CHARACTERISTICS OF THREE-DIMENSIONAL GROUND MOTIONS ALONG PRINCIPAL AXES, SAN FERNANDO EARTHQUAKE

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SYNOPSIS

Using the concept of an orthogonal set of principal axes and applying both the time and the frequency domain moving-window technique to the accelerograms recorded during the San Fernando earthquake of February 9, 1971, characteristics of three-dimensional ground motions along principal axes are determined. It is concluded from the resulting intensity functions and the time dependent frequency contents that realistic three components of ground motion can be generated stochastically as nonstationary random processes along their corresponding principal axes without cross correlation with one another in a statistical sense.

INTRODUCTION

It is becoming increasingly evident that responses of some important structural systems such as three-dimensional piping systems, nuclear power plants, highway structures and earthfill dams are significantly affected by more than one component of ground motion excitation. Because of this awareness, there will be an increasing demand, in the future, for dynamic analyses of such structural systems using multi-dimensional ground motions.

A number of stochastic models for earthquake ground motions have been employed in the one-dimensional form; however, when extending the use of such models to the two- or three-dimensional form, the question "Should the components of motion be cross correlated statistically?" immediately arises. If correlated, one must establish appropriate cross-spectral density functions in addition to realistic power spectral density functions.

Similar to the stress state problem, an orthogonal set of principal axes exists for ground motions along which the components of motion have maximum, minimum and intermediate values of variance and have zero values of covariance. This property suggests that components of motion need not be cross correlated statistically provided they are directed along principal axes.

METHOD OF ANALYSIS

Principal Axes of Ground Motions [1,2] Suppose three translational components of ground motion at point 0 along an arbitrary set of orthogonal axes x, y and z are represented by the relation

$$a_i(t) = \zeta_i(t)\ b_i(t) \qquad i = x,y,z \tag{1}$$

where $b_i(t)$ and $\zeta_i(t)$ $(i = x,y,z)$ are stationary random processes and deterministic intensity functions, respectively. If the processes are considered to be Gaussian, covariance functions defined by

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$$E[a_i(t)\ a_j(t+\tau)] = \zeta_i(t)\ \zeta_j(t+\tau)\ E[b_i(t)\ b_j(t+\tau)] \quad i,j = x,y,z \quad (2)$$

characterize completely the processes in a probabilistic sense [3]. Since real earthquake accelerograms demonstrate a very rapid loss in correlation with increasing values of $|\tau|$, the influence of coordinate directions on the covariance functions can be investigated substituting $\tau=0$ in Eqs. (2). Principal variances and the directions of principal axes can be obtained respectively as eigenvalues and corresponding eigenvectors of the covariance matrix defined by Eqs. (2).

Moving-Window Procedure Variances and covariances (μ_{ij} $i,j = x,y,z$) are obtained as continuous functions of time t_o using the so-called "moving-window" technique, i.e. using the relation [4]

$$\mu_{ij}(t_o,\Delta T) = < [a_i(t)-\bar{a}_i][a_j(t)-\bar{a}_j] >_{t_o - \frac{\Delta T}{2}}^{t_o + \frac{\Delta T}{2}} \quad i,j = x,y,z \quad (3)$$

where the time averages are taken over the interval ΔT centered at time t_o but where the mean values $\bar{a}_i$ and $\bar{a}_j$ are found by averaging $a_i(t)$ and $a_j(t)$ over the entire duration of motion. The values of time window length ΔT in Eqs. (3) should be taken sufficiently long so that the higher frequency fluctuations are essentially removed but the slower time dependent characteristics are retained, i.e. the time average over duration ΔT will be essentially equal to the average taken across the ensemble. This formulation allows one to obtain the time dependency of principal variances and their corresponding directions of principal axes.

Using the Fourier transformation, the moving-window technique, as applied in the time domain above, can be applied in the frequency domain as well. In this case, however, variances and covariances are evaluated as continuous functions of frequency f_o. In the frequency domain formulation, one can investigate principal variances, directions of principal axes, etc. associated with those frequencies of ground motion in the range $(f_o-\Delta f/2) < f < (f_o+\Delta f/2)$. Hopefully, this approach can be used to reveal certain characteristic features of the various types of seismic waves associated with strong ground motions.

Time Dependent Frequency Content Variation of frequency content of ground motions with time is examined by Fourier amplitude spectra generated by the moving-window technique as follows

$$A(\omega,t_o,\Delta T) = \left| \int_{t_o - \frac{\Delta T}{2}}^{t_o + \frac{\Delta T}{2}} a(t)\ e^{-i\omega t} dt \right| \quad (4)$$

Since intensity of ground motion can be expressed in terms of variances [5], the moving-window Fourier amplitude spectra can be normalized with respect to the maximum values generated for time t_o when determining frequency variations with time.

RESULTS OF ANALYSIS

Principal Variances and Directions of Principal Axes Both the time domain and frequency domain moving-window analysis described above has been applied to the ground motions recorded during the San Fernando earthquake of February 9, 1971. The time- and frequency-dependent principal variances and directions of principal axes have been evaluated for acceleration records at 99 stations. The direction of a principal axis is represented by direction angles ϕ and θ in three-dimensional space as shown in Fig. 1.

In this paper, the results for station No. 264, basement of the Millikan Library, CALTECH, are presented. Figure 2 shows the time-dependent principal variances and directions of principal axes as determined using discrete values of t_o one-half second apart and using 5 seconds for ΔT in Eqs. (3). Similarly, Fig. 3 shows the results of frequency-dependency using a frequency bandwidth Δf nearly equal to 0.98 Hz placed at uniform frequency intervals of 0.49 Hz. The solid, short-dashed and intermediate-dashed curves in these figures represent the major, minor and intermediate principal axes, respectively and the horizontal long-dashed straight line represents the direction θ_E to the reported epicenter. Although, the functions of principal transformation shown in Figs. 2 and 3 have numerous unexplainable features, certain correlations in the time domain moving-window formulation should be noted as follows:

(1) Usually during the early periods of low intensity motion, either the major or the intermediate principal axis is nearly vertical, i.e. the vertical component represents a large amount of energy in comparison with the horizontal components.
(2) Later, the major and intermediate principal axes shift towards horizontal positions with the minor axis taking the nearly vertical position.
(3) Following the shift of the major principal axis towards a horizontal position, the horizontal directions of the major and intermediate axes can suddenly interchange. This interchange which occurs after the period of high intensity motion is due to a corresponding change in the direction along which the seismic waves have maximum energy.
(4) Usually during the period of high intensity motion, the horizontal direction of either the major or the intermediate principal axis is towards the fault slip zone. This characteristic suggests that the direction along which seismic waves contain maximum energy either coincides with the direction to the fault slip zone or is at right angle to it.

Figure 4 shows the horizontal directions (θ) of the major and intermediate principal axes for stations in the extended Los Angeles area at the period when the motions are of highest intensity. While the correlation is not strong, there is a tendency of the directions of the major principal axis or, in some cases, the intermediate principal axis to point in the general direction of the fault slip zone [6] which is also the general direction towards the previously reported locations of surface fault traces south of the epicenter [7]. Since the concept of intensity defined here is identical to that defined by Arias [8], one can speculate that the direction of the major principal axis coincides with the direction to maximum energy release in the fault slip zone.

<u>Time Dependent Frequency Content along Principal Axes</u> Applying the moving-window technique, the Fourier spectral amplitude analysis has been employed to evaluate variations of frequency content for the principal components of motion with time. Spectral amplitudes have been evaluated using a time window length of 5 seconds which is identically the same as that used in the time domain formulation of principal axes. To determine the general characteristics, the amplitude spectra are smoothed by weighed convolutions. In Fig. 5, the time dependencies of frequency content for

station No. 264 are shown using a three-dimensional spectral diagram in which the contour lines represent levels of the normalized Fourier amplitude. The shaded zones in the diagram represent the highest range of spectral magnitude; thus, indicating the corresponding range of dominant frequencies at time t_0. The characteristic features of these diagrams representing variation of frequency content with time can be summarized as follows:

(1) The dominant frequency is found to have decreasing values with time. This tendency coincides with the results reported in the literature [9].
(2) In some cases, the dominant frequency changes its value suddenly at a fixed time which may indicate the arrival of different types of seismic waves.
(3) Generally, it is observed that spectral density functions derived from the three-dimensional diagrams are less sharply peaked for motions along the minor axis than for motions along the major and intermediate axes. This tendency agrees with the results of the moving-window analysis in the frequency domain which show that principal variances along the minor principal axis are more uniform than those along the major and intermediate principal axes.

SIMULATION OF THREE-DIMENSIONAL GROUND MOTIONS

It was concluded previously that three components of ground motion are independent of one another, provided they are directed along a set of principal axes. Therefore, one can simulate the three components by generating independent motions having appropriate intensity and frequency content and directing them along principal axes.

Sample accelerograms, having properties of spectral density and deterministic intensity function as shown in Fig. 6, have been simulated using Toki's method [10] along a set of principal axes. Assuming the horizontal and vertical direction angles (θ,ϕ) of principal axes to be time dependent as shown in Fig. 6, the simulated components are transformed to three components along the principal axes of the structural system for use in dynamic analysis. The resulting components of motion are shown in Fig. 7. Assuming that these motions were recorded by an accelerograph installed in the structural system, principal variances, directions of principal axes and frequency content of motion along the principal axes have been evaluated. The resulting properties of principal variances and directions of principal axes are presented in Fig. 8 and those of the corresponding time dependency of frequency content are represented in Fig. 9. The properties of the simulated motions generally coincide well with the prescribed properties.

CONCLUDING STATEMENT

The objective of the study described herein is to establish an appropriate stochastic model for three-dimensional ground motions which reflect the significant statistical properties of the motions of the San Fernando earthquake.

The resulting model is represented by the product of a deterministic intensity function and a constant intensity process having a variable frequency content with time. The properties of the simulated motion reveal complexities similar to the characteristics of the motions recorded during the San Fernando earthquake. Therefore, this simple model as prescribed appears to be adequate.

ACKNOWLEDGEMENT

The results presented in this paper have been taken from reference No. 2 which contains similar data for many stations and presents much more detailed observations and analyses.

The financial support provided by the National Science Foundation under Grant No. GI-36387 is acknowledged with sincere thanks and appreciation.

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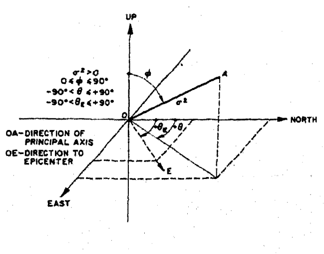

Fig. 1 Directions of principal axes in three-dimensional space.

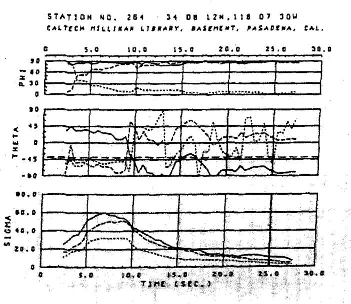

Fig. 2 Time dependent directions of principal axes and square roots of principal variances.

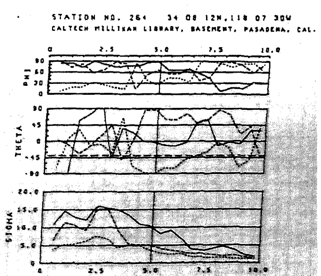

Fig. 3 Frequency dependent directions of principal axes and square roots of principal variances.

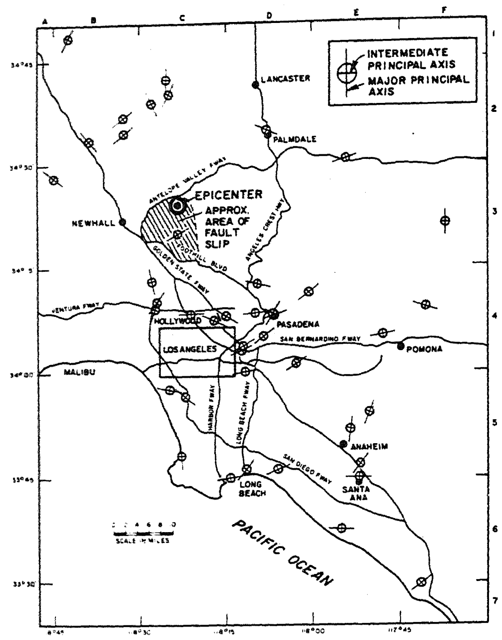

Fig. 4 Directions of major and intermediate principal axes of stations in the extended Los Angeles area.

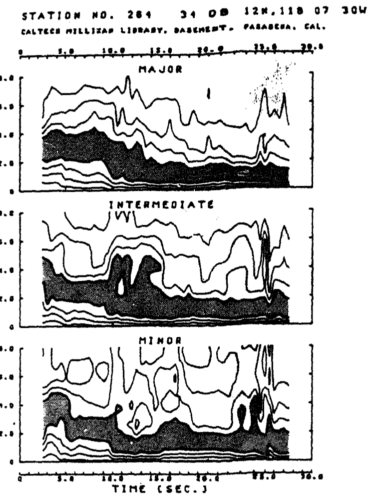

Fig. 5 Time dependent frequency distribution.

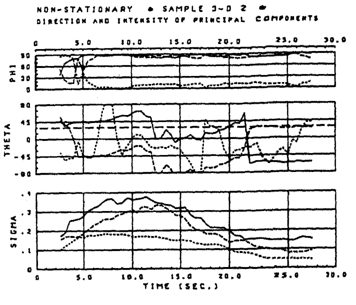

Fig. 8 Time dependent directions of principal axes and square roots of principal variances for simulated motion.

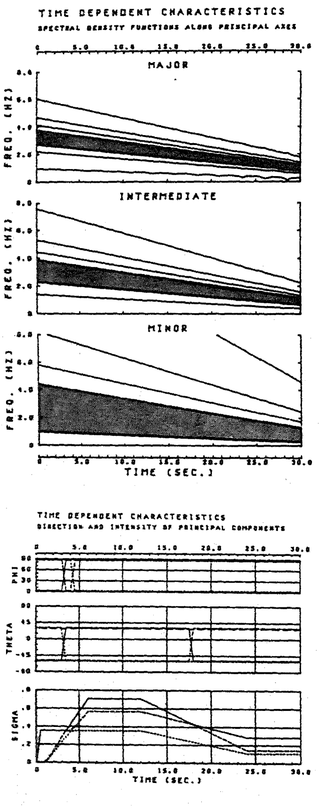

Fig. 6 Prescribed intensity and spectral density function.

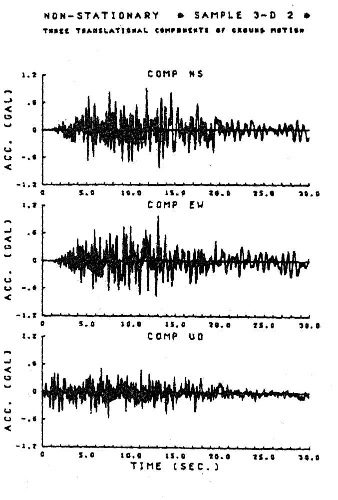

Fig. 7 Time histories of simulated motion.

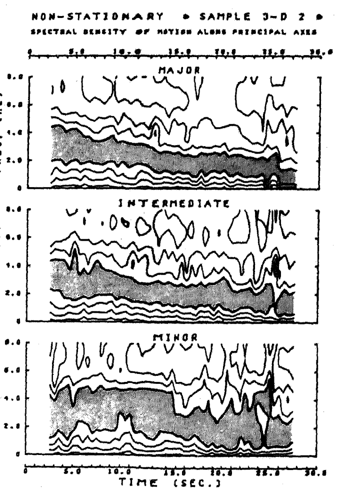

Fig. 9 Time dependent frequency distribution for simulated motion.