

THE EFFECT OF EMBEDMENT ON VIBRATION OF FOOTINGS AND STRUCTURES

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SYNOPSIS

An approximate analytical solution for footing vibrations was developed, making it possible to consider embedment, layering, backfill and coupled motions. The analytical solution was compared with finite element solutions and with the results of field experiments.

INTRODUCTION

There is enough experimental evidence to prove that embedment greatly affects the vibration of footings in that it increases resonant frequencies and reduces resonant amplitudes. These effects are particularly important in the case of embedment into a shallow layer and with rocking or torsional vibration modes.

The response of embedded footings is a very difficult problem to solve in a rigorous analytical way. The most promising approach seems to be the finite element technique (5, 9). However, even this technique cannot embrace all the practical problems and approximate corrections may often be necessary. It would also be a loss to entirely discard the already numerous and sophisticated solutions of the surface footings because of the embedment effect. Instead, it may often be sufficient to apply an approximate correction for the effect of embedment to the solutions of surface footings. An approximate analytical approach of this kind is reported in this paper.

APPROXIMATE ANALYTICAL SOLUTION

The approximate analytical solution is based on the assumption that the soil reactions acting at the footing base are equal to those of surface footings, and the reactions acting on footing sides are equal to those of an independent overlying layer. As well, this overlying layer is considered to be composed of a series of infinitesimally thin independent layers. Such an approach was first adopted by Baranov (3) and has been extended by the writer and his associates to include torsion, layering and coupled vibration modes and also to express the soil reactions in a form directly applicable in the dynamic analysis of structures supported by embedded footings (4, 6, 7).

The formulation of soil reactions is compatible with the standard approaches of structural dynamics. If a rigid cylindrical body undergoes horizontal displacement $u(t)$ and rocking in vertical plane $\psi(t)$, the frequency dependent stiffness constants are typically expressed as

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$$k_{\psi\psi} = G r_o^3 \left[C_{\psi 1} + \left(\frac{z_c}{r_o} \right)^2 C_{u1} + \frac{G_s}{G} \delta S_{\psi 1} + \frac{G_s}{G} \delta \left(\frac{\delta^2}{3} + \frac{z_c^2}{r_o^2} - \delta \frac{z_c}{r_o} \right) S_{u1} \right]$$

etc.; the frequency dependent damping constants are similarly expressed. Here, G = shear modulus of soil underneath the footing, G_s = shear modulus of the backfill (side layer), r_o = equivalent base radius, z_c = height of C.G. above the base, $\delta = l/r_o$ and l = embedment depth. Parameters C and S relate to base or side reactions respectively and depend on dimensionless frequency $\alpha_o = \omega r_o \sqrt{\rho/G}$, in which ρ = soil density; they are given for all vibration modes in (4, 6, 7).

With stiffness and damping constants defined as indicated, the response curves of embedded footing vibrations can be easily calculated for any vibration mode and any embedment depth.

Vertical vibration: With Awojobi's base reactions (2), vertical response of footings embedded into strata of almost any thickness can be obtained. Resonant amplitudes calculated with these base reactions are plotted in dimensionless form in Fig. 1 vs. relative stratum thickness h/r_o for several values of mass ratio $b = m/\rho r_o^3$ and embedment ratio δ . It can be seen from Fig. 1 that embedment reduces the effect of layering upon the increase in the resonant amplitudes and that the latter effect increases with increasing mass ratio. With Warburton's (8) base reactions an even more pronounced effect of layering is observed.

Also plotted in Fig. 1 and indicated by circles are results of the finite element solution by Lysmer and Kuhlemeyer (5). The agreement between the two solutions appears quite satisfactory.

Torsional vibration: Awojobi's base reactions (1) yield resonant amplitudes of rotation (torsion) plotted in Fig. 2 for several values of inertia ratio $B_c = I/\rho r_o^5$ (I = mass moment of inertia about vertical axis) and embedment ratio. The effect of layering is very pronounced, increases with inertia ratio and can be reduced due to embedment. Circles indicate the finite element solution by Wass and Lysmer (9). The agreement is again satisfactory. The agreement with experiments is much worse and indicates the desirability of the inclusion of slippage in the regions of high shear.

Coupled vibration of embedded footings can be readily solved with difficulties no greater than with surface footings. An example of coupled response is shown in Fig. 3 in which the rocking component of a coupled response involving horizontal translation, rocking and torsion is plotted. Embedment ratio δ varies from 0 (surface footing) to 1.0. The three natural frequencies of the surface footing are shown in dashed lines and denoted by α_1 , α_2 and α_3 . (Details on the analysis can be found in 7). The effect of embedment is particularly marked in the region of the first resonance and is very small in the region of the third resonance in which there is, in most cases, no appreciable resonant increase in amplitudes. The second resonance is dominated by torsion.

For coupled horizontal translation and rocking, comparison of the theory with field experiments indicated a much better agreement for embedded footings than for surface footings (4).

CONCLUSIONS

The approximate analytical solution makes it possible to predict forced vibrations of embedded footings in any vibration mode, including coupled modes, at negligible computing costs. The soil reactions can be defined in a way readily applicable in the analysis of structures. The approach agrees quite well with the finite element solutions and with field experiments except for torsional vibration with which the experiments indicate a need to incorporate the effects of slippage into the theory.

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BIBLIOGRAPHY

1. Awojobi, A.O., "Torsional Vibration of a Rigid Circular Body on an Infinite Elastic Stratum," *Int. J. of Solids and Structures*, 1969, Vol. 5, pp. 369-378.
2. Awojobi, A.O., "Vertical Vibration of a Rigid Circular Body and Harmonic Rocking of a Rigid Rectangular Body on an Elastic Stratum," *Int. J. of Solids and Structures*, 1972, Vol. 8, pp. 759-774.
3. Baranov, V.A., "On the Calculation of Excited Vibrations of an Embedded Foundation," (in Russian), *Voprosy Dynamiki i Prochnosti*, No. 14, Polytechnical Institute of Riga, 1967, pp. 195-209.
4. Beredugo, Y., Novak, M., "Coupled Horizontal and Rocking Vibration of Embedded Footings," *Canadian Geotechnical Journal*, November 1972.
5. Lysmer, J., Kuhlemeyer, R.L., "Finite Dynamic Model for Infinite Media," *J. of Engineering Mechanics Division, Proceedings ASCE*, Vol. 95, No. EM4, August, 1969, pp. 859-877.
6. Novak, M., Beredugo, Y., "Vertical Vibration of Embedded Footings," *J. of Soil Mechanics and Foundations Division, Proceedings ASCE*, Vol. 98, No. SM12, December, 1972, pp. 1291-1310.
7. Novak, M., Sachs, K., "Torsional and Coupled Vibrations of Embedded Footings," *Int. J. of Earthquake Engineering and Structural Dynamics*, J. Wiley & Sons (to appear).
8. Warburton, G.B., "Forced Vibration of a Body on an Elastic Stratum," *J. of Applied Mechanics*, March, 1957, pp. 55-58.
9. Wass, G., Lysmer, J., "Vibrations of Footings Embedded in Layered Media," *Proceedings of the WES Symposium on Applications of the Finite Element Method in Geotechnical Engineering*, U.S. Army Engineer Waterways Experiment Station, Vicksburg, Miss., May, 1972.

