THE INITIAL WAVE OF A TSUNAMI NEAR ITS SOURCE

by

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SYNOPSIS

Solutions based on a linearized description of wave motion are presented for the initial waves near the source region generated in a two and three-dimensional fluid domain of uniform depth by a deformation of the bottom. The "sea-bed" deformations are assumed to consist of a block section of the bed moving vertically (either up or down) to a position of permanent deformation; the time-displacement history of the bed motion is varied. Experiments have been conducted using a unique laboratory facility to determine the validity of the linear theory for the two-dimensional fluid domain.

INTRODUCTION

Tsunami hazards have become especially acute in recent years as a result of the increased land use in coastal regions. Predicting the potential tsunami hazard at a specific coastal site requires an understanding of the following processes: (1) generation of the tsunami, (2) propagation of the tsunami across the variable depth ocean, and (3) the response characteristics of the coastal region to the incident wave system. The study presented herein is an effort to provide additional insight into the tsunami generation process for certain idealized tectonic features of the source mechanism. Both a two and three-dimensional model of generation are examined theoretically using a linear theory. Experiments have been conducted for the two-dimensional model which enables the applicability of the linear theory, which is only an approximation to the complete (nonlinear) description of wave motion, to be determined. The general structure of the tsunami generated by the Alaskan earthquake of 1964 near the source region is hypothesized using the results of this study.

BED DEFORMATION MODELS

Although numerous tsunamigenic earthquakes have occurred, very little information is available in the literature regarding the tectonic deformations of the sea bed which are responsible for generating tsunamis. Because of this lack of information regarding prototype bed deformations, and to gain a more basic understanding of the generation process, only simple theoretical and experimental models of bed deformations are considered.

The Two-Dimensional Model – The first model of tsunami generation to be investigated consists of a two-dimensional fluid domain with a uniform depth, h, and of infinite extent in the direction of wave

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propagation. The fluid is assumed to be initially at rest. For time, t, greater than zero a block section of the bed, symmetric about x=0 (see Fig. 1a) and of half-width, b, is permitted to move vertically (either up or down) to a position $\zeta = \pm\zeta_o$. Two time-displacement histories, $\zeta=\zeta(t)$, of the bed motion are considered and are shown in Fig. 1b. The first bed motion is denoted as ζ_e in Fig. 1b and will be referred to as the exponential bed displacement. With this time-displacement history the block section moves from $\zeta=0$ to $\zeta = \pm\zeta_o$ in an asymptotic manner (a discontinuity in the bed velocity exists at t=0). The second bed motion of interest shown in Fig. 1b is denoted as ζ_s and will be referred to as the half-sine bed displacement. During this bed motion, the block section of the bed moves to a position of permanent deformation ($\zeta = \pm\zeta_o$) in a finite time, T (the bed velocity is a smooth function for all time).

The exponential and half-sine bed displacements can be characterized by three quantities: a characteristic amplitude, ζ_o, a characteristic size, b, and a characteristic time, t_c. An obvious choice of the characteristic time for the half-sine bed motion is the total time of the displacement, i.e., $t_c=T$. Since the exponential bed motion theoretically requires an infinite period of time before completion, a characteristic time has been chosen as the time required for two-thirds of the displacement to be completed, i.e., $t=t_c$ when $\zeta/\zeta_o=2/3$. (These characteristic times are identified on the abscissa in Fig. 1b.)

Experiments have been conducted in a unique laboratory facility consisting of a wave tank (31.6m long, 39.4 cm wide, and 61 cm deep) with a moveable section of the bottom adjacent to the upstream end of the tank. This section of the bed can be moved either upwards or downwards in a programmed motion by a hydraulic-servo mechanism (see Hammack (1972) for details).

The Three-Dimensional Model–Since the actual tsunami problem is inherently three-dimensional (3-D), it is also of interest to investigate, theoretically, certain aspects of a simple 3-D model of generation. In order to make a direct comparison of the resulting wave behavior between a two and a three-dimensional case, the following 3-D model of generation has been investigated: in a 3-D fluid domain of uniform depth, h, a circular section of the bed (radius, r_o) moves vertically (either up or down) through a distance ζ_o in accordance with the time-displacement history ζ_e, shown in Fig. 1b. The primary difference between the two and three-dimensional models of generation is the radial spreading of the wave energy during propagation in the latter.

THE GENERATION PARAMETERS

For the models of tsunami generation under consideration, the resultant wave properties are functionally related to five independent quantities which define the generation process. The quantities are the uniform water depth, h, the gravitational acceleration, g, and three characteristics of the bed motion: ζ_o, b or r_o, and t_c. It is well known from the methods of dimensional analysis that from these five quantities there exist three nondimensional parameters that characterize the generation process. Hammack (1972) has shown that the proper nondimensional parameters are: ζ_o/h which represents an amplitude scale,

b/h or r_o/h which represent a size scale, and $t_c\sqrt{gh}/b$ or $t_c\sqrt{gh}/r_o$ which incorporates the time as well as size scale of the bed motion. The quantity $t_c\sqrt{gh}$ is simply the distance a long gravity wave of small amplitude will travel in time t_c; hence, if the time-size ratio, $t_c\sqrt{gh}/b$ or $t_c\sqrt{gh}/r_o$, is much less than unity a major portion of the bed motion occurs before elevations (or depressions) of the water surface have an opportunity to propagate from the generation region. This results in an initial water surface deformation similar to the deformed bed. Motions of this type are termed impulsive. When $t_c\sqrt{gh}/b$ is much greater than unity the water surface elevations (depressions) have sufficient time to leave the generation region during the bed motion; hence, the displaced water volume is distributed over a larger region of the fluid domain resulting in initial waves of smaller amplitude. Bed motions of this type will be referred to as creeping; bed motions which occur such that the time-size ratio is about unity will be termed transitional.

PRESENTATION OF RESULTS

The results presented are primarily concerned with the wave profile at the boundary of the generation region, i.e., the initial wave at $x=b$ in the 2-D model (see Fig. 1a) and at $r=r_o$ for the 3-D case. One of the important characteristics of the initial wave propagating from the generation region is its maximum amplitude, η_o. Experimental and theoretical results for the variation of the maximum wave amplitude (which has been normalized by the total bed displacement, ζ_o) with the time-size ratio, $t_c\sqrt{gh}/b$, are presented in Fig. 2 for the 2-D model; Fig. 2a is for an exponential motion and Fig. 2b is for a half-sine motion. Theoretical results alone are presented for the 3-D model with an exponential bed motion and a size scale $r_o/h=12.2$; the abscissa should be interpreted as $t_c\sqrt{gh}/r_o$ for this result.

The variation of the relative wave amplitude, η_o/ζ_o, with the time-size ratio shown in Fig. 2 is similar for each size scale and bed motion investigated. For rapid motions of the bed such that $t_c\sqrt{gh}/b << 1$ (impulsive bed motions) the relative wave amplitude is maximum for each size scale and remains constant with decreasing time-size ratios. The maximum amplitude of the initial wave for these impulsive bed motions is equal to one-half of the total bed displacement for the larger size scales; as b/h decreases, the maximum wave amplitude also decreases. As the time-size ratio of the bed motion becomes very large, i.e., $t_c\sqrt{gh}/b >> 1$ (creeping bed motions), the relative wave amplitude, η_o/ζ_o, decreases and becomes inversely proportional to the time-size ratio.

Examination of the experimental results indicates that nonlinear effects based on the magnitude of the disturbance-amplitude scale, ζ_o/h, become significant for impulsive and transitional bed motions when $|\zeta_o/h| > 0.2$. No significant nonlinear effects are observed for creeping motions regardless of the magnitude of the disturbance-amplitude scale (which is unity for some experiments). In addition, Fig. 2 shows that the theoretical variation of the relative wave amplitude becomes independent of the size scale for each bed motion when $b/h > 6$.

The theoretical variation shown in Fig.2a for the 3-D model with $r_o/h=12.2$ is similar to the general behavior found for the 2-D model with $b/h=12.2$. However, the magnitude of η_o/ζ_o is smaller for the 3-D model at an equivalent time-size ratio.

In addition to the maximum wave amplitude, it is also of interest to examine the temporal structure of the initial waves resulting from these bed deformations. Typical wave profiles at x=b are shown in Fig.3a and 3b for impulsive, transitional, and creeping bed motions with an exponential and half-sine time-displacement-history, respectively; both experimental and theoretical profiles are shown for the 2-D model. Theoretical results alone are presented in Fig. 3c for the 3-D model with an exponential bed uplift.

For impulsive bed motions in the 2-D model (Figs.3a and 3b) the water surface elevation becomes maximum, remains at this position for an interval of time, and then returns to the still water level (SWL) and oscillates about this level. The initial wave resembles the permanent deformation of the bed and is affected only weakly by the time-displacement history of the bed. The time-displacement history of the bed becomes more important in determining the structure of the initial wave as the time-size ratio increases; this is especially obvious for creeping bed motions. In fact, the shape of the wave during a creeping motion appears to be proportional to the time-history of the bed velocity. The primary difference between the initial waves shown in Fig.3c for the 3-D model and the results for the 2-D model is the presence of large amplitude negative waves trailing the initial positive wave for an impulsive bed uplift.

For the complex wave profiles shown in Figs.3a and 3b no single period exists which describes the initial wave. Three times are required: (1) a rise time t_r at which the wave reaches its maximum displacement, (2) a fall time t_f at which the water level begins to return to the SWL, and (3) a nodal time t_n at which the water surface again reaches the SWL (all times being measured from t=0). A knowledge of these three times as well as the maximum wave amplitude, η_o, provides a reasonable description of the structure of the initial wave.

Experimental and theoretical results for the variation of each of these times (t_r, t_f, and t_n) as a function of the time-size ratio are shown in Fig.4 for the 2-D model with a half-sine bed displacement; the times have been normalized by the characteristic time of the bed motion, t_c, and are shown separately for each size scale (b/h). The rise-time ratio, t_r/t_c, approaches unity for impulsive bed motions and one-half for creeping bed motions. Since the nodal-time ratio, t_n/t_c, also approaches unity for $t_c\sqrt{gh}/b >> 1$, a symmetric wave results for creeping bed motions (see Fig.3b). The linear theory agrees reasonably well with the experimental data for the rise-time ratio regardless of the magnitude of the disturbance-amplitude scale; however, nonlinear effects do appear to be significant when $|\zeta_o/h| > 0.2$ for the nodal and fall-time ratio. Again it is important to note that the rise time and nodal-time ratios have become independent of the disturbance-size scale, b/h, for the larger size scales investigated. The difference $(t_n - t_f)/t_c$ decreases for impulsive bed motions as the size scale increases; hence, for a given time-size ratio, the rear portion of the wave becomes steeper.

CONCLUSIONS AND APPLICATION OF RESULTS

These results have established the importance of the generation parameters: ζ_o/h, b/h, and $t_c\sqrt{gh}/b$, in determining the profile of the initial wave. For impulsive bed motions ($t_c\sqrt{gh}/b << 1$) the initial wave resembles the final shape of the deformed bed; the exact time-displacement history of the motion appears to have minor effects on the resulting wave behavior. The wave profile for creeping bed motions ($t_c\sqrt{gh}/b >> 1$) is strongly dependent on the time-displacement history of the movement; in fact, the resulting waves resemble the time-history of the bed velocity. Nonlinear effects become significant for impulsive or transitional bed motions ($t_c\sqrt{gh}/b \leq 1$) when the total bed displacement exceeds approximately 20% of the water depth, i.e., for $|\zeta/h| > 0.2$. Nonlinear effects are not observed for creeping bed motions regardless of the magnitude of the disturbance-amplitude scale. The primary difference between the initial waves of the 2-D and analogous 3-D model appears to be in the formation of large amplitude negative waves during the bed uplift (or vice versa) in the 3-D model.

In order to extrapolate these conclusions to prototype phenomena, typical magnitudes of these generation parameters for tsunamigenic earthquakes are required. The most well documented tsunamigenic earthquake for which information of the tectonic deformations exists is the Alaskan earthquake of 27 March 1964; details of the tectonics for this earthquake have been presented by Plafker (1969). Hammack (1972) has shown from an examination of typical cross-sections for the sea bed uplift through the source region of the Alaskan tsunami (presented by Plafker) that the disturbance-amplitude scales of the bed deformation are small ($\zeta_o/h < 0.1$) and an approximate disturbance-size scale is given by $b/h \approx 450$. No instrument records exist which indicate the time-displacement history of the ground motion during this earthquake; however, based on measurements of an atmospheric gravity wave also generated by the Alaskan earthquake, Van Dorn (1964) suggests that the ground motion must have occurred in 2 to 6 minutes. Using characteristic times, t_c, of 2 and 6 minutes, b=50 mi, and h=600 ft, the time-size ratio is found to lie in the range $0.06 < t_c\sqrt{gh}/b < 0.18$.

If the approximate values of the generation parameters presented above are indeed typical of tsunamigenic earthquakes, the following conclusions may be stated: (1) since the bed motions are impulsive, the permanent spatial deformation of the sea bed and not the time-displacement history of its motion is of primary importance in determining wave behavior, (2) nonlinear effects are not significant in defining initial wave behavior, and (3) the maximum amplitude of the wave system propagating from the source region is equal to one-half the maximum bed displacement. If the spatial deformation of the sea bed consists primarily of uniform uplift or downthrow, the lead wave characteristics (η_o, t_r, t_f, t_n) can be approximated from the results presented in Figs. 2 and 4 once estimates are made of ζ_o, b, h, and t_c. The results presented in these figures for b/h=100 can be used for larger size scales, since the characteristics of the lead wave were found to be insensitive to increases in size scale for b/h > 12.2.

As a first approximation Hammack (1972) has applied these results to determine the characteristics of the initial wave near the seaward boundary of the source region (near Middleton Island) of the tsunami generated by the Alaskan earthquake of 1964. For time-size ratios of 0.06 and 0.18 the following range of the temporal characteristics for the initial wave are found from Fig. 4: $2 < t_r < 6$ mins, $64 < t_f < 66$ mins, $68 < t_n < 72$ mins. The large nodal time is in general agreement with the measured nodal period of the lead wave of the Alaskan tsunami found at Wake Island (see Van Dorn (1964)). For an impulsive bed motion such as this, Fig. 2 implies that the amplitude of the lead wave leaving the generation region would be of the order of one-half of the maximum displacement of the bed. Hence, from Plafker's (1969) results the amplitude of the lead wave should be about 8 ft in the region of Middleton Island.

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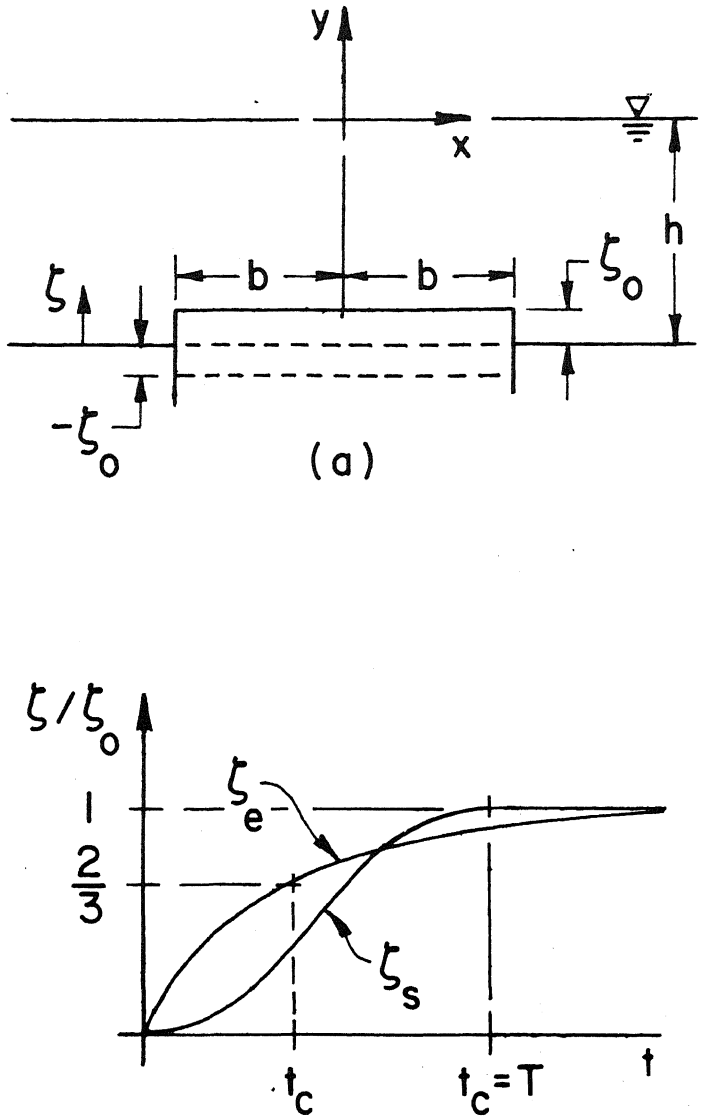

$$\zeta_e(t) = \zeta_o(1 - e^{-\alpha t})$$

$$\zeta_s(t) = \begin{cases} \zeta_o\left[\frac{1}{2}\left(1 - \cos\frac{\pi t}{T}\right)\right], & t \leq T \\ \zeta_o & , t > T \end{cases}$$

(b)

Fig. 1 a) Definition sketch of the bed deformation. b) Time-displacement histories of the bed motion.

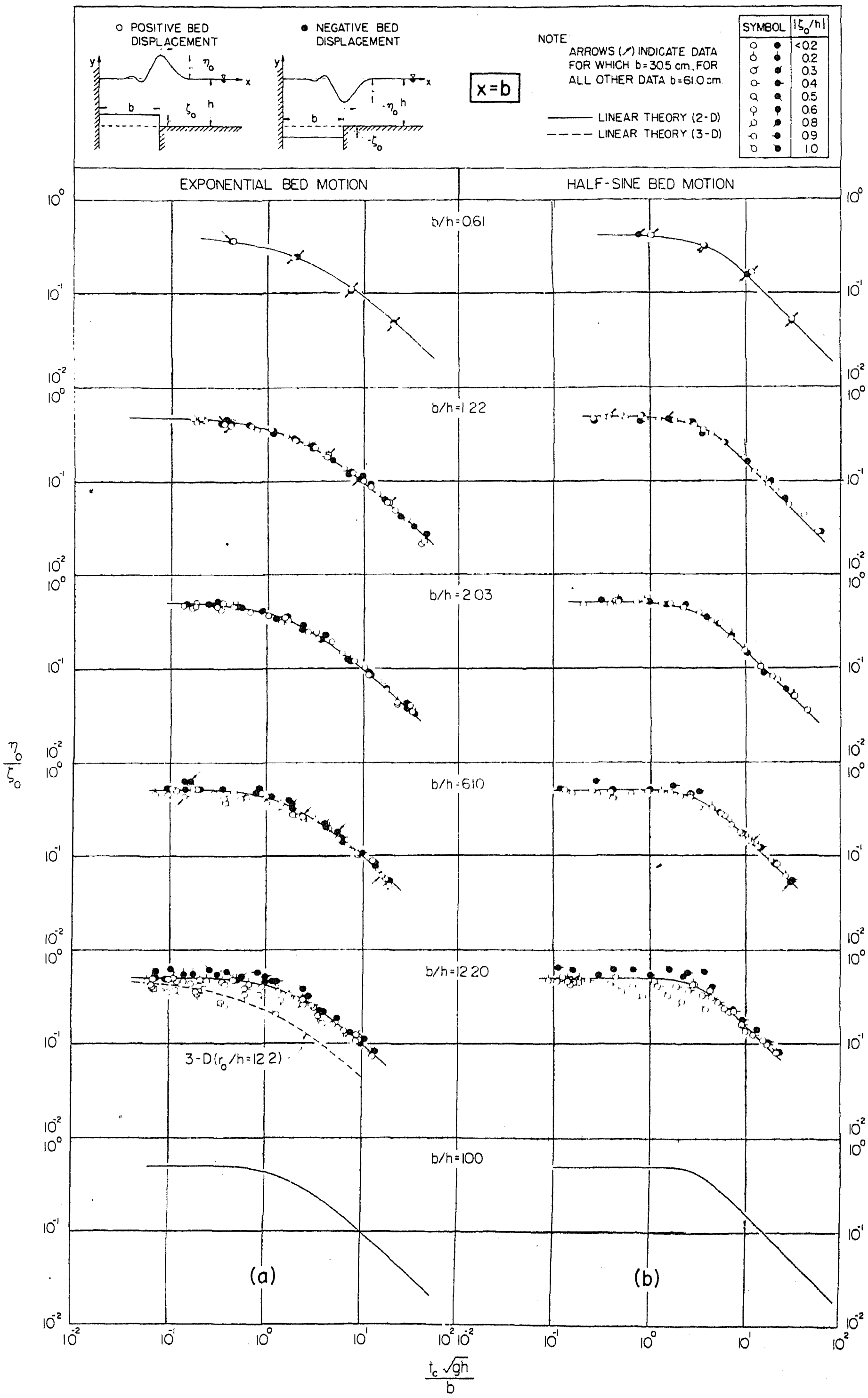

Fig. 2 Variation of the relative wave amplitude, η_o/ζ_o, with the time-size ratio, $t_c\sqrt{gh}/b$, at x=b; a) exponential bed motion, b) half-sine bed motion.

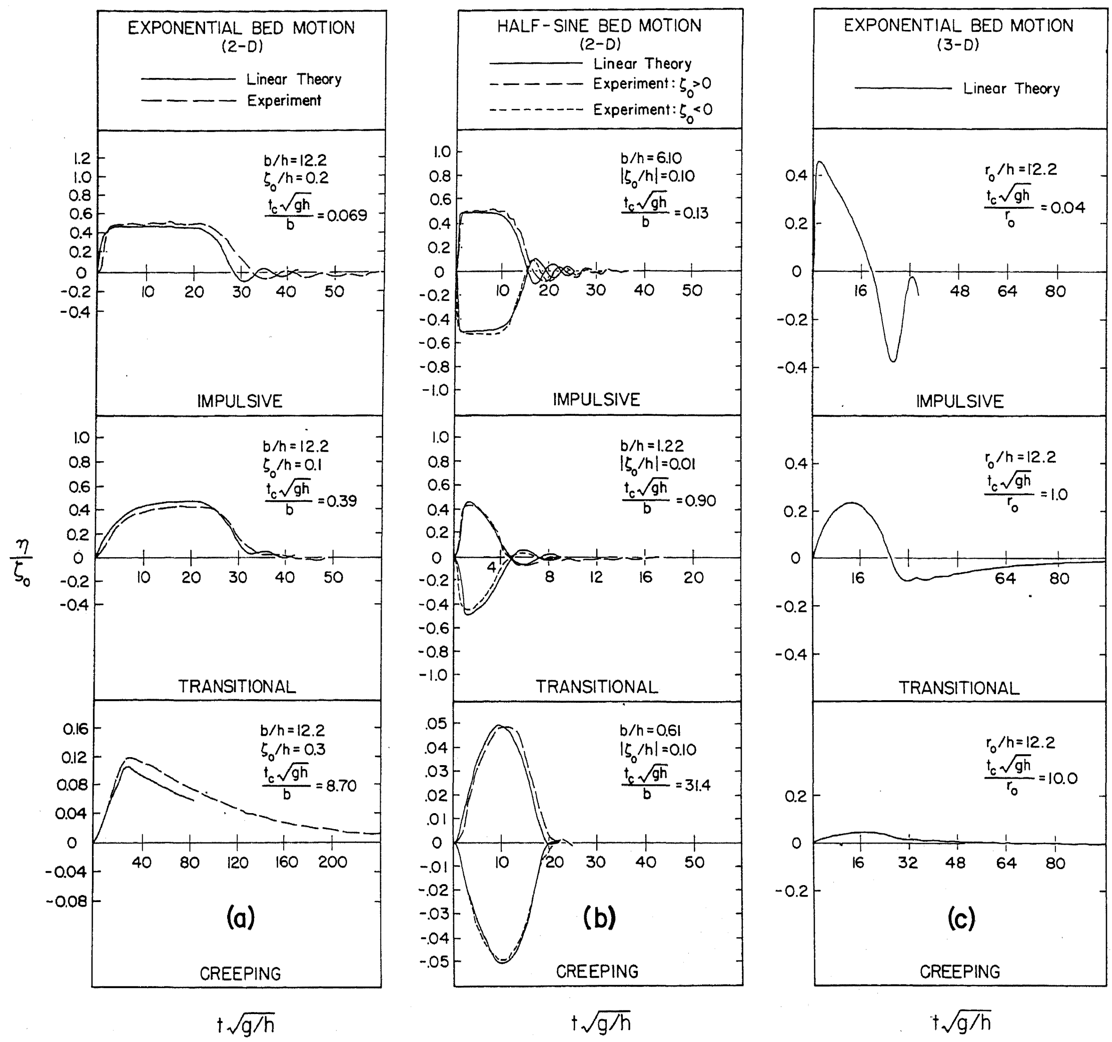

Fig. 3 Typical wave profiles at x=b for impulsive, transitional, and creeping bed motions; a) exponential motion (2-D), b) half-sine motion (2-D), c) exponential motion (3-D).

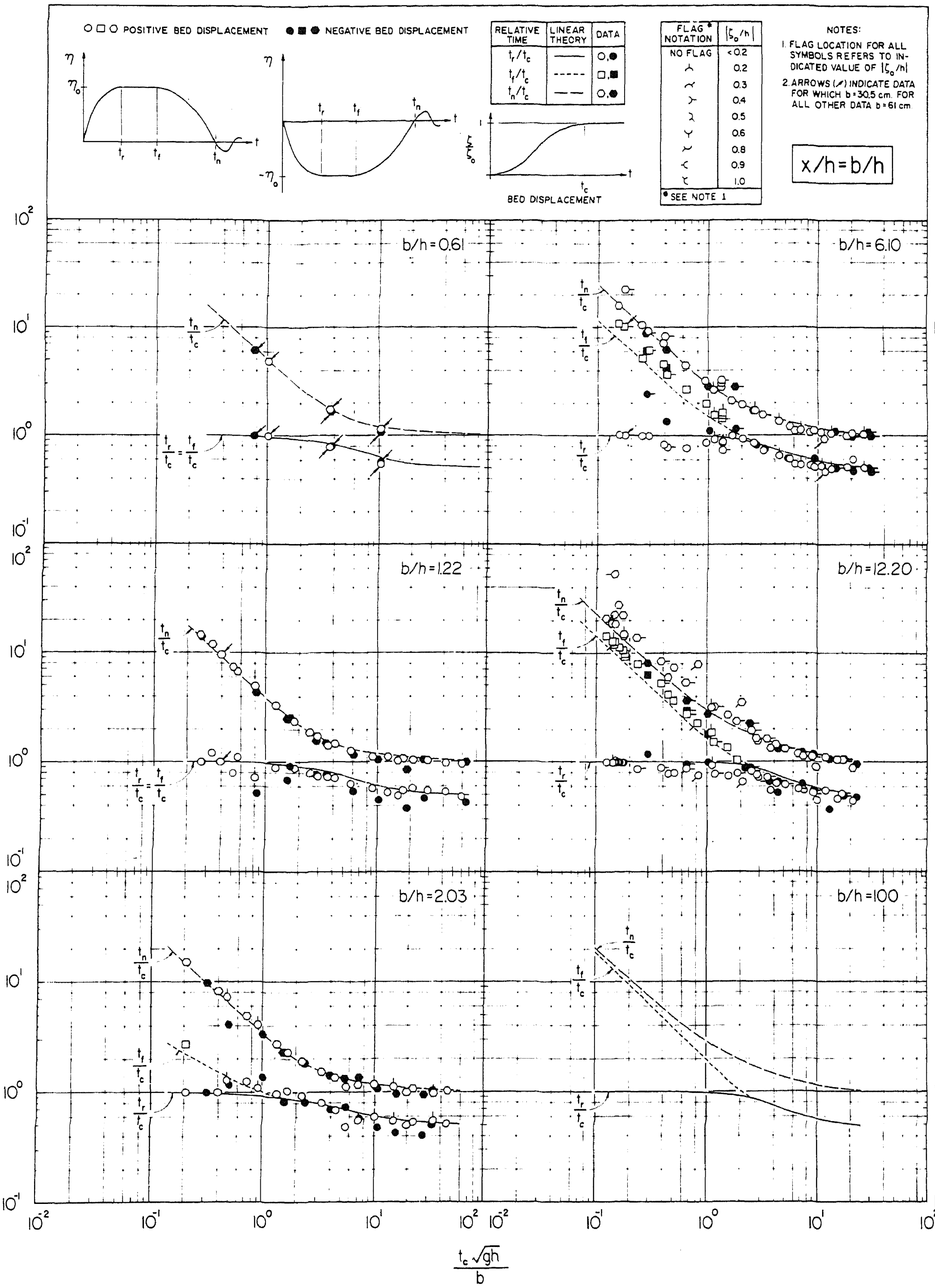

Fig. 4 Variation in t_r/t_c, t_f/t_c, and t_n/t_c of the lead wave with the time-size ratio, $t_c\sqrt{gh}/b$, at $x/h=b/h$ for half-sine bed motion.