

OPTIMIZATION OF FRAMED STRUCTURES  
UNDER EARTHQUAKE LOADS

by  
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SYNOPSIS

Automatic, computerized design of plane frames for different combinations of vertical and horizontal loads, yielding optimum stiffness and weight distribution, may be obtained using mathematical programming techniques. In the case of earthquake loading, however, difficulties arise because of period-dependency of the earthquake design loads. The design variables define the stiffness of the frame and hence its period, whereby the design earthquake loads need to be redefined in each step. In this study, a stepwise, iterative design process, applying linear programming techniques, is described. The solution yields a set of design values for the cross section areas of all the frame elements which minimizes the steel weight of the frame and simultaneously gives optimum stiffness distribution for the frame to resist the prescribed earthquake loads. The elements of the analytical and numerical methods are reviewed and a numerical example is given.

ANALYSIS

To recapitulate the mathematical programming formulation of any structural frame design problem, consider a structure which is fully determined by the  $n$ -dimensional design vector

$$X = \{X_1, \dots, X_n\} \dots \dots \dots (1)$$

The design variables  $\{X_i\}$  may be taken as a parameter description of each structural element (e.g. area, moment of inertia and so on) or a description of different features such as the degree of fixation of element ends and so on.

Once all the design variables have been assigned values, there exists one complete structure and its weight or even the total cost of erection can be expressed as a function (to be minimized)

$$Z(X) \rightarrow \text{minimum} \dots \dots \dots (2)$$

termed the object function of the problem.

The admissible values for the design variables are to be located in the design space which is defined as follows.

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The deflexion of the frame and the maximum stresses in the frame must be within the acceptable limits, set by design philosophy and material strength. This will result in the following inequalities or constraints.

The deflexion or drift limitations

$$|Y(X)| \leq YD \dots \dots \dots (3)$$

where Y are the critical deflexion values to be checked and YD is the corresponding vector of the admissible values

$$S(X) \leq SD \dots \dots \dots (4)$$

Eq.(4) refers to the stress constraints where S(X) is the matrix of critical stresses sought after among the critical sections of the frame and SD is the corresponding matrix of the allowable stresses.

The constraints in Eq.(3) and (4) together define subspace, the design space, in the n-dimensional variable space. The boundaries of the design space are the above constraints treated as equalities which corresponds to n-dimensional hypersurfaces. The optimum design vector is thus located at a boundary point of the design space at which the function, Eq.(2), is minimum.

The constraints of Eqs.(3) and (4) are because of the effect of all possible load combinations the structural frame is to accomodate. Here, the design philosophy being studied amounts to determining the optimum values of the design variables so that all load combinations can be withstood through linear deformation. Therefore, the design earthquake forces are selected as suitable for elastic behaviour. However, the thus determined optimum structure can afterwards be checked for non-linear behaviour because of more realistic earthquake forces and possibly put through a dynamic analysis.

#### NUMERICAL EXAMPLE

A two bay, 30 storey steel frame, was selected for analysis (Fig.1). Modified Californian SEAOC design loads are taken as the lateral earthquake loads and moreover, the maximum deflexion of the top storey is restricted to be 1/300 the total height of the frame.

It is noteworthy, that these two effects are contradictory with respect to the frame stiffness. The earthquake loads call for more flexible design, whereas the deflexion constraint demands more rigid design. Ostensibly, the optimization process selects a design which yields the best compromise available.

The numerical procedure now starts with selecting an initial design vector  $X^1$  which, in the present case, consists of the cross section areas of all the elements. The global optimum, which yields the lowest admissible value of the object function within the design space, is then sought through a step-by-step approach which consists of solving a linear programming problem for each step.

The original mathematical programming problem (Eqs.(2)-(4)) is, in general, strongly non-linear. Therefore, for each step the functions in Eqs.(2)-(4) are linearized by a first order Taylor approximation. This involves the numerical computation of the left-hand sides of Eqs.(3) and (4) which in turn requires the computation of the fundamental period, the force envelope, the mass matrix and the local and global stiffness matrices. Further, the partial derivatives of all these quantities with respect to all the design variables are needed. Hereafter, it is easy to formulate a linear programming problem which gives the optimum incremental vector  $\Delta X$  which results in maximum decrease of the object function. More details about the computational process can be found in a recent report by the authors [1].

Now, the structural frame of Fig.1 may consist of 90 different elements, requiring overwhelming computational effort and cost. The frame is therefore optimized in stages as follows: In the first stage, the storeys are grouped 6 at a time, regarding the columns, whereas all the beams are equal. Because of symmetrical design, there are only two different column types and thus with 11 design variables a crude weight and stiffness optimization is obtained.

In the following stage, the storeys are grouped 3 at a time regarding the columns and 6 at a time regarding the beams, thus allowing for a total of 25 design variables. Starting with the results of the preceding stage the optimization process now finds a better optimum design.

The technique of stage optimization, each new stage allowing a larger number of individual variables, may be continued until a sufficiently detailed design has been reached.

The changes in steel volume and largest natural period through 3 stages of optimization are respectively: 12.06 m<sup>3</sup> - 9.83 m<sup>3</sup> - 9.20 m<sup>3</sup> and 3.77 sec - 4.60 sec - 4.80 sec.

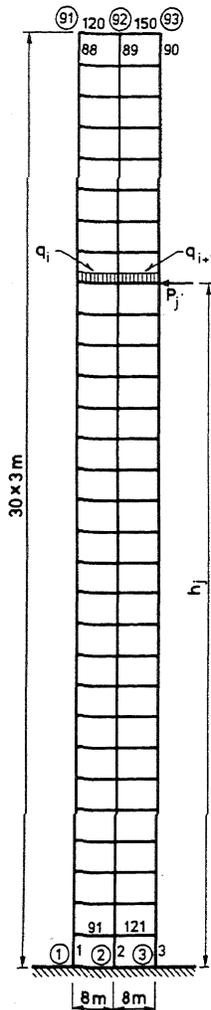
The distributions of section moduli in the optimum structure, for which the continuous design values have been assigned IPE or HE-B profile values, is shown in Fig.2 while Fig.3 displays the profile numbers. Fig.4 shows the deflection of the frame exposed to the modified SEAOC design loads.

The computer processing time, of course, is large. The first stage took about 10 minutes CPU-time on a IBM 370/165 computer, and the following stage needed about 20 minutes CPU-time for full processing. However, taking into account the information obtained, the computer costs are negligible.

#### REFERENCES

- [1] Solnes, J. And Holst, O. Weight Optimization of Framed Structure under Earthquake Loads. Report No.33/1972, Struct. Res. Laboratory, Techn. Univ. Denmark.

FIG. 1. 30 STOREY STEEL FRAME: LOADS.



**LOAD CASE NO. 1**

$q_i = 25 \text{ kN/m}^*$  ( $i=91,160$ )  
 $P_j = 0$  ( $j=1,93,3$ )

**Allowable stresses:**  
 Beams: (IPE profiles)  
 $r_{\text{bending}} = 1.57 \text{ kN/m}^2$   
 $r_{\text{compression}} = 1.25 \text{ kN/m}^2$

Columns: (HE-B profiles)  
 $r_{\text{bending}} = 2.36 \text{ kN/m}^2$   
 $r_{\text{compression}} = 1.89 \text{ kN/m}^2$

**LOAD CASE NO. 2**

$q_i = 18.75 \text{ kN/m}$  ( $i=91,160$ )

$P_j = \frac{0.0675}{\sqrt[3]{T}} \cdot \frac{h_j w_j}{\sum h_j w_j} \cdot \sum w_j$

where  $w_j$  is the total weight concentrated at the  $j$ -th floor, and  $T$  is the fundamental period of the frame.

Lateral deflection of node no.91 (93) is equal to or less than 1/300 times height of frame.

**Allowable stresses:**

Beams: (IPE profiles)  
 $r_{\text{bending}} = 2.16 \text{ kN/m}^2$   
 $r_{\text{compression}} = 1.73 \text{ kN/m}^2$

Columns: (HE-B profiles)  
 $r_{\text{bending}} = 3.24 \text{ kN/m}^2$   
 $r_{\text{compression}} = 2.59 \text{ kN/m}^2$

$E = 2.1 \cdot 10^8 \text{ kN/m}^2$  (Young's modulus)

\* SI units

FIG. 3. 30 STOREY STEEL FRAME: FINAL OPTIMUM DESIGN.

1. Beams	: IPE	400
2. Center columns	: HE-B	240
3. Outer columns	: HE-B	180

1	400
2	260
3	200

1	450
2	260
3	240

1	450
2	280
3	240

1	450
2	400
3	280

1	500
2	500
3	300

1	500
2	600
3	340

1	550
2	700
3	400

1	550
2	900
3	450

FIG. 2. SECTION MODULI ( $10^3 \text{ cm}^3$ ) DISTRIBUTIONS.

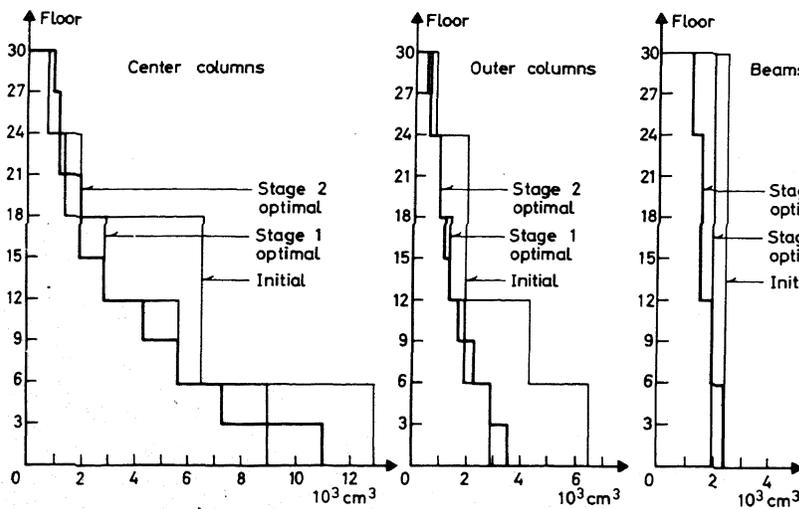


FIG. 4. DEFLEXIONS ( $\times 18$ ).

