OPTIMAL DESIGNING OF TOWER STRUCTURES UNDER SEISMIC ACTION

by

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SYNOPSIS

The present paper concerns a cantilever under seismic action. The basic equations of a variable cross-section and variable mass along the height of the cantilever are given hereto. The solution of the non-linear equation is sought through successive approximation; firstly it is assumed the bending rigidity to be uniform and the optimal shape of the cantilever is determined at the first and at the second mode of vibration, proportionally to the second and the third derivative of the deflection.

INTRODUCTION

The optimal designing of tower structures is the problem of the day. It is of special importance when designing structures in seismic areas. The different modes of vibration have to be taken into consideration.

The following features have to be defined when considering the question of the optimal designing: material properties and properties of the structure and the stages of its work - elastic, viscoelastic or elastoplastic; the most probable seismic ground motion and the properties of the soil; optimization of the structure and so on.

In the case of tower structures we choose the criterion of the strength capacity according to the bending moments and the shear forces.

The theoretical model of a tower structure is accepted to be a cantilever fixed on a fundament with uniformly distributed mass under a definite law (fig. 1).

Furthermore we assume that the structural material is homogeneous,

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isotropic and elastic; the vibrations are small and $\varphi << 1$

The equations of motion of the structure in horizontal and vertical direction and of bending are:

$$(1)\quad \begin{aligned} &\frac{\partial Q(z,t)}{\partial z} + N(z,t)\frac{\partial^2 u(z,t)}{\partial z^2} + \frac{\partial N(z,t)}{\partial z}\frac{\partial u(z,t)}{\partial z} + \rho A(z)\frac{\partial^2 (u_0(t)+u(z,t))}{\partial t^2} = 0;\\ &\frac{\partial N(z,t)}{\partial z} - Q(z,t)\frac{\partial^2 u(z,t)}{\partial z^2} - \frac{\partial Q(z,t)}{\partial z}\frac{\partial u(z,t)}{\partial z} + \gamma A(z) + \rho A(z)\frac{\partial^2 w(z,t)}{\partial t^2} = 0;\\ &\frac{\partial M(z,t)}{\partial z} - Q(z,t) = 0; \end{aligned}$$

After some simplifications and using the geometric and constitutive equations we obtain a general equation of the following form:

$$(2)\quad \begin{aligned} &\frac{\partial^2}{\partial z^2}\left[EJ(z)\frac{\partial^2 u(z,t)}{\partial z^2}\right] - \left[\gamma A(z) - \rho A(z)\frac{\partial^2 w(z,t)}{\partial t^2}\right]\frac{\partial u(z,t)}{\partial z}\\ &-\int_z^{\ell-z}\left[\gamma A(z) - \rho A(z)\frac{\partial^2 w(z,t)}{\partial t^2}\right]dz + \rho A(z)\partial^2 u(z,t) = \rho A(z)\frac{\partial^2 u_0(t)}{\partial^2 t} \end{aligned}$$

Let us consider that equation in its simplified form:

$$(3)\quad \frac{\partial^2}{\partial z^2}\left[EJ(z)\frac{\partial^2 u(z,t)}{\partial z^2}\right] + \rho A(z)\frac{\partial^2 u(z,t)}{\partial t^2} = \rho A(z)\frac{\partial^2 u_0(t)}{\partial t^2};$$

where

$$A(z) = \pi r^2 = A_0\frac{\partial^2 u(z,t)}{\partial z^2}$$

$$J(z) = \frac{\pi}{4}r^4 = cA_0^2\left(\frac{\partial^2 u(z,t)}{\partial z^2}\right)^2$$

For the case of free vibration equation (2) has homogeneous form;

$$(4)\quad EcA_0\frac{\partial^2}{\partial z^2}\left[\left(\frac{\partial^2 u(z,t)}{\partial z^2}\right)^2\frac{\partial^2 u(z,t)}{\partial z^2}\right] + \frac{\partial^2 u(z,t)}{\partial z^2}\;\frac{\partial^2 u(z,t)}{\partial t^2} = 0$$

It is non-linear partial differential equation. Its solution is obtainable through successive approximation.

We base on the condition that EJ = const. and A = const. and use the known equation

$$(5)\quad EJ\frac{\partial^4 u(z,t)}{\partial z^4} + \rho A\frac{\partial^2 u(z,t)}{\partial t^2} = 0 \quad\text{or}\quad \frac{EJ}{m}\frac{\partial^4 u(z,t)}{\partial z^4} + \frac{\partial^2 u(z,t)}{\partial t^2} = 0$$

The particular solution of this differential equation is presented in the following form:

(6) $$u(z,t)=Z_k(z)\,T_k(t),$$

where

(7) $$Z_k(z)=C_1S(\alpha z)+C_2T(\alpha z)+C_3U(\alpha z)+C_4V(\alpha z);$$

$S(\alpha z), T(\alpha z), U(\alpha z), V(\alpha z)$ are Crilov's functions;

p_k - natural frequency of vibration

The successive derivatives of the $Z(z)$ function are the following:

$$Z'(z)=\alpha\left[C_1V(\alpha z)+C_2S(\alpha z)+C_3T(\alpha z)+C_4U(\alpha z)\right];$$

(8) $$Z''(z)=\alpha^2\left[C_1U(\alpha z)+C_2V(\alpha z)+C_3S(\alpha z)+C_4T(\alpha z)\right];$$

$$Z'''(z)=\alpha^3\left[C_1T(\alpha z)+C_2U(\alpha z)+C_3V(\alpha z)+C_4S(\alpha z)\right]$$

The constants C_i are defined by the following boundary conditions:

at the fixed end $Z(0)=0$; $Z'(0)=0$;

at the free end $Z=\ell$; $Z''(\ell)=0$; $Z'''(\ell)=0$;

Substituting these boundary conditions in the expressions (7, 8) it is obtained:

(9) $$\begin{aligned}&C_1=0; \quad C_2=0;\\ &C_3S(\alpha\ell)+C_4T(\alpha\ell)=0;\\ &C_3V(\alpha\ell)+C_4S(\alpha\ell)=0;\end{aligned}$$

The condition for nonzero solutions for the constants C_3 and C_4 is

$$\begin{bmatrix} S(\alpha\ell); & T(\alpha\ell)\\ V(\alpha\ell); & S(\alpha\ell)\end{bmatrix}=0$$

By these constants the equation of the normal mode of vibration and the corresponding derivatives are:

$$Z(z)=(\operatorname{sh}\alpha\ell+\sin\alpha\ell)(\operatorname{ch}\alpha z-\cos\alpha z)-(\operatorname{ch}\alpha\ell+\cos\alpha\ell)(\operatorname{sh}\alpha z-\sin\alpha z);$$

$$Z'(z)=\alpha\left[(\operatorname{sh}\alpha\ell+\sin\alpha\ell)(\operatorname{sh}\alpha z+\sin\alpha z)-(\operatorname{ch}\alpha\ell+\cos\alpha\ell)(\operatorname{ch}\alpha z-\cos\alpha z)\right];$$

$$Z''(z)=\alpha^2\left[(\operatorname{sh}\alpha\ell+\sin\alpha\ell)(\operatorname{ch}\alpha z+\cos\alpha z)-(\operatorname{ch}\alpha\ell+\cos\alpha\ell)(\operatorname{sh}\alpha z+\sin\alpha z)\right];$$

$$Z'''(z) = \alpha^3[(\operatorname{sh}\alpha l + \sin\alpha l)(\operatorname{sh}\alpha z - \sin\alpha z) - (\operatorname{ch}\alpha l + \cos\alpha l)(\operatorname{ch}\alpha z + \cos\alpha z)];$$

The respective diagrams for the case in question - fig.2 are determined by these expressions and at $\alpha_1 l = 1{,}875$ and $\alpha_2 l = 4{,}694$

The cross-section $A(z)$, respectively $\tau(z)$ could be obtained from the requirement of satisfying strength capacity, taking into account the bending moments $M(z)$ and the shear forces $Q(z)$. In view of that requirement the obtained forms are also shown on fig.2. At the second mode of vibration it occurs that the cantilever must be thicker at the upper end where usually the fracture of a smoke tube occurs. It is interesting to note that such thickness appears just on that part of some palm trees.

Using the obtained expression for $A(z)$ as a first approximation, the second approximation could be found.

References:

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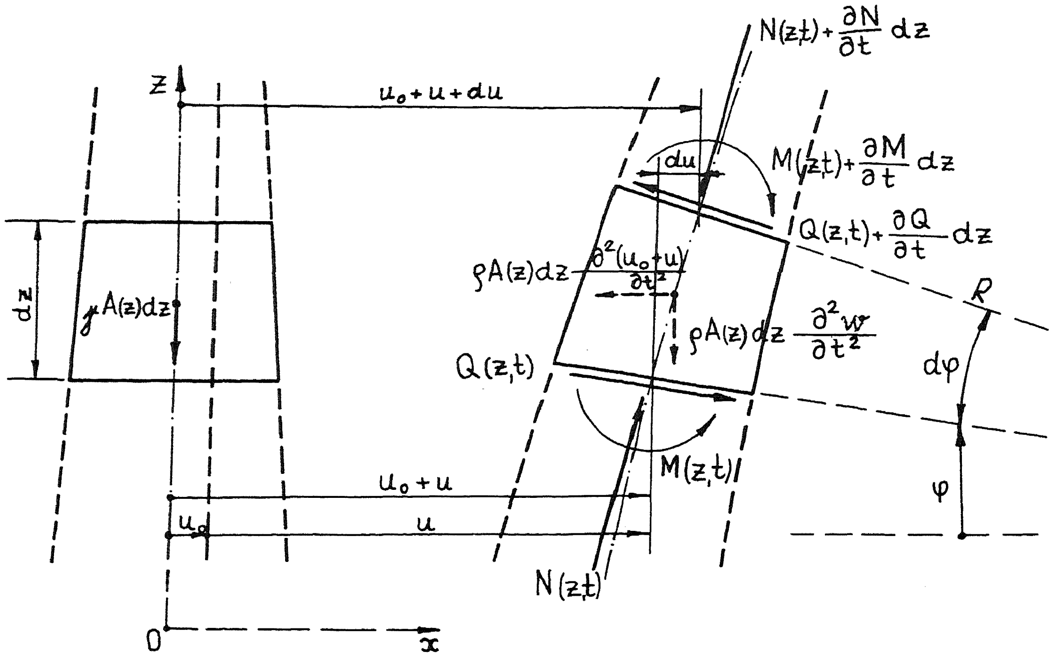
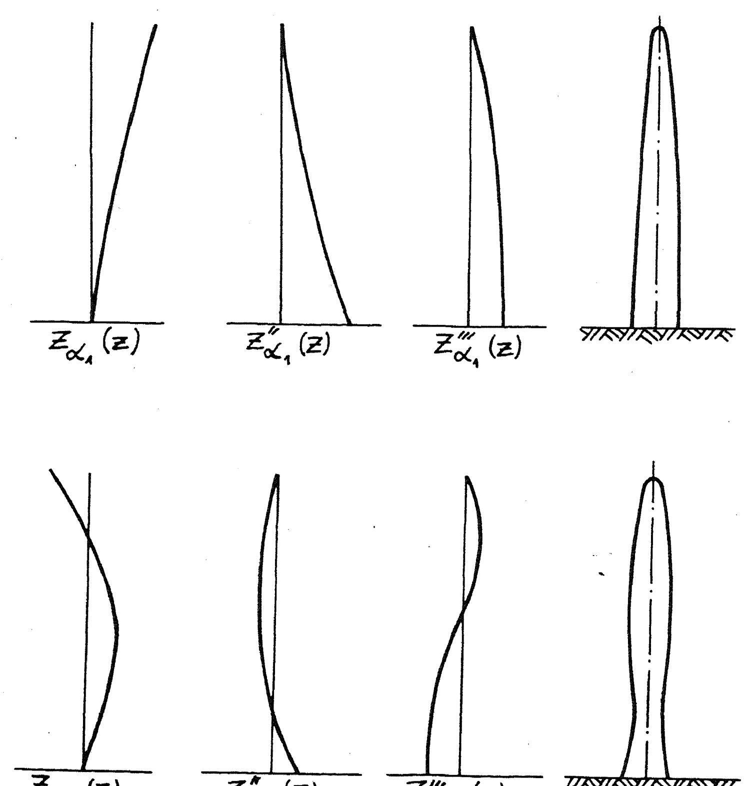


$N(z,t)+\frac{\partial N}{\partial t}dz$
u_o+u+du
du
$M(z,t)+\frac{\partial M}{\partial t}dz$
$Q(z,t)+\frac{\partial Q}{\partial t}dz$
dz
$\gamma A(z)dz$
$\rho A(z)dz\,\frac{\partial^2(u_o+u)}{\partial t^2}$
$\rho A(z)dz\,\frac{\partial^2 w}{\partial t^2}$
R
$Q(z,t)$
$d\varphi$
$M(z,t)$
u_o+u
u_o
u
φ
$N(z,t)$
0
x
z

Fig. 1



$Z_{\alpha_1}(z)$
$Z''_{\alpha_1}(z)$
$Z'''_{\alpha_1}(z)$
$Z_{\alpha_2}(z)$
$Z''_{\alpha_2}(z)$
$Z'''_{\alpha_2}(z)$

Fig. 2