sup> and by HOUSNER<sup>(9)</sup>. New definitions are herein proposed. In Ref.(1) it has been shown that

$$I_{a,xx} \leq \frac{1}{2g} \int_0^\infty S_{pv}^2(\omega_c, 0) d\omega_c \quad (4.1)$$

where

$$S_{pv}(\omega_c, 0) \equiv \sup_t \{ \omega_c |x(t)| \} \quad (4.2)$$

is the pseudo-velocity spectra for zero damping expressed as a function of the undamped circular frequency  $\omega_0$ . LANGE<sup>(2)</sup> has found a very good correlation between  $I_{a,xx}$  and the integral on the right hand side of (4.1). Therefore we define a spectral density  $I_{S,a}$  by the equation

$$I_{S,a} \equiv \frac{1}{2g} \int_0^\infty S_{pv}^2(\omega_c, 0) d\omega_c \quad (4.3)$$

By a procedure similar to that employed in Ref. (1) it may be shown that

$$I_{v,xx} \leq \frac{1}{2\beta} \int_0^\infty S_{pv}^2(\tau_c, 0) d\tau_c \quad (4.4)$$

Here  $S_{pv}(T_0, 0)$  is the pseudovelocity spectrum for zero damping expressed as a function of the undamped natural period,  $T_0$ . There is also good correlation between the two sides of relation (4.4). Therefore a second definition of spectral intensity is possible. We define  $I_{S,v}$  by the integral on the right hand side of (4.4).

These definitions give spectral intensities along a given direction. Unfortunately, it is not possible to combine spectral intensities along different directions to define a spectral intensity tensor, as it was the case with the two measures of intensity defined in Section 1 and 2. We have already stated some reasons to prefer  $I_{a,xx}$  as compared with  $I_{v,xx}$ , when used in connection with earthquake engineering problems. Given that  $I_{S,a}$  correlates well with  $I_{a,xx}$ , it is plausible to prefer  $I_{S,a}$  as compared with  $I_{S,v}$ .

Through repeated integrations by parts of the well known Duhamel's formula it is relatively easy to obtain the two following expansions in powers of  $\omega_0$  for the relative displacement  $x(t)$  of a viscously damped linear oscillator, valid, respectively, for large and for small values of  $\omega_0$  :

$$\omega_0^2 x(t) = -a(t) + 2\zeta a^{(1)}(t) \cdot \frac{1}{\omega_0} + (1-4\zeta^2) a^{(2)}(t) \cdot \frac{1}{\omega_0^2} - 4\zeta(1-2\zeta^2) a^{(3)}(t) \cdot \frac{1}{\omega_0^3} + O\left(\frac{1}{\omega_0^4}\right) \quad (4.5)$$

$$x(t) = -s(t) + 2\zeta s^{(-1)}(t) \cdot \omega_0 + (1-4\zeta^2) s^{(-2)}(t) \cdot \omega_0^2 - 4\zeta(1-2\zeta^2) s^{(-3)}(t) \cdot \omega_0^3 + o(\omega_0^4) \quad (4.6)$$

Here  $a^{(n)}(t)$  is the  $n^{\text{th}}$ . derivate of  $a(t)$ , and  $s^{(-n)}(t)$  is defined recursively by

$$s^{(-n)}(t) = \int_0^t s^{(-n+1)}(\tau) d\tau \quad ; \quad s^{(-1)}(t) = \int_0^t s(\tau) d\tau \quad (4.7)$$

where  $s(\tau)$  is the ground displacement at the instant  $\tau$ . These expansions may be obtained formally from the differential equation of motion by expanding the differential operator in a Taylor series.

From Eqs. (4.5) and (4.6) it is obvious that there is a perfect duality between the behavior of response spectra at very large and at very small frequencies. From these equations it follows that the integrals employed to define  $I_{S,a}$  and  $I_{S,v}$  are both convergent. Furthermore, from the same equations it follows that

$$\int_0^\infty \tilde{S}_{pv}(\omega, \zeta) d\omega \quad , \quad \text{and} \quad \int_0^\infty \tilde{S}_{pv}(T_0, \zeta) dT_0$$

are both divergent. Hence, the restriction of the interval of integration to a range of finite natural periods in Housner's definition of spectral intensity is unavoidable.

##### 5. Characteristic Frequencies of Ground Motion and Classification of Accelerograms.

Consider the integrals

$$J_v \equiv \int_0^{t_0} v^{-2} dt \quad ; \quad J_a \equiv \int_0^{t_0} a^2 dt \quad ; \quad J_{\ddot{a}} \equiv \int_0^{t_0} \ddot{a}^2 dt \quad (5.1)$$

We may define two frequencies,  $\Omega_V$  and  $\Omega_a$  as follows

$$\Omega_V^2 \equiv J_a : J_V \quad , \quad \Omega_a^2 \equiv J_a : J_a \quad (5.2)$$

These will be called the characteristic frequencies of ground motion. From Schwarz's inequality it follows that  $\Omega_a^2 \geq \Omega_V^2$ . The following conjecture is advanced : if the ground motion is time-limited,  $\Omega_a^2 \geq 3\Omega_V^2$ . We have not been able to prove this relation with full generality.

Parseval's relation allows to rewrite (5.2) in the alternative form :

$$\Omega_V^2 \equiv \int_{-\infty}^{\infty} \omega^2 |F_V(\omega)|^2 d\omega : \int_{-\infty}^{\infty} |F_V(\omega)|^2 d\omega \quad , \quad \Omega_a^2 \equiv \int_{-\infty}^{\infty} \omega^2 |F_a(\omega)|^2 d\omega : \int_{-\infty}^{\infty} |F_a(\omega)|^2 d\omega \quad (5.3)$$

ACUÑA<sup>(10)</sup> has computed  $\Omega_a$  and  $\Omega_V$  for a total of 53 accelerogram components of real earthquakes. The results are shown on Fig. 1. Here  $T_a = 2\pi/\Omega_a$ ,  $T_V = 2\pi/\Omega_V$ . This diagram may be used to classify strong-motion accelerograms. Three distinct classes are apparent according to the values of  $T_a$  : I)  $T_a > 1.0$  sec.; II)  $0.2 < T_a < 1.0$  sec.; III)  $T_a < 0.2$  sec. The boundaries of these classes are somewhat arbitrary. A sharper definition may be perhaps be obtained in the future through the study of a larger sample of records. For the time being, it may be remarked that for each one of the records considered, the two components fall in the same class. Class I consists of accelerograms recorded in downtown Mexico City (Torre Latinoamericana and Parque Alameda). Class III contains only accelerograms recorded in Lima (Perú) and Santiago de Chile. The intermediate Class II contains records obtained in the Western United States. Downtown Mexico City on one side, and Lima and Santiago on the other, represent two extremes as to local soil conditions. It may, therefore, be inferred that the soil condition is the chief single factor reflected in this classification. This must not be construed as an assertion that this is the only important factor. Certainly, earthquake magnitude, epicentral distance and focal mechanism are factors that have influence on  $\Omega_a$  and  $\Omega_V$ . Unfortunately the data available did not allow a factorial analysis to decide on the relative importance of all these factors. We must also guard the reader against the all too hasty conclusion that  $\Omega_a$  and  $\Omega_V$  reflect in some way characteristic pertaining to the site exclusively.

Borrowing a term from the theory of light, the non negative dimensionless number  $r \equiv \Omega_a^2/\Omega_V^2 - 1$  is a measure of the incoherence of the ground motion. The value  $r = 0$  corresponds to a sinusoidal motion of infinite duration. For motions of finite duration  $r > 0$ .

Table 1 gives the range and average values of  $\Omega_a$ ,  $\Omega_V$  and  $r$  for the three classes of accelerograms. Accelerograms of Class I may be characterized as slightly incoherent. Whereas those in Class III are strongly incoherent. It is worthy to note that there is a significant difference as regards coherence, between the records obtained in the basement of Torre Latinoamericana and Parque Alameda. The records of the first site show higher coherence. This may be explained by the effect of soil-structure interaction.

Finally, let us remark that  $\Omega_a$  and  $\Omega_v$  are related with the shape of the autocorrelation functions for ground velocity and for ground acceleration for small values of  $\xi$ . By a series expansion in powers of  $\xi$  and subsequent integration it may be shown that the normalized autocorrelation functions can be written in the form :

$$\hat{R}_v(\xi) = 1 - \frac{\Omega_v^2}{2!} \xi^2 + \frac{\Omega_a^2 \Omega_v^2}{4!} \xi^4 + \dots$$

$$\hat{R}_a(\xi) = 1 - \frac{\Omega_a^2}{2!} \xi^2 + \dots$$

## 6. Smoothed Fourier Spectra and Frequency-Shaping Function.

A theoretical expression for smoothed Fourier amplitude spectra may be obtained in two steps. In the first, it is assumed that the motion of the ground arises from the superposition of a large number of elementary motions satisfying the following assumptions : 1) for each elementary motion the kinetic energy per unit volume of the ground at a given point can be represented in the frequency space  $-\infty < \omega < \infty$  by two symmetrical narrow rectangles centered at  $+\omega$  and  $-\omega$  respectively; 2) all elementary motions convey the same amount of kinetic energy; 3) the abscissa  $\omega$  of the middle points of the rectangles is a random variable with variance  $\Omega^2$ ; 4) the elementary motions have zero cross-correlation. By the central limit-theorem, the distribution of the kinetic energy of the resulting motion in the frequency space is normal, i.e.,

$$|F_v(\omega)|^2 = A e^{-\omega^2/2\Omega^2} \quad \text{and, therefore,} \quad |F_a(\omega)|^2 = A \omega^2 e^{-\omega^2/2\Omega^2} \quad (6.1)$$

Taking into account Eq. (1.4) it follows that

$$A = \frac{4g I_a}{\sqrt{2\pi} \Omega^3} \quad (6.2)$$

The characteristic frequencies of the ground motion considered in this first step of the derivation are given by  $\Omega_v^2 = \Omega^2$ ,  $\Omega_a^2 = 3\Omega^2 = 3\Omega_v^2$ . Therefore, the analysis in this first step is applicable only to earthquakes satisfying the condition  $T_v : T_a = \sqrt{3}$ . This corresponds to the smallest values of the ratio  $T_v : T_a = 1.64$ , obtained by Acuña. Or to express it in another way, only accelerograms with very slight incoherence have spectra of the form given in (6.1).

In the second step the spectra obtained in the first step are superposed assuming that  $\Omega$  is uniformly distributed in the interval  $\Omega_1, \Omega_2$  and that the kinetic energies of the component motions are additive; i.e., motions obtained in the first step are superposed admitting that they are mutually uncorrelated. This gives the final result

$$|F_a(\omega)|^2 = \frac{4g I_a}{\sqrt{2\pi} (\Omega_2 - \Omega_1)} \left\{ e^{-\frac{\omega^2}{2\Omega_1^2}} - e^{-\frac{\omega^2}{2\Omega_2^2}} \right\} \quad (6.3)$$

This result contains three parameters :  $I_a, \Omega_1, \Omega_2$ . The first is the earthquake intensity defined in Section 1. The other two are related to  $\Omega_a$  and  $\Omega_v$  by the equations

$$\Omega_v^2 = \Omega_1 \Omega_2, \quad \Omega_a^2 = \Omega_1^2 + \Omega_1 \Omega_2 + \Omega_2^2 \quad (6.4)$$

Therefore

$$\Omega_a^2 - 3\Omega_v^2 = (\Omega_2 - \Omega_1)^2 \geq 0 \quad ; \quad \text{i.e.,} \quad \Omega_a \geq \Omega_v \sqrt{3} \quad (6.5)$$

The function defined in (6.3) may be used as a frequency-shaping function (power spectral density) for the development of artificial accelerograms.

## 7. Smoothed Response Spectra.

Smoothed response spectra may be obtained by using two guiding principles : 1) the approximate proportionality between the pseudovelocity spectrum and the Fourier amplitude spectrum for accelerations; 2) the duality property established in Section 4. We omit the derivation and give only the final results

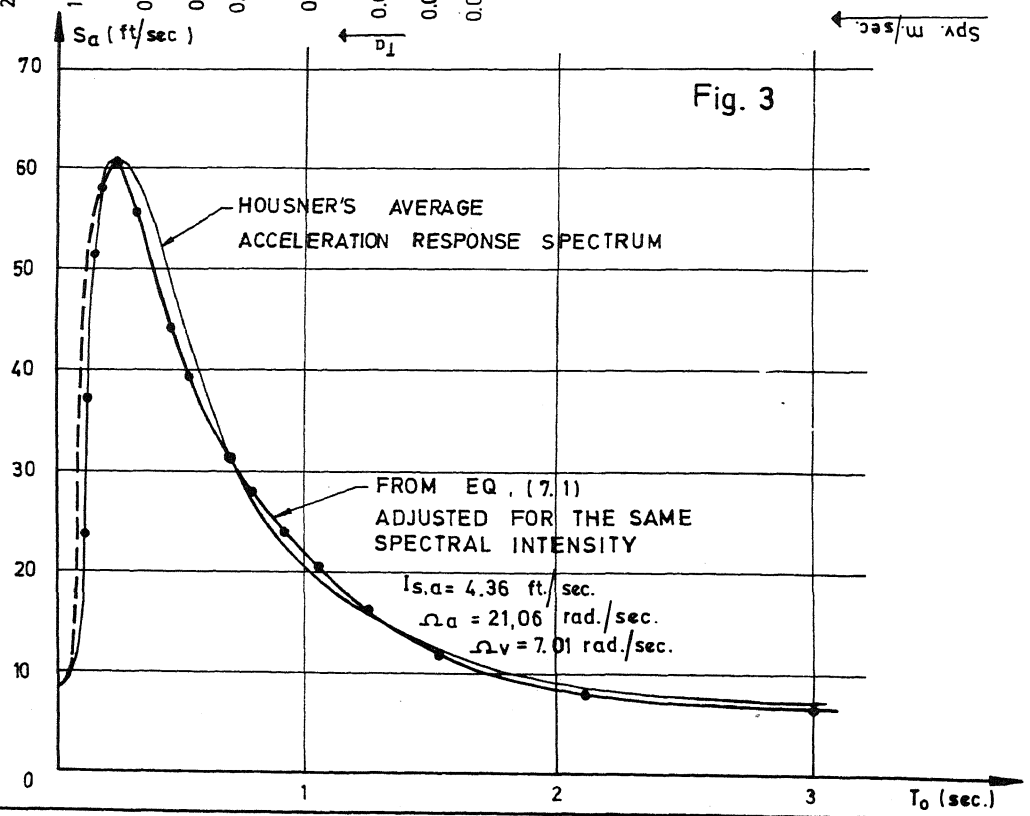
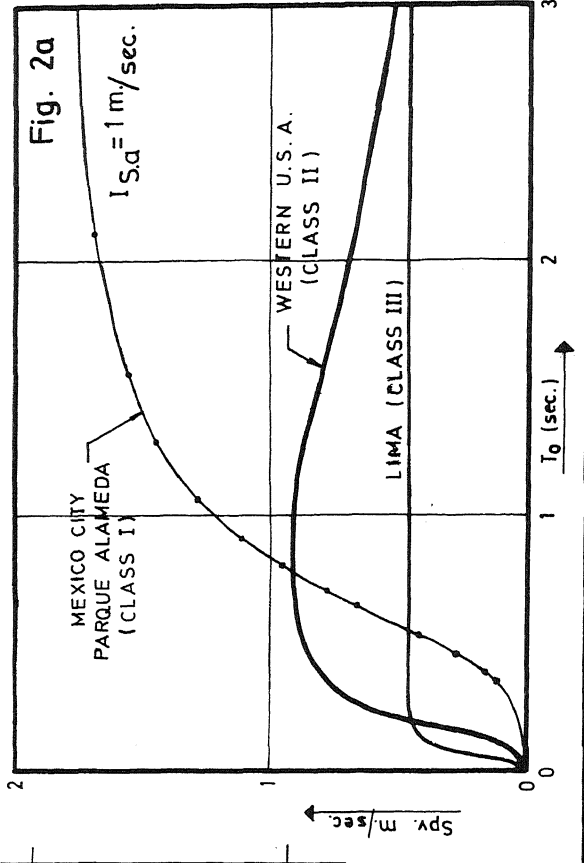
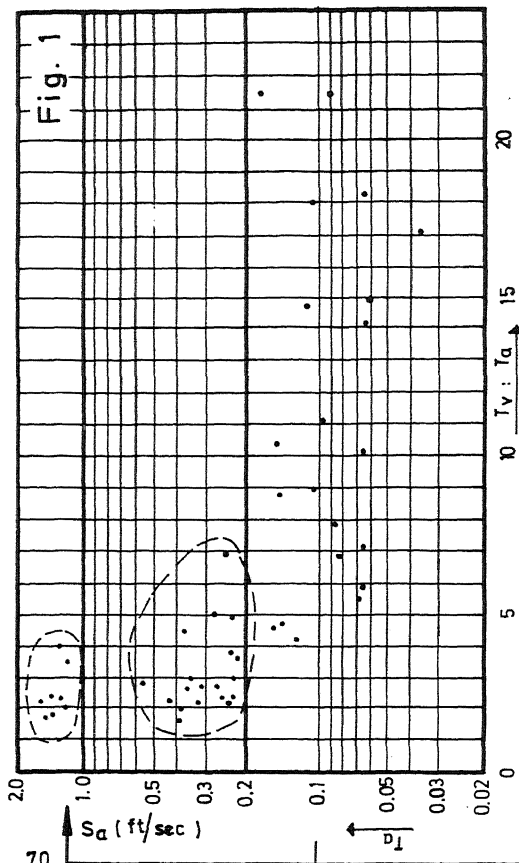
$$S_{pv}^2(\omega_c, 0) = \frac{4g I_{S,a}}{\sqrt{2\pi} (\Omega_2 - \Omega_1)} \left\{ \exp\left(-\frac{\omega_c^2}{2\Omega_1^2}\right) - \exp\left(-\frac{\omega_c^2}{2\Omega_2^2}\right) \right\} + \quad (7.1)$$

$$+ \frac{a_{max}^2}{\Omega_2^2 - \Omega_1^2} \left\{ \exp\left(-\frac{2\Omega_1^2}{\omega_c^2}\right) - \exp\left(-\frac{2\Omega_2^2}{\omega_c^2}\right) - 2 \left[ \exp\left(-\frac{\omega_c^2}{2\Omega_2^2}\right) - \exp\left(-\frac{\omega_c^2}{2\Omega_1^2}\right) \right] \right\}$$

Fig. 2 gives the spectra for Class I-II-III accelerograms, all for the same spectral intensity  $I_{S,a} = 1$  m/sec. It has been assumed that  $a_{max} = 0.25g\sqrt{I_{S,a}}$ . This is an empirical relation derived from results obtained by LANGE<sup>(2)</sup>. Fig. 3 compares the theoretical result with Housner's average spectrum

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T A B L E 1

CLASS OR SITE	$\Omega_a$ Rad./Sec.		$\Omega_v$ Rad./Sec.		r	
	RANGE	av.	RANGE	av.	RANGE	av.
CLASS I	4.07 - 5.38	4.70	1.31 - 2.68	2.08	1.55 - 14.2	5.49
CLASS II	14.6 - 29.0	21.06	3.20 - 12.56	7.01	1.75 - 60.5	11.30
CLASS III	35.0 - 174.0	74.9	1.63 - 17.00	7.11	16.4 - 461.0	155.0
MEXICO CITY (TL.)	4.07 - 4.50	4.27	1.88 - 2.68	2.25	1.55 - 4.7	2.92
MEXICO CITY (PA.)	4.50 - 5.38	5.14	1.31 - 2.53	1.91	3.05 - 14.2	8.06
SANTIAGO	40.5 - 96.6	75.7	8.26 - 17.00	11.82	16.1 - 101.0	43.4
LIMA	35.0 - 174.0	74.5	1.63 - 13.85	6.03	19.4 - 461.0	199.6

