SIMPLE MODELS FOR FOUNDATIONS IN LATERAL AND ROCKING MOTION

by

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SYNOPSIS

An approximate analysis is presented for the steady-state response of a rigid massless disk which is supported at the surface of an elastic halfspace and is excited either by a horizontal force $Pe^{i\omega t}$ or by an overturning moment $Te^{i\omega t}$. The analysis is based on the assumption that only a portion of the halfspace in the form of a semi-infinite truncated cone is effective in transmitting the energy imparted to the disk. For each mode of excitation, the dynamic force-displacement relationship for the appropriate disk-cone model is shown to be in good agreement with the corresponding result for the actual disk-halfspace system. It is further shown that the force-displacement relationship for each cone is identical to that for a simple oscillator having frequency independent properties.

ANALYSIS OF APPROXIMATING CONES

The appropriate cones for the two modes of excitation considered are shown in Figs. 1a and 1b. For the horizontally excited system, the apex of the cone is taken at a distance z_0 above the surface, whereas for the system in rocking motion, the apex is taken at the center of the disk. The angles of opening of the cones, α_x and α_θ, are different in the two cases.

Each cone is assumed to deform only in shear. For the horizontally excited system, horizontal planes are presumed to displace horizontally without distortion, and for the disk in rocking motion, spherical segments having centers at the apex are presumed to rotate about the apex without distortion. Representative surfaces of no distortion are identified by the horizontal lines in Fig. 1a and by the circular arcs in Fig. 1b. The upper boundary of the shear cone in rocking motion is taken at $r = r_0$, and the small conical segment between this boundary and the apex, represented by the dotted area in Fig. 1b, is assumed to rotate as a rigid body with the disk.

Let $x(z)\,e^{i\omega t}$ be the horizontal displacement at a distance z of the horizontally excited cone, and $\theta(r)\,e^{i\omega t}$ be the rotation at r of the cone in rocking motion. The equations of motion for these cones are, respectively,

$$\frac{1}{z^2}\frac{d}{dz}\left[z^2\frac{dx}{dz}\right] + \frac{\omega^2}{c_s^2}\,x = 0 \quad \cdots\cdot \quad (1a) \quad \text{and} \quad \frac{1}{r^4}\frac{d}{dr}\left[r^4\frac{d\theta}{dr}\right] + \frac{\omega^2}{c_s^2}\,\theta = 0 \quad \cdots\cdot \quad (1b)$$

in which $c_s = \sqrt{G/\rho}$ = the shear wave velocity in the soil, G = the shear modulus of elasticity, and ρ = the mass density of the soil. Making use of the boundary conditions at infinity, the solutions of these equations can be written as

$$x = x_0\,\frac{z_0}{z}\,e^{-i\frac{\omega(z-z_0)}{c_s}} \quad \cdots\cdot \quad (2a) \qquad \theta = \frac{\theta_0}{1 + i a_0}\left[\left(\frac{r_0}{r}\right)^3 + i a_0\left(\frac{r_0}{r}\right)^2\right] e^{-i\frac{\omega(r-r_0)}{c_s}} \quad (2b)$$

where $a_0 = \omega r_0/c_s$ = a dimensionless frequency parameter, and x_0 and θ_0 are,

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respectively, the amplitudes of the horizontal and angular displacements of the disk.

The boundary conditions at the top surface of the cones in translational and rotational motions are, respectively,

$$P + GA \frac{dx}{dz}\Big|_{z=z_o} = 0 \cdots\cdots \quad (3a) \qquad \text{and} \qquad T + \omega^2 I_o \theta_o + GA_o r_o^2 \frac{d\theta}{dr}\Big|_{r=r_o} = 0 \quad (3b)$$

where $A = \pi r_o^2$ = the area of the disk; $A_o r_o^2$ = the moment of inertia of the top spherical surface of the shear cone in Fig. 1b about the axis of rotation; and I_o = the effective mass moment of inertia of the conical segment which is assumed to rotate as a rigid body with the disk.

Substituting Eq. 2a into Eq. 3a leads to Eq. 4 for the dynamic translational stiffness of the disk-cone system, $Q_x = P/x_o$,

$$Q_x = \frac{GA}{z_o}\left[1 + i a_o \frac{z_o}{r_o}\right] \cdots\cdots \quad (4) \qquad\qquad Q_j = K_j [k_j + i a_o c_j] \cdots\cdots\cdots\cdots \quad (5)$$

The static value of this stiffness is thus GA/z_o. Finally, by replacing the static stiffness with the corresponding stiffness of the actual disk-halfspace system, $K_x = 8\,Gr_o/(2-\nu)$, Eq. 4 may be rewritten in the form of Eq. 5, where $j = x$, $k_x = 1$, $c_x = (2-\nu)\pi/8$, and ν = Poisson's ratio for the material in the halfspace.

In a similar manner, by substituting Eq. 2b into Eq. 3b and replacing the resulting expression for the static rotational stiffness of the cone, $3GA_o r_o$, with the corresponding stiffness of the disk-halfspace system, $K_\theta = 8\,Gr_o^3/(3-3\nu)$, it is found that $Q_\theta = T/\theta_o$ also is given by Eq. 5, where in this case $j = \theta$

$$k_\theta = 1 - \frac{1}{3}\frac{a_o^2}{1+a_o^2} - B_o a_o^2 \cdots \quad (6a) \qquad\qquad c_\theta = \frac{1}{3}\frac{a_o^2}{1+a_o^2} \cdots\cdots\cdots\cdots\cdots \quad (6b)$$

and B_o is a dimensionless measure of I_o, defined as

$$B_o = \frac{I_o c_s^2}{K_\theta r_o^2} = \frac{3(1-\nu)}{8}\frac{I_o}{\rho r_o^5} \qquad (7)$$

The angles of opening, α_x and α_θ, are determined from the following expressions, obtained by equating the static stiffnesses of the cones, GA/z_o and $3\,GA_o r_o$, to the corresponding stiffnesses of the halfspace

$$\tan\frac{\alpha_x}{2} = \frac{8}{(2-\nu)\pi} \cdots\cdots\cdots \quad (8a) \qquad \cos\frac{\alpha_\theta}{2}\left[3 + \cos^2\frac{\alpha_\theta}{2}\right] = 4 - \frac{8}{3(1-\nu)\pi} \quad (8b)$$

For $\nu = 0$, $\alpha_x = 104^o$ and $\alpha_\theta = 64^o$, whereas for $\nu = 0.5$, $\alpha_x = 119^o$ and $\alpha_\theta = 96^o$.

Eq. 5 is of precisely the same form as the corresponding expression for the harmonically vibrating disk-halfspace system (1). As an indication of the accuracy with which the shear cones represent the halfspace, the values of k_x and c_x for the two systems are compared in Fig. 2 for the two limiting values of ν, and a similar comparison is made in Fig. 3 for k_θ and c_θ. The data for the halfspace were obtained from Ref. 1 neglecting the effect of coupling between the horizontal and rocking motions. In Fig. 3, the halfspace solutions corresponding to $\nu = 0$ and $\nu = 0.5$ are compared with the results obtained from Eq. 6a taking $B_o = 0$ and $B_o = 0.027$, respectively.

For the horizontally excited system, the agreement between the two sets of results is more than adequate for most practical purposes. For the system in rocking motion, the agreement is excellent for $\nu = 0.5$, although not as satisfactory for $\nu = 0$. It is important to note, however, that improved agreement may be achieved in each case by empirically modifying the coefficients in the expressions for k_j and c_j for the cone systems and by appropriately selecting the value of B_o in Eq. 6a. As a matter of fact, the more accurate expressions for k_j and c_j for the halfspace presented in Ref. 2 were developed by this approach.

DISCRETE MODELS

For the horizontally excited disk-cone system, for which both k_x and c_x are frequency-independent quantities, it is well known that the expression for Q_x is identical to that obtained for the massless spring-dashpot system shown in Fig. 4a, provided $K = K_x$ and $C = c_x K_x r_o / c_s$.

It can further be shown that Q_θ for the rotationally excited disk-cone system is identical to that for the mass-spring-dashpot oscillator shown in Fig. 4b, where

$$K = \frac{K_\theta}{r_o^2}, \quad C = \frac{1}{3}\frac{K_\theta}{c_s r_o}, \quad M = \frac{1}{3}\frac{K_\theta}{c_s^2}, \quad \text{and} \quad I_o = B_o\frac{K_\theta r_o^2}{c_s^2} = B_o\frac{8}{3(1-\nu)}\rho r_o^5 \tag{9}$$

Note that all parameters in Eq. 9 are independent of the exciting frequency, ω, and that, unlike previous models in which the mass was connected directly to the spring, in the present model these elements are connected through the dashpot.

By considering the equilibrium of the forces on the rigid bar and the mass, and by eliminating the y-coordinate, the following expression is obtained for

$$Q_\theta = \frac{T}{\theta_o} = K r_o^2\left[\left(1 - \frac{\omega^2 M/K}{1+(\omega M/C)^2} - \frac{\omega^2 I_o}{K r_o^2}\right) + i\,\frac{\omega M}{C}\,\frac{\omega^2 M/K}{1+(\omega M/C)^2}\right] \tag{10}$$

For the values of K, C, M and I_o specified in Eq. 9, Eq. 10 is identical to the relation defined by Eqs. 5, 6a, 6b, and 7.

CONCLUSION AND ACKNOWLEDGMENT

Although the problem considered in this paper has been the subject of several comprehensive previous studies (e.g. Ref. 1), the approximate analysis presented herein provides further valuable insight into the behavior of the system and a basis for generalizations that would not be possible otherwise. An example of such an extension is given in Ref. 2.

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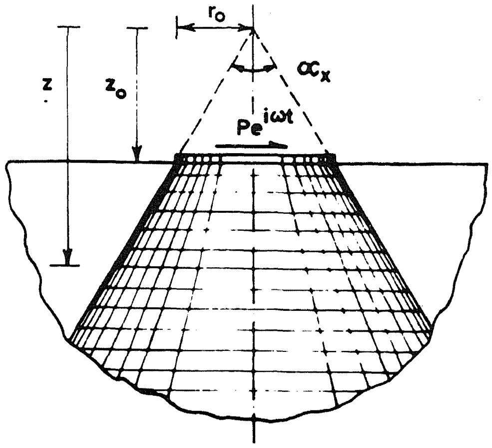
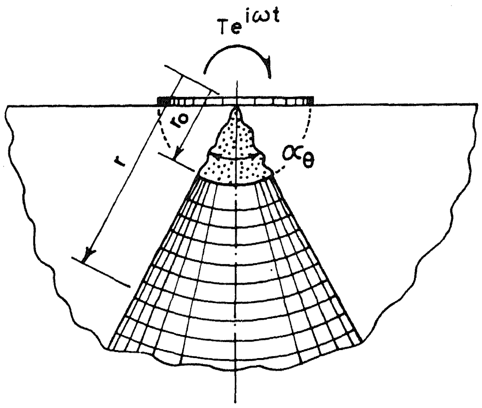
REFERENCES

1. Veletsos, A.S. and Wei, Y.T., "Lateral and Rocking Vibrations of Footings," Jour. of Soil Mech. and Found. Div., ASCE, Vol. 97, Sept. 1971, pp. 1227-48.

2. Verbic, B., and Veletsos, A.S., "Impulse Response Functions for Elastic Foundations," Struc. Research at Rice, No. 15, Rice Univ., Houston, Texas, Sept. 1972.



(a) For Translation



(b) For Rotation

FIG. 1 SHEAR-CONE MODELS

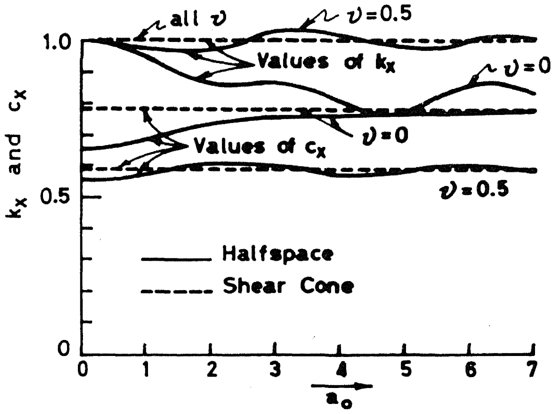


FIG. 2 COMPARISON OF k_x AND c_x FOR HALFSPACE AND CONE MODEL

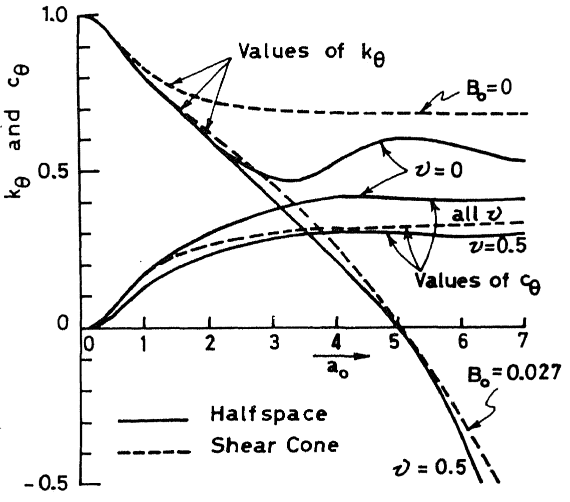
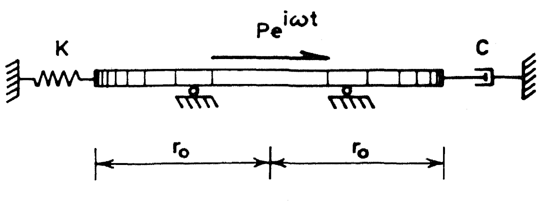
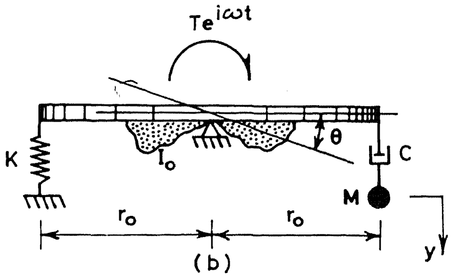


FIG. 3 COMPARISON OF k_θ AND c_θ FOR HALFSPACE AND CONE MODEL



(a)



(b)

FIG. 4 EQUIVALENT DISCRETE MODELS