THE αβγ METHOD FOR THE CHARACTERIZATION OF EARTHQUAKE ACCELEROGRAMS.

G. Rodolfo Saragoni (I)

SYNOPSIS

A characterization method for the time evolution of acceleration amplitudes of earthquake accelerograms using three parameters is presented. The method considers that mean square acceleration $E\{a^2(t)\}$ tends to the chi-square function

$$E\{a^2(t)\} = \beta e^{-\alpha t} t^{\gamma} ,$$

where parameters α, β and γ characterize the time evolution of acceleration amplitudes for each type of record. These parameters are easily estimated using a time moment technique. The method has been applied to 32 accelerograms fron U.S.A., México, Perú and Chile with satisfactory results.

In addition a duration of strong motion region is defined in terms of α and γ. Using this definition the distribution of expected energy among build-up, strong motion and end-up is studied for different values of the parameters. Expressions for the parameters α, β and γ in terms of epicentral distance and Richter magnitude are also presented for earthquake design in the U.S. West Coast. Finally, expression for the estimation of the expected maximum ground acceleration and instrumental intensities in terms of the parameters are also given.

REPORT

1. Introduction

Actual characterization methods for earthquake accelerograms present some limitations for studies of nonlinear structural responses and for forecasting of seismic risk. Characterization are actually done in qualitative terms by using the name of the record or in qualitative ways by considering only maximum acceleration, duration of strong motion region or instrumental intensities (Housner's or Arias (1),(2)). In this work a method which characterizes the time evolution of acceleration amplitudes in terms of three parameters is presented, the method has physical meaning and it avoids some of the limitations of actual characterization procedures. The method can be particularly powerful for design methods that consider simulated earthquake accelerograms.

2. The αβγ Method

Kanai (3) and Duke et al. (4) have proposed to represent the physical effect of the path between source and local site (see Figure 1) by a linear and time invariant system consisting in a set of filters acting in cascade. By assuming that the number of these filters is relatively large, the mean square acceleration $E\{a^2(t)\}$ recorded at the site can be approximated to a chi-square function of the type (5)

(I) Professor of Earthquake Engineering, Department of Civil Engineering, University of Chile, Casilla 5373, Santiago, Chile.

$$E\{a^2(t)\} = \beta\, e^{-\alpha t}\, t^{\gamma} \qquad (1)$$

where the parameters α, β and γ characterize the evolution of acceleration amplitude with time t.

3. The Estimation of the Parameters

Defining the "energy" function $W_a(t)$ of an accelerogram a(t) by

$$W_a(t) = \int_0^t a^2(\tau)\, d\tau \qquad (2)$$

the parameters α, β and γ can be estimated for each earthquake accelerogram using a time moment technique (5) from the following expressions :

$$\frac{\gamma + 1}{\alpha} = \frac{\int_0^{t_o} \tau\, a^2(\tau)\, d\tau}{W_a\,(t_o)} \quad , \qquad (3)$$

$$\frac{(\gamma+1)(\gamma+2)}{\alpha^2} = \frac{\int_0^{t_o} \tau^2 a^2(\tau)\, d\tau}{W_a\,(t_o)} \quad , \qquad (4)$$

and

$$\beta = \frac{\alpha^{\gamma+1}}{\Gamma(\gamma+1)}\, W_a\,(t_o) \qquad (5)$$

where t_o is the duration of the record and $\Gamma(\cdot)$ denotes gamma function.

The parameters α, β and γ were determined for 32 accelerograms, the values are given in references (6) and (7). The normalized expected energy is compared en Fig. 2 with the calculated from direct integration of the values of the record. In this figure the expected total energy function $E\{W_a(t_o)\}$ has been normalized equal to 1. The time scale has also been normalized by setting the total duration t_o=1. The corresponding figures for all 32 earthquake accelerograms are given in reference (6). The similarity between the smooth theoretical curve and the real one is good considering that only a single sample from each acceleration process $\{a(t)\}$ was used.

However it was found that the function of Eq. (1) fails in the case of accelerograms obtained at very short epicentral distances and on rocky soils. Arias (8) has suggested to consider a beta function for the mean square acceleration of these type of records. However this new function does not improve significatively the results and gives very similar functions that the chi-square approximation (9). The behavior of the mean square acceleration function for records obtained at very short epicentral distances and on rocky soil may be due to the fact that in these cases the condition that the number of filters must be moderately large is not satisfied since the medium between source and site is relatively homogeneous.

The chi-square approximation can be also used to estimate mean square velocity and displacement of earthquake records. The corresponding parameters of these functions for the 32 accelerograms are given in references (6) and (7). The chi-square function can be also applied to estimate the time evoulution of predominant frequencies of acceleration records (6),(7).

4. Definition of a Strong Motion Region

The duration of the strong motion region of an accelerogram can be conveniently defined between the time t_1^* and t_2^* at which the chi-square function of Eq. (1) has its inflection points (5). These points occur at $t_1^* = (\gamma - \sqrt{\gamma})\alpha^{-1}$ for $\gamma > 1$ or zero for $\gamma \leqslant 1$ and $t_2^* = (\gamma + \sqrt{\gamma})\alpha^{-1}$. In this region the mean square acceleration remains approximately constant since the values of $E\{a^2(t)\}$ at t_1^* and t_2^* and its maximum do not significantly differ from their average values (5).

The duration of the strong motion region Δt_s can be estimated from the parameters α and γ considering the following expressions.

$$\Delta t_s = \begin{cases} t_2^* - t_1^* = \dfrac{2\sqrt{\gamma}}{\alpha} & ; \text{ for } \gamma > 1 \\ t_2^* = \dfrac{\gamma+\sqrt{\gamma}}{\alpha} & ; \text{ for } \gamma \leqslant 1 \end{cases} \qquad (6)$$

In addition the build-up region of the record goes from 0 up to t_1^* for $\gamma < 1$ or it is equal to 0 for $\gamma \leqslant 1$ and the end-up region starts at t_2^*.

5. Accelerogram Energy Distribution

The distribution of the expected total energy $E\{W_a(t_o)\}$ among the three regions of an acceleration process : build-up, strong motion and end-up can be estimated using Eq. (2), (see Ref.(5)). The distribution of the normalized total expected energy as a function of γ is shown in Fig. 3. From this figure it follows that only about 50 to 70% of the total expedted energy of an acceleration process is in the "stationary" strong motion region. Therefore, different results in structural responses must be expected between analysis which uses "equivalent stationary" and nonstationary processes, specially in the case of nonlinear structures.

6. Macroseismic Relations for α, β and γ

Macroseismic relations between epicentral distance D (Km) and Richter magnitude and the parameters α, β and γ have been studied for accelerograms recorded in the West Coast of US. (10).

The relation for the duration of the strong motion region Δt_s (secs)was found to be (10) :

$$\Delta t_s = \begin{cases} 1.26 \times 10^{-4}\, e^{1.51M} + 0.044\, MD & ; D < 30 \text{ Km.} \\ \dfrac{e^{0.80M}}{D^{0.86}} & ; D > 30 \text{ Km.} \end{cases} \qquad (7)$$

Relations between α and γ and Δt_s were established by assuming that t_1^* has an average value of 3,5 secs. The relations are (10) :

$$\alpha = \frac{14}{\Delta t_s^2} + \frac{2}{\Delta t_s} \quad (8)$$

$$\gamma = (\frac{7}{\Delta t_s} + 1)^2 \quad (9)$$

An average value for t_1^* of 3.5 secs. was considered in Eqs. (8) and (9) since it is difficult to estimate t_1^* exactly from real accelerograms due to the starting time of the record.

The expected total energy $E\{W_a(t_o)\}$ of acceleration processes was found to be related with M and D by the following expression (10) :

$$E\{W_a(t_o)\} = \frac{8.71 \times 10^{-6}\ e^{2.77M}}{D^{0.25M}} \quad (10)$$

where acceleration amplitudes are measured in 0.10 g., and $E\{W_a(t_o)\}$ in $10^{-2}g^2$ sec. and g is the acceleration of gravity.

Since α, γ and $E\{W_a(t_o)\}$ are known from Eqs. (8), (9) and (10) the value of β is determined from Eq. (5) :

$$\beta = \frac{\alpha^{\gamma+1}}{\Gamma(\gamma+1)} E\{W_a(t_o)\}, \quad (5)$$

Figures showing the variation of the parameters α, β and γ with D and M are given in reference (10).

7. Expected Maximum Accelerations and Instrumental Intensities in Terms of α, β and γ.

Considering that acceleration processes are approximately stationary *in the strong motion region, the maximum expected acceleration can be* estimated from (11) :

$$E\{a(t)_{max}\} = (\lambda' + \frac{0.5772}{\lambda'})\ \sigma_o, \quad (11)$$

with

$$\lambda' = (2\ln \frac{\nu_o\ \Delta t_s}{2})^{1/2} \quad (12)$$

and where

$$\sigma_o = \left| \frac{\beta}{2} (\frac{\sqrt{\gamma}}{\alpha})^\gamma e^{-\gamma} \{(\sqrt{\gamma})^\gamma + (1+(\sqrt{\gamma})^\gamma)\ e^{-\sqrt{\gamma}}\} \right|^{1/2} \quad (13)$$

and ν_o is the predominant frequency of the accelerogram in the strong motion region.

Finally, the expected Arias intensity $E\{I_A\}$ (2) and the expected Housner intensity $E\{I_H\}$ can be estimated in terms of the parameters from

$$E\{I_A\} = \frac{\beta\Pi}{2g} \cdot \frac{\Gamma(\gamma+1)}{\alpha^{\gamma+1}} \quad (14)$$

and (?)

$$E\{I_H^2\} = \frac{21\Pi\beta}{2g}\ \frac{\Gamma(\gamma+1)}{\alpha^{\gamma+1}} \quad (15)$$

where I_H is in ft. and the acceleration amplitudes are in m/sec^2.

8. Bibliography

1. Housner, G.W., "Intensity of Ground Motion During Strong Earthquakes", Earthq. Rsearch Lab. California Institute of Technology, Pasadena, Calfifornia, 1952.
2. Arias, A., "A Measured of Earthquake Intensity", in Seismic Design for Nuclear Power Plants, R. Hansen (Editor), MIT Press, Massachusetts, 1970.
3. Kanai, K., "An Empirical Formula for the Spectrum of Strong Earthquake Motion", 2WCEE, Tokyo, Japan, 1960.
4. Duke, C.M., J.E. Luco, A. Carriveau, P. Hradilek, R. Lastrico and D. Ostrom, "Strong Earthquake Motion and Site Conditions", BSSA, Vol. 60, N°4, 1970.
5. Saragoni, G.R., "Nonstationary Analysis and Simulation of Earthquake Ground Motions", Ph.D. Dissertation, University of California at Los Angeles, L.A., California, June 1972.
6. Joannon, J.G., A. Arias and G.R. Saragoni, (in Spanish) "Time Variation of the Frequency Content of Earthquake Accelerograms", Dept. of Civil Engineering, Structure Division, Report SES I-1/75, University of Chile, Santiago, Chile, 1975.
7. Joannon, J.G., A. Arias and G.R. Saragoni, "The Time Variation of the Predominant Frequency of Earthquake Motions", 6WCEE, New Delhi, India, 1977.
8. Arias, A., "Earthquake Intensity and Smoothed Earthquake Spectra", 5WCEE, Rome, Italy, 1973.
9. Holzapfel, A., A. Arias and G.R.Saragoni, "An Approximated Expression for the Mean Square Acceleration of Earthquake Ground Motions", Fifth Symposium on Earthquake Engineering, Roorkee, India, 1974.
10. Labbé, J.C. and G.R. Saragoni, (in Spanish) "Macroseismic Relations for an Earthquake Acceleration Model", Dept. of Civil Engineering, Structure Division, Report SES I-X/76, University of Chile, Santiago, Chile, 1976.
11. Davenport, A.G., "Note on the Distribution of the Largest Value of a Random Excitation with Application to Gust Loading", Proceedings Institution of Civil Engineers, 1964.

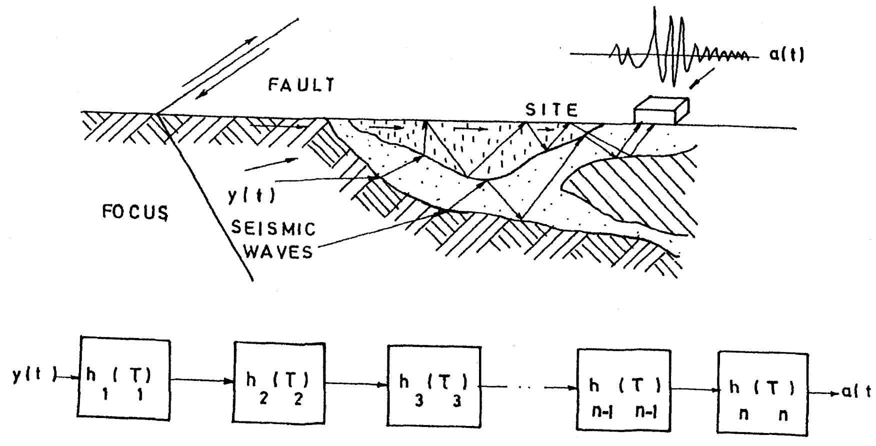

FIG. 1. Seismic Waves Path and its Idealized Model

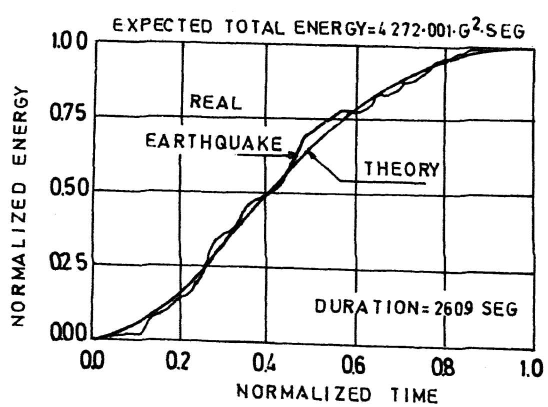

FIG. 2. 8244 Orion 1971 E W

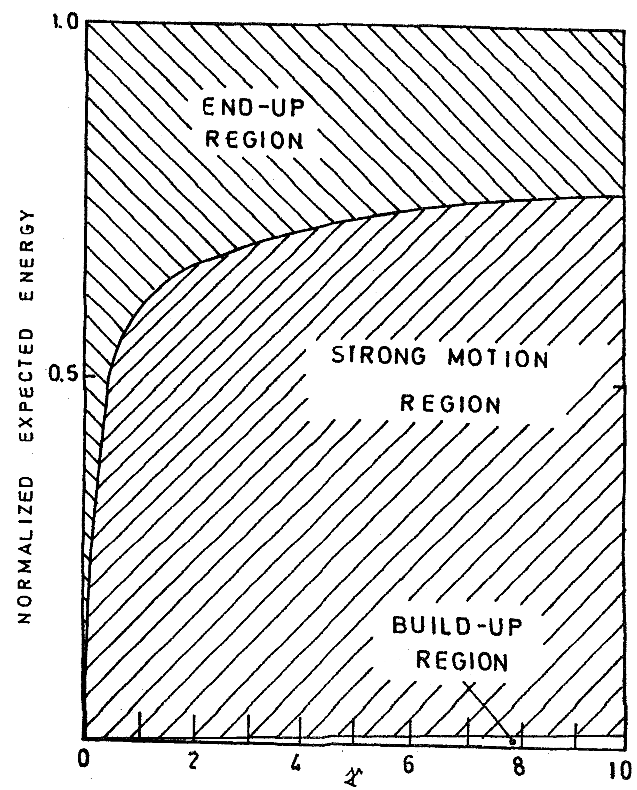

FIG. 3. Accelerogram Energy Distribution

DISCUSSION

K.L. Kaila (India)

The acceleration recorded at any one site depends on the characteristics of the earthquake source (in your paper shown by different attenuation laws for different magnitudes), the travel path distance, of the soil conditions at the recording site. The effects of the soil conditions is very predominant as has been shown in the paper by Tsuchida et al (2.173) strong attenuation of acceleration with depth. Therefore each accelerogram is an individual record pertaining to a particular earthquake, epicentral distance and the soil conditions at the recording site. How can one generalise the results obtained from such a record to other sites and for future earthquakes in design of earthquake proof structures.

Author's Closure

With regard to the question of Mr. Kaila, we wish to state that the attenuation laws for the duration of the strong motion region Δt_s given by Eq. (7) and for the expected total energy $E\{W_a(t_o)\}$ of Eq. (10) are only valid for the seismicity of the West Coast of U.S.

The regression expression for the duration of the strong motion region was estimated using 28 accelerograms of earthquakes with $M \geqslant 5.8$ and $D \leqslant 60$ Km.

The attenuation expression for the expected total energy was estimated from 54 accelerograms with $M \geqslant 5.5$ and $D \leqslant 100$ Km.

Most of these accelerograms were obtained on alluvium.

Accelerograms with $D > 100$ Km. and $M < 5.5$ were considered irrelevant for earthquake design purposes and spurious in the regression analyses.

The correlation coefficients for the regression expressions were found higher than 0.80 for Eq. (7) and equal to 0.94 for Eq.(10).

Considering the good levels of correlations obtained for both expressions and the scarcity of accelerograms recorded in different types of soils with $M \geqslant 5.5$ no intent was done in the study to include local site effects through some soil classification.

The dynamic amplification of body waves travelling through multilayered soils, as you suggest, is a debatable matter in the case of accelerograms produced by earthquakes with focal mechanisms of the type presents at the West Coast of U.S. Many cases of strong accelerograms recorded in this region which can not been satisfactory explained by this effect has been reported in the literature for earthquakes former to San Fernando, 1971 earthquake (Brady (1), Trifunac (2)) and for San Fernando (Crouse (3)).

Earthquake accelerograms recorded in this region suggest in many cases significant contributions from dispersive surface waves and many good papers has been presented in this same session trying to consider the effect of surface waves or trying to separate surface from body waves.

Apparently the prominent effect of soil amplification is really significant when there are high differences of shear wave velocities among bedrock and layered soils as it is the the case reported by Trushida et al. where accelerograms with amplitudes less than 0.02 g. were recorded on 5 m. of soil fill.

References

1. Brady, A.G., "Studies of Response to Earthquake Ground Motion", California Institute of Technology, Earthquake Engineering Research Laboratory, Report, Pasadena, California 1966.

2. Trifunac, M.D., "Response Envelope Spectrum and Interpretation of Strong Earthquake Ground Motion", Bulletin of the Seismological Society of America, Vol. 61, N°2, pp. 343-356, April 1971.

3. Crouse, C.B., "Engineering Studies of the San Fernando Earthquake", California Institute of Technology, Report EERL 73-04, Pasadena, California, March 1973.