

EARTHQUAKE RESPONSE SPECTRA OF SYSTEMS
PROVIDED WITH NONLINEAR AUXILIARY MASS DAMPERS *

by

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SYNOPSIS

Results of an analytical and experimental investigation, employing analog and digital computers, into the effectiveness of various types of passive nonlinear auxiliary mass dampers in controlling the response of structures to earthquake-like excitations are presented. Properly designed dampers of this type are found to be effective in controlling the response of primary systems even with a small mass ratio.

GLOSSARY

- m_i = i^{th} concentrated mass, $i = 1, 2$;
- k_i = i^{th} linear spring constant, $i = 1, 2, 3$;
- c_i = i^{th} viscous damping coefficient, $i = 1, 2, 3$;
- ζ_i = $c_i / (2\sqrt{k_i m_i})$ = fraction of critical damping for m_i , $i = 1, 2$,
 $\zeta_3 = c_3 / (2\sqrt{k_3 m_2})$;
- ω_i = $\sqrt{k_i / m_i}$ = natural frequency of m_i , $i = 1, 2$, $\omega_3 = \sqrt{k_3 / m_2}$
- x_i = absolute displacement of m_i , $i = 1, 2$;
- z_i = $x_i - y$ = relative displacement of m_i with respect to base;
- $\ddot{y}(t)$ = absolute base acceleration;
- T_i = $2\pi / \omega_i$ = natural period, $i = 1, 2, 3$;
- μ = m_2 / m_1 = damper mass ratio;
- d = clearance of deadspace zone related to k_2 and c_2 , (ft);
- S_v = maximum relative velocity response spectrum of primary mass m_1 , (ft/sec);

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S_d = maximum relative displacement response spectrum of primary mass m_1 , (ft).

INTRODUCTION

The model of the system under discussion is shown in Fig. 1. It consists of a base-excited viscously-damped single-degree-of-freedom (SDOF) primary mass m_1 that is coupled to an auxiliary mass m_2 by means of a spring k_3 and dashpot c_3 in addition to a nonlinear element whose restoring force and damping characteristics are given by functions g and h , respectively.

The equation of motion for the system of Fig. (1) is

$$\ddot{z}_1 = -\ddot{y} - \omega_1^2 z_1 - 2\zeta_1 \omega_1 \dot{z}_1 + \mu Q \quad (1)$$

$$\ddot{z}_2 = -\ddot{y} - Q$$

where $Q = \omega_3^2 u + 2\zeta_3 \omega_3 \dot{u} + \omega_2^2 g(u) + 2\zeta_2 \omega_2 h(u, \dot{u})$

and $u = z_1 - z_2$.

By proper adjustment of its parameters, this system can be made to represent the conventional form of the dynamic vibration neutralizer (DVN), the Lanchester damper, the impact damper (ID), a combination of these dampers, or various types of dampers with arbitrary nonlinearities.

The multi-degree-of-freedom system under discussion was subjected to actual earthquake ground motion (El Centro, 1940; Taft, 1952; San Fernando, 1972) as well as to artificial earthquakes. Digital as well as electronic analog computers were used to solve the governing differential equations of motion (1). In addition to obtaining the exact relative velocity and relative displacement spectra of both masses that constitute the system, the root-mean-square value of the response and the probability distribution of the relative velocity peaks and the relative displacement peaks of both masses were determined.

DISCUSSION OF RESULTS

The effects of all system parameters were determined for a wide range of practical values. Typical findings of this investigation are presented graphically in the form of standard relative velocity and relative displacement response spectra as well as in the form of frequency histograms for peak distribution (see Figs. 2-7).

Results indicate that, in general, auxiliary mass dampers are less effective in reducing the peak response of earthquake excitation, than they are in the case of steady-state deterministic excitation or stationary random excitation. Among the class of conventional auxiliary mass dampers, Lanchester dampers (which rely exclusively on mechanical energy dissipation) are the least effective in attenuating the re-

sponse of earthquake-excited primary systems; the dynamic vibration neutralizer is moderately effective; and the impact damper (which employs the principle of momentum transfer) is the most effective. However, in regard to reducing the peak response of structures subjected to earthquake-like excitation, a nonlinear damper combining features that are characteristic of each of the above mentioned dampers was found to be significantly more effective than the aforementioned dampers in reducing the velocity response as well as the displacement response spectra level over a wide range of system parameters.

It was found that even when auxiliary mass dampers have moderate or slight effect on the maximum amplitude of the response, they still reduce the overall root-mean-square response level substantially, and they "contract" and shift the amplitude probability density curve to significantly lower levels.

The passive nonlinear dampers under discussion are moderately effective even when their mass ratio is on the order of a few percent of the primary mass. Also, in addition to being easy to design and maintain, their performance is relatively insensitive to slight variations of system parameters.

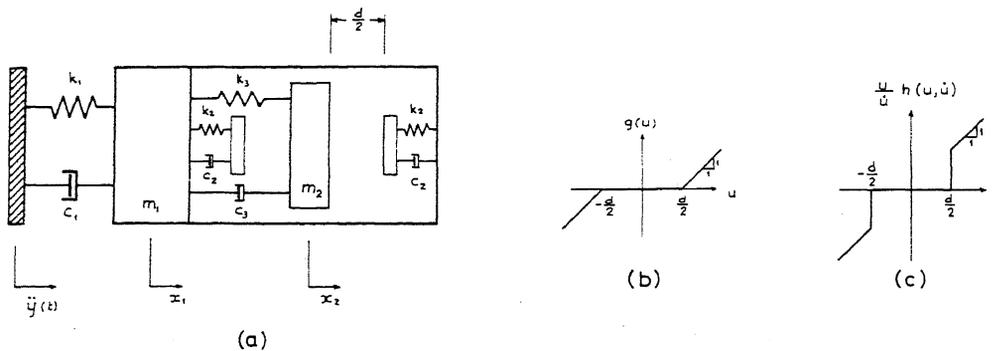
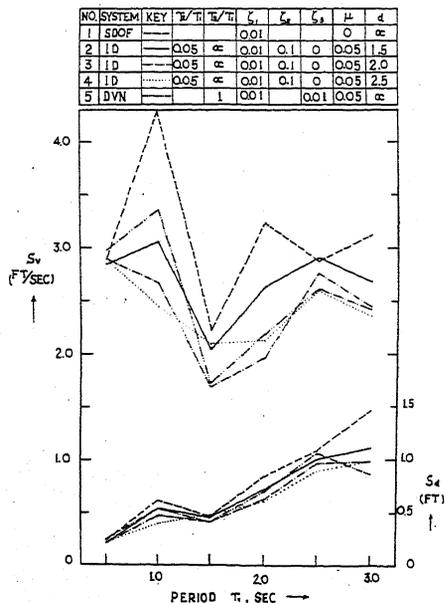


FIG. 1. MODEL OF SYSTEM



2. RESPONSE SPECTRA OF A SDOF SYSTEM WITH AN IMPACT DAMPER; (EL CENTRO, 1940)

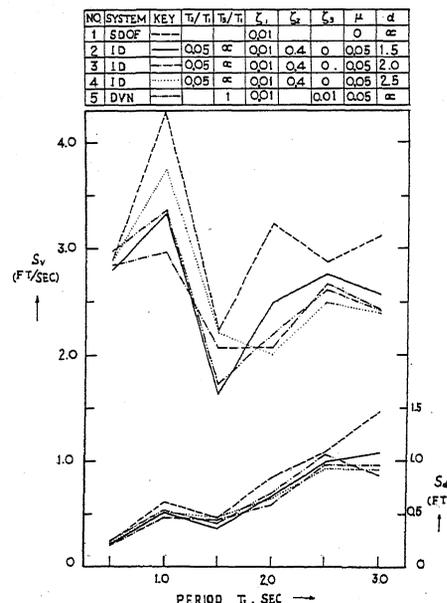


FIG. 3. RESPONSE SPECTRA OF A SDOF SYSTEM WITH AN IMPACT DAMPER; (EL CENTRO, 1940)

NO	SYSTEM	KEY	T_2/T_1	T_3/T_1	ζ_1	ζ_2	ζ_3	μ	d
1	SDOF	---			0.02			0	0.8
2	DVN	---	1		0.02		0.01	0.05	0.8
3	DVN	---	1		0.02		0.10	0.05	0.8
4	DVN	---	1		0.02		0.25	0.05	0.8
5	DVN	---	1		0.02		0.50	0.05	0.8

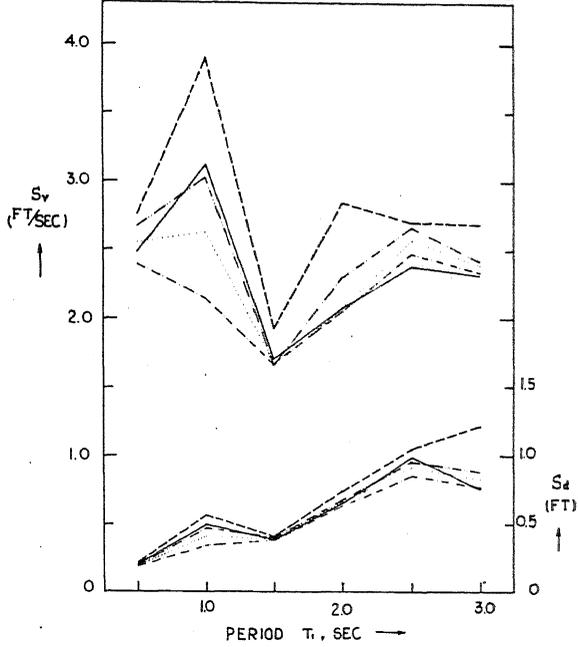


FIG. 4. RESPONSE SPECTRA OF A SDOF SYSTEM WITH A DYNAMIC VIBRATION NEUTRALIZER; (EL CENTRO, 1940)

NO	SYSTEM	KEY	T_2/T_1	T_3/T_1	ζ_1	ζ_2	ζ_3	μ	d
1	SDOF	---			0.02			0	0.8
2	ID	---	0.5		0.02	0.1	0	0.025	1.5
3	ID	---	0.5		0.02	0.1	0	0.05	1.5

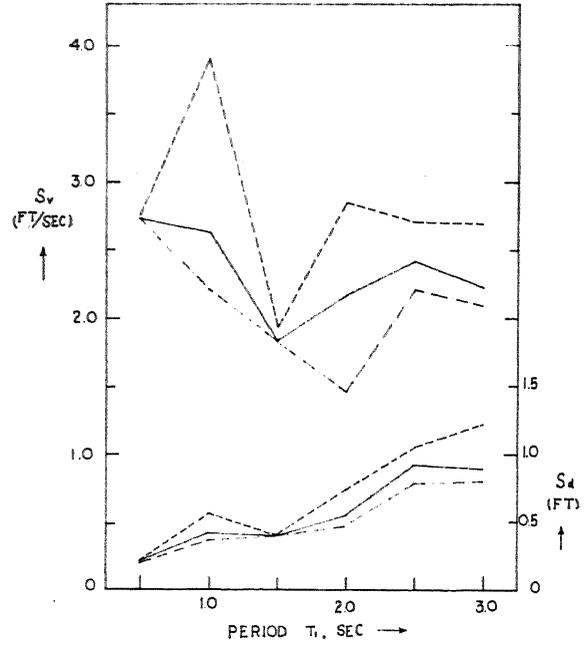


FIG. 5. RESPONSE SPECTRA OF A SDOF SYSTEM WITH A NONLINEAR DAMPER; (EL CENTRO, 1940)

NO	SYSTEM	KEY	T_2/T_1	T_3/T_1	ζ_1	ζ_2	ζ_3	μ	d
1	SDOF	---			0.05			0	0.8
2	ID	---	0.5		0.05	0.1	0	0.025	1.5
3	ID	---	0.5		0.05	0.1	0	0.05	1.5

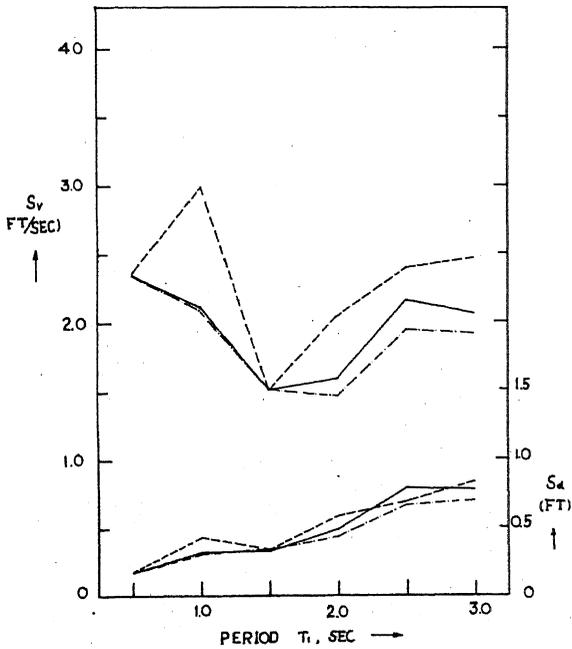


FIG. 6. RESPONSE SPECTRA OF A SDOF SYSTEM WITH A NONLINEAR DAMPER; (EL CENTRO, 1940)

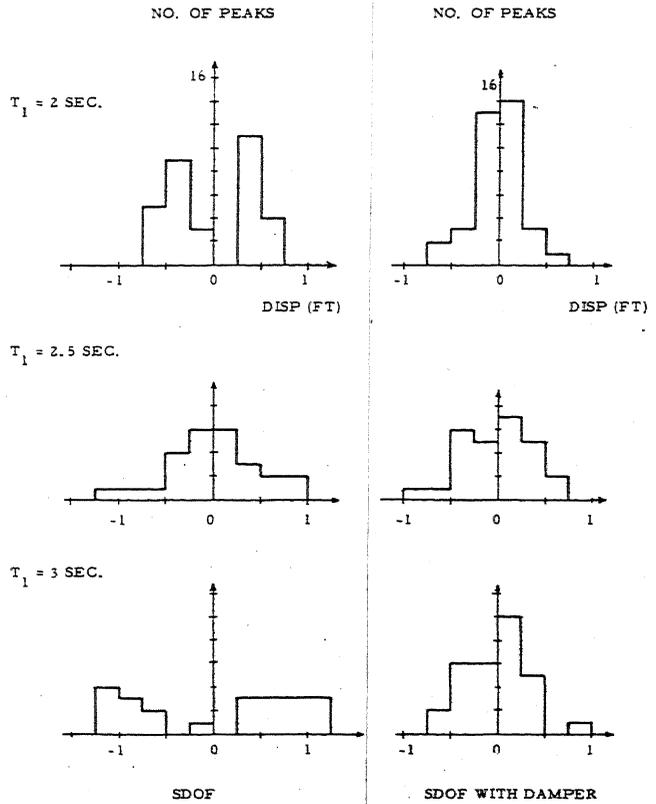


FIG. 7. HISTOGRAM OF PEAK DISTRIBUTION; (EL CENTRO, 1940)