

DYNAMIC COEFFICIENTS FOR EVALUATING THE  
SEISMIC RESPONSE OF SOIL-STRUCTURE INTERACTION SYSTEMS

by

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In a previous parametric study of seismic soil-structure interaction (5) it was established that the response of an elastic single story structure situated on the surface of an elastic foundation medium is dependent upon two interaction indices. The resulting design curves were based on the use of a dynamic model in which the flexible foundation medium is characterized by constant stiffness and associated damping coefficients (3). These dynamic foundation coefficients were developed by utilizing Bycroft's (1) elasticity solutions for a dynamically loaded continuum, and have been evaluated for a Poisson's ratio,  $\nu$ , of zero only.

Recently, Gladwell (2) studied the actual mixed boundary value problems for a dynamically loaded rigid circular disc on an elastic half-space, and the results are presented for the range  $0 \leq \nu \leq 0.5$ . These elasticity solutions can be used to establish constant coefficients for the stiffness and associated damping properties of the flexible foundation medium (4). The translational dynamic stiffness and the associated translational radiation damping coefficients are:

$$K_T = 6.77 \rho V_s^2 r / (1.79 - \nu) \quad ; \quad D_T = 6.21 \rho V_s r^2 / (2.54 - \nu) \quad (1,2)$$

in which  $\rho$  = the mass density of the foundation medium;  $r$  = the radius of the base mass of the structure; and  $V_s$  = the shear wave velocity of the foundation medium. The coefficients for the rotational mode are:

$$K_R = 2.52 \rho V_s^2 r^3 / (1.00 - \nu) \quad ; \quad D_R = 0.136 \rho V_s r^4 / (1.13 - \nu) \quad (3,4)$$

These equations represent a consistent set of stiffness and damping coefficients which characterize the dynamic properties of the elastic half-space. The variations of these coefficients with respect to Poisson's ratio are shown in Fig. 1. For comparison the coefficients as obtained for the Bycroft solutions (3), and the static cases, are also shown.

Utilizing the dynamic coefficients for the foundation medium as defined by Eqs. 1 through 4 a parametric study of a wide range of single story soil-structure systems was carried out to investigate the influence of various factors on the seismic response of the structures. The results of the study can be described by two dimensionless interaction indices:  $\theta^*$ , and the ratio  $T_v/T_r$ . The interaction index  $\theta^*$  is defined as the ratio of the maximum flexural displacement of the structure on a flexible foundation to the maximum flexural displacement of a single mass oscillator with a period  $T_v$ , which is the fundamental period of the three degree-of-freedom soil-structure system while  $T_r$  is the period of the structure when situated on a rigid foundation. Since  $\theta^*$  is the ratio of the maximum displacements

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of two systems having the same fundamental frequency, its value indicates the amount of energy which is dissipated into the foundation medium by the interaction phenomenon. Thus, the maximum flexural response of a soil-structure system can be evaluated by means of the relationship

$$u_{\max} = \theta^* S_d$$

in which  $S_d$  = the seismic spectral displacement for a structure having a period  $T_v$ .

By studying a wide variety of structures on various foundation media the corresponding interaction indices were evaluated. The foundation medium had a density of 110 lb. per cu. ft., while the structure was assumed to have viscous damping equal to one percent of critical (i.e.,  $\lambda = 0.01$ ). It was observed that the interaction indices  $T_v/T_r$  and  $\theta^*$  were similar for structures having the same value of the ratio  $h/T_r$  (expressed in ft. per sec.). Therefore, families of average curves were developed for the interaction indices as functions of  $V_s$  and the structural index  $h/T_r$ . The resulting curves for the case of  $\nu = 0.0$  are shown in Fig. 2.

A detailed study of the results revealed that the interaction indices are also dependent upon other system parameters, but to a lesser degree. It is of interest to note that the interaction indices are insensitive to practical variation in the ratio of the two masses of the structure.

The influence of these additional parameters on the values of the interaction indices as obtained from Fig. 2 can be seen from the plots shown in Figs. 3 and 4. From these figures it can be seen that the most significant interaction parameters are  $V_s$ ,  $h/T_r$  and  $\Gamma$ . The parameter  $\Gamma$  = the "effective top mass density" =  $mg/\pi r^2 h$ , and, for the purposes of this study has units of lbs. per cu. ft. Thus,  $\Gamma$  can be thought of as the density of a material having the weight of the top mass,  $m$ , and a volume equal to that enclosed by the structure.

The results of this study show that for single story structures on flexible foundation media that the dynamic coupling phenomenon can be characterized by the interaction indices  $T_v/T_r$  and  $\theta^*$ . These indices indicate how the natural period of the system is modified by soil-structure interaction, and the extent to which the structural response and the mechanism of energy exchange between the soil and the structure are effected.

#### ACKNOWLEDGEMENT

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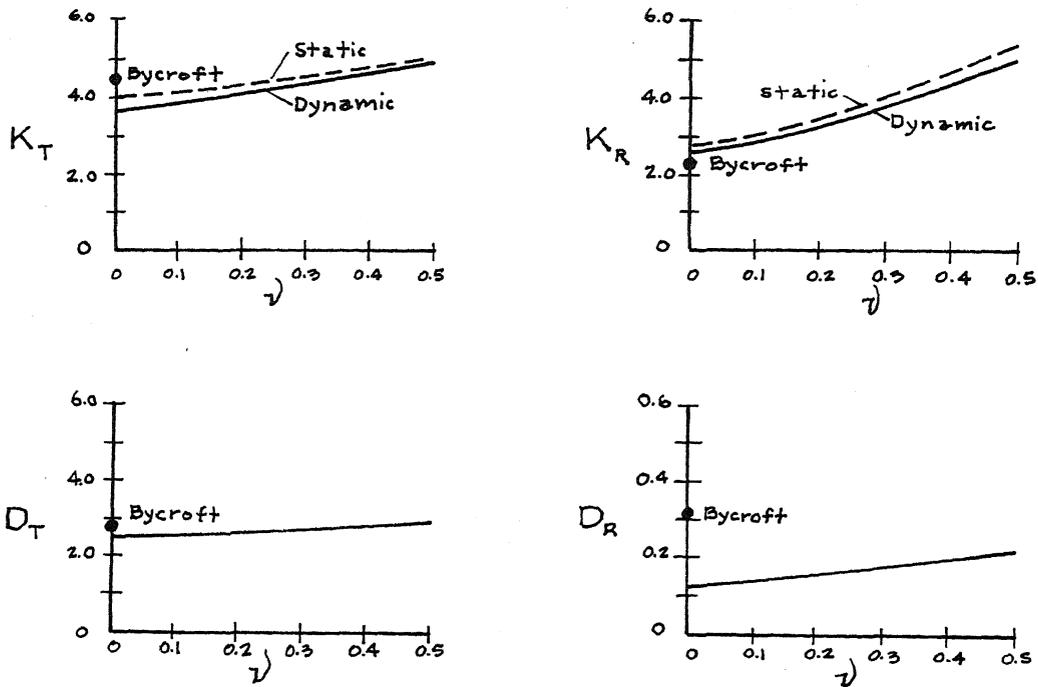


FIG. 1 - COEFFICIENTS FOR THE DYNAMIC STIFFNESS AND ASSOCIATED RADIATION DAMPING OF THE FOUNDATION MEDIUM.

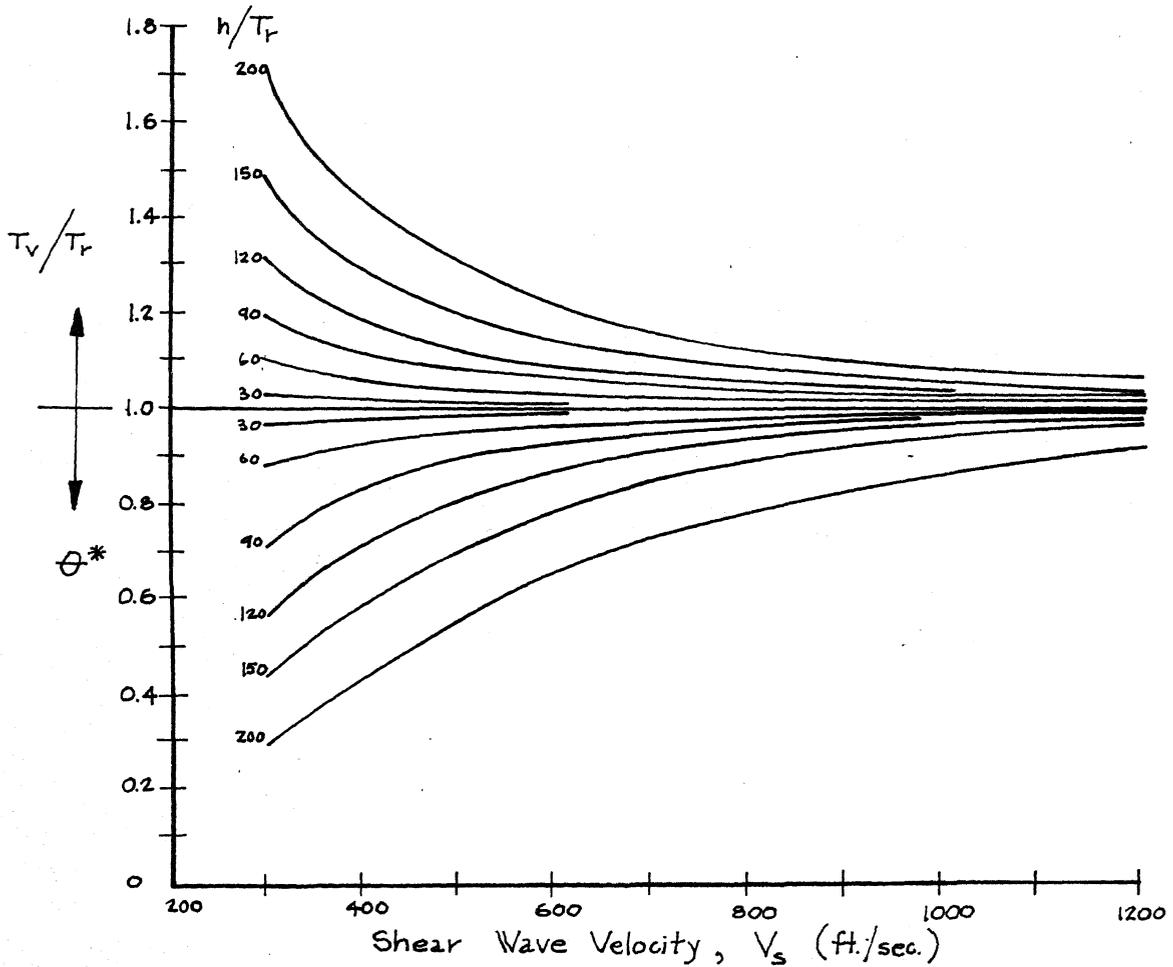


FIG. 2 - INTERACTION INDICES

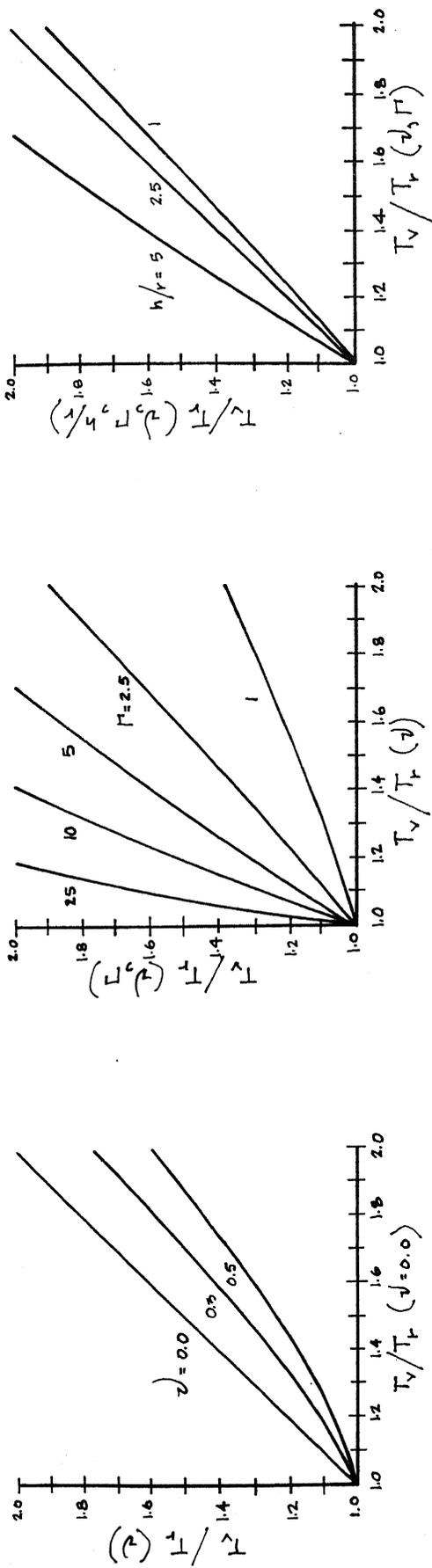


FIG. 3 - PARAMETRIC VARIATION OF  $T_v/T_r$  INTERACTION INDEX

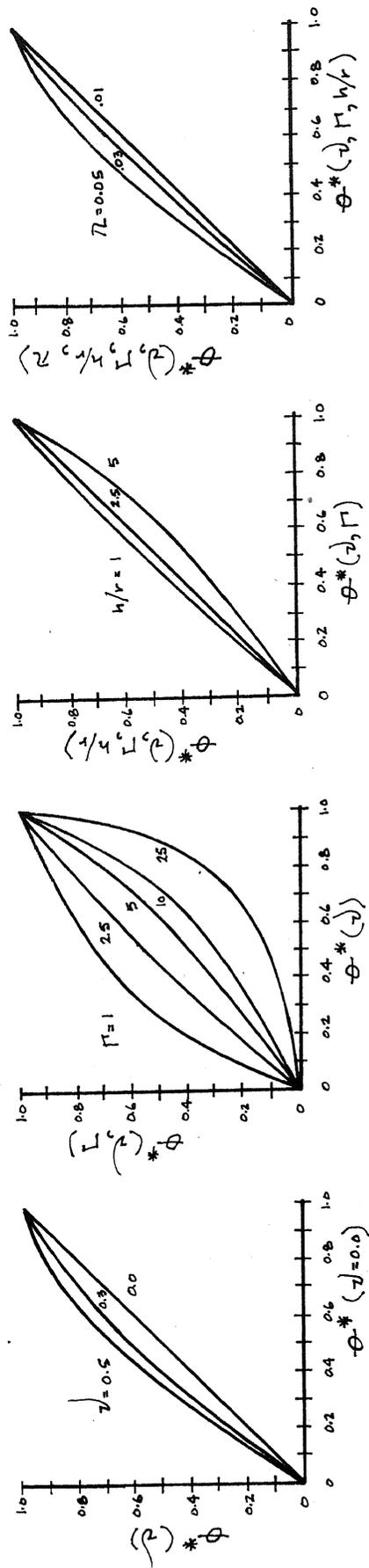


FIG. 4 - PARAMETRIC VARIATION OF  $\theta^*$  INTERACTION INDEX