PROBABILISTIC ASSESSMENT OF SEISMIC RISK ON LOCAL SOIL SEDIMENTS

by

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Synopsis

Probability distributions of earthquake parameters on local soil sediments are derived from corresponding distributions for rock or hard ground conditions. The method is primarily intended for cases where the soil response spectrum is dominated by site amplification effects. As an example, seismic design parameters for Mexico City lacustrine clay sediments are presented and compared with recorded data.

Outline of the method

Let $x(t)$ denote ground velocity or acceleration, y the peak value of $|x(t)|$ in an interval of duration τ, and let subscripts 1, 2 refer to rock and soil motions, respectively; $x(t)$ is regarded as a zero-mean stationary Gaussian process.

For a soil deposit acting as a narrow-band filter on incident base motions, the probability that y_2 remains below level r over an interval τ can be expressed as[1]

$$p_N[y_2 \leq r \mid \Sigma_2 = \sigma_2] = F_{Y_2 \mid \Sigma_2}(r \mid \sigma_2; N) = exp\,[-2N\,exp\,(-r^2/2\sigma_2^2)] \qquad (1)$$

where N, the number of "apparent" cycles of motion in τ, and σ_2, the standard deviation of $x_2(t)$, are taken as deterministic parameters. Eq 1 is exact only when $r \to \infty$, its error being on the safe side for narrow-band processes and finite r. More accurate estimates of p_N require knowledge of higher moments of the response power spectral density (PSD), which may be difficult to obtain.[1]

If real soil behavior is considered, σ_2 will be a non-linear function of the standard deviation of bedrock motion, say $\sigma_2 = \varphi(\sigma_1)$; in the absence of recorded data, this dependence must be determined through numerical models. A recent stochastic method of one-dimensional amplification analysis with Ramberg-Osgood soil description, seems

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well suited for this purpose.[2] Seismic excitation is specified as an acceleration or velocity PSD and response is characterized through an equivalent soil amplification function $A(\omega)$. The function φ can, however, be determined by any other method, including step-by-step numerical simulation, provided adequate statistical representativeness of the results is ensured. Deterministic methods of finite elements are the only option available when two- or three-dimensional modelling of the deposit becomes necessary.

Marked differences in calculated responses of stratigraphic profiles with strong impedance contrasts may arise from moderate variations in the subsoil's initial shear moduli. The resulting scatter in response parameters is a consequence both of spatial variability of the moduli and of current field and laboratory testing procedures, and it can be estimated by a second-moment probabilistic technique.[3] In the simplest case, namely when perfect correlation is assumed to exist among the initial moduli of all subsoil layers, the method calls for two separate analyses for a specified earthquake input.[4]

If the distribution of y_1 is known and y_1 is taken to be proportional to σ_1, say $y_1 = \beta\,\sigma_1$, the cumulative distribution function (CDF) of y_2 is derived from eq 1 as

$$p_N[y_2 \leqslant r] = F_{Y_2}(r; N) = \int_0^{\infty} F_{Y_2|\Sigma_2}(r\,|\,\varphi(\sigma_1); N)\, f_{\Sigma_1}(\sigma_1)\, d\sigma_1 =$$

$$= \beta \int_0^{\infty} exp\,\{-2N\, exp\,[r^2/2\,\varphi^2(\sigma_1)]\}\, f_{Y_1}(\beta\,\sigma_1)\, d\,\sigma_1 \qquad (2)$$

A way of performing the numerical integration is discussed elsewhere.[4] Results tend to be sensitive to the form of $\varphi(\sigma_1)$ for small σ_1 values but practically insensitive to it as $\sigma_1 \to \infty$, provided σ_2^2 is a non-decreasing function of σ_1. Sufficiently high, physical upper bounds imposed on y_1 and/or y_2 do not appreciably affect the results.

If $f_{Y_1}(y_1)$ is specified for a single random event and earthquake occurrence is assumed to follow a Poisson process with rate of exceedance ν, the probability that y_2 remains below level r over an assigned period T is given by

$$F_{Y_2}(r; N, \nu, T) = exp\,\{-\nu T\,[1 - F_{Y_2}(r; N)]\} \qquad (3)$$

For a given soil site, one can use eqs 2 and 3 to calculate the first two moments of F_{Y_2} as a function of T and thus obtain a design basis for a variety of structures.

When site effects introduce a prominent peak in the response spectrum, standard statistical values of spectral amplification factors are no longer applicable[5] and knowledge of the peak ordinate and its frequency is indispensable for defining the spectral shape.

Denoting by s_1 the design spectral ordinate on rock or hard ground and by S_2 the soil peak spectral ordinate, we assume that

$$S_2 = s_1\, A_{max} = y_1\, h_1\, A_{max} \qquad (4)$$

where h_1 is the spectral amplification factor applicable to peak rock velocity or acceleration for a prescribed damping value, according to current procedures. $A_{max} = A_{max}(y_1)$ is the largest value of the soil amplification function for a smooth excitation spectrum. By heuristic considerations, the approximate relationship $A_{max} = (1 + b\, y_1^{-c})^{1/2}$ can be shown to hold up to large values of y_1, at least in one-dimensional profiles; y_1 is peak acceleration and b, c are constants depending on the soil system. The dominant frequency ω_2 associated with A_{max} can be obtained numerically as a function of y_1; ω_2 must be known in order to specify whether eq 4 applies in the velocity or acceleration portion of the rock spectrum.

Taking h_1 to be lognormally distributed[6] and knowing the distribution of y_1, that of S_2 is calculated from eq 4 through standard techniques. The distribution of ω_2 can be derived from the numerically established relation $\omega_2 = \omega_2(y_1)$. If y_1 is approximately lognormal and A_{max} is taken in the form described, S_2 will also be lognormally distributed and its first two moments are easily obtainable from those of y_1 and h_1.

Example

The foregoing theory has been applied to the determination of seismic design parameters on the Mexico City clay deposits. CDFs for hard ground conditions were derived on the assumption that the mean rate of exceedance is given by $\nu = K\, y_1^{-\delta}$ for $y_1 \geqslant y_0$ and $\nu = \nu_0 = K(y_0)^{-\delta}$ for $0 \leqslant y_1 < y_0$. Values $K = 2 \times 10^{-3}$, $\delta = 2.7$, $y_0 = 0.2$ and $K = 1.18 \times 10^{-4}$, $\delta = 2.39$, $y_0 = 0.05$ were used for accelerations (in m/sec^2) and velocities (in m/sec), respectively. Parameters for velocity were obtained from data of ref 7 without modification, whereas those for acceleration result from a reduction of 50 percent in the evidently conservative values given in the same reference. CDFs for hard ground conditions derived from the foregoing hypotheses are well approximated by lognormal distributions having the same first two moments.

The assumed soil profile is described in a previous paper[4]; response analyses were carried out for six different levels of excitation intensities (up to a peak rock acceleration of 2.33 m/sec^2 and velocity of 0.53 m/sec) using the method of ref 2 with a Kanai-Tajimi type PSD. A value $\beta = 3$ was adopted in eq 2. Fig 1 shows the calculated mean soil peak amplification factor A_{max} as a function of σ_1 and also the mean $\pm$ one standard deviation curves corresponding to a standard deviation equal to 20 percent of the mean value assigned to the initial shear moduli of the subsoil. Small circles give the peak ratios of undamped pseudovelocity spectra from horizontal accelerogram components of moderate intensity earthquakes simultaneously recorded on soft and hard ground in the Mexico City area. Figs 2 and 3 display, on lognormal paper, cumulative probability values of peak ground velocity (y_2) and peak spectral velocity (S_2) on soft soil, obtained from 22 horizontal components of strong-motion accelerograms recorded in Mexico City from December 1961 to August 1968. Straight lines represent lognormal distributions which closely fit those calculated numerically by present method for $T =$ 6 yr, and actually have the same first two moments; the mean and mean ± one standard deviation lines have the same meaning as in fig 1. Due to the small sample size, points corresponding to probabilities greater than 0.8 in figs 2 and 3 are not considered statistically representative. Fig. 4 illustrates, as a function of recurrence period T, the median values $\check{m}_{Y_1}$, $\check{m}_{Y_2}$ and $\check{m}_{S_2}$ of the fitted lognormal CDFs for peak ground velocity and peak spectral ordinate at different damping values ζ. The $\check{m}_{S_2}$ curves are included in this figure because the response peak always occurs in the velocity portion of the spectrum. Observe that the ratio $\check{m}_{S_2}/\check{m}_{Y_2}$ markedly decreases with increasing T for all damping values, presumably because of non-linear soil effects. For $T = 50$ yr, a recurrence period commonly used in design, the calculated median spectral amplification factors on soft soil are 1.7 times greater than those suggested for average design spectra.[6] Finally, fig 5 illustrates the variation of the variance of peak ground velocity and peak spectral ordinate with recurrence period, whereas fig 6 depicts that of peak ground acceleration.

Acknowledgments

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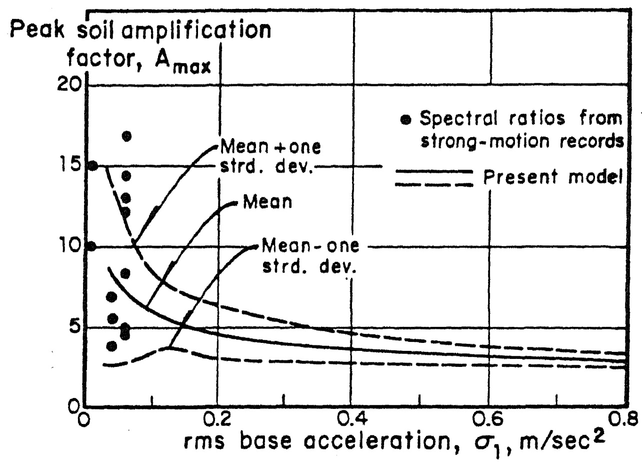

Fig 1. Peak soil amplification vs excitation intensity for Mexico City soft clay sediments

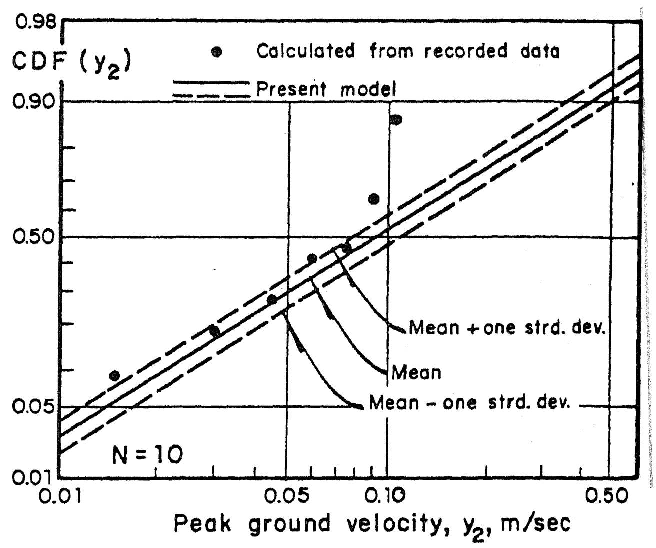

Fig 2. Probability distribution (CDF) of peak ground velocity on sediments; T=6 yr

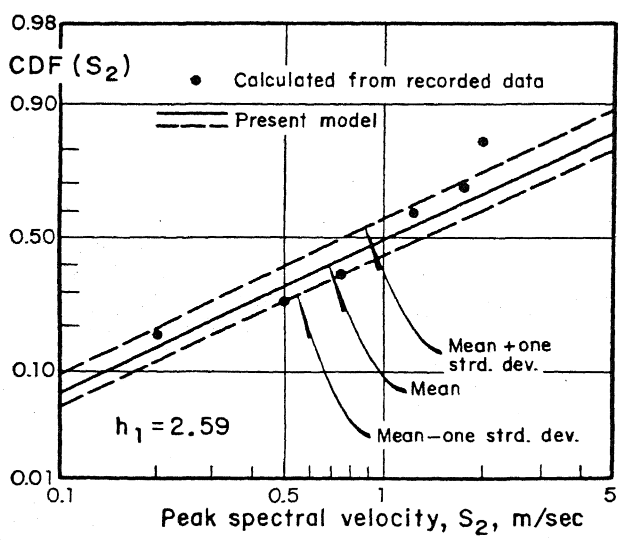

Fig 3. Probability distribution of peak spectral velocity on sediments; T=6 yr

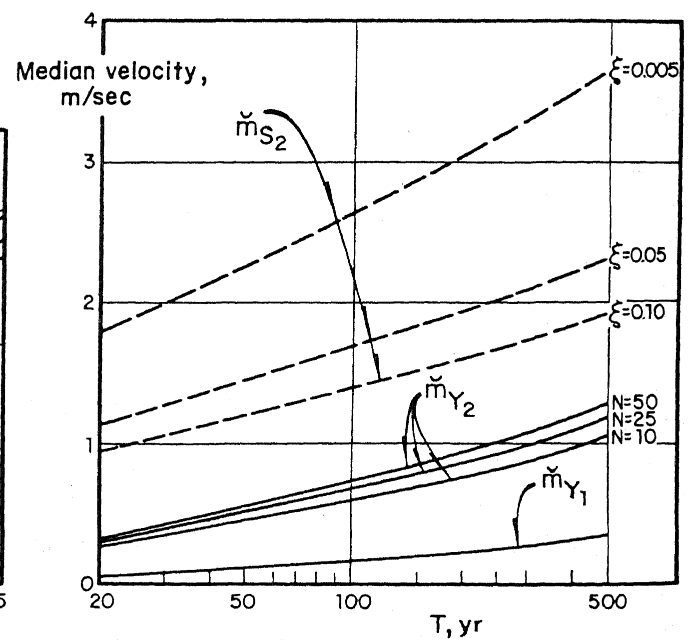

Fig 4. Median velocity values vs recurrence period on hard ground ($\check{m}_{Y_1}$) and sediments ($\check{m}_{Y_2}$, $\check{m}_{S_2}$)

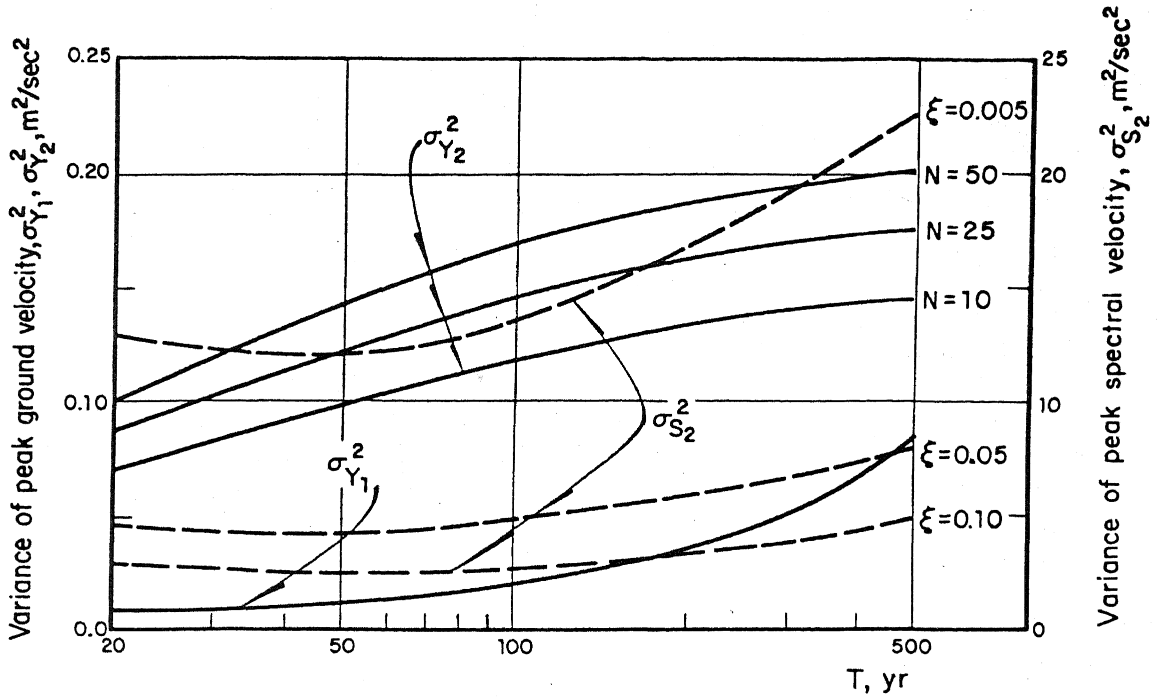

Fig 5. Variance of peak velocity values vs recurrence period on hard ground ($\sigma^2_{Y_1}$) and sediments ($\sigma^2_{Y_2}$, $\sigma^2_{S_2}$)

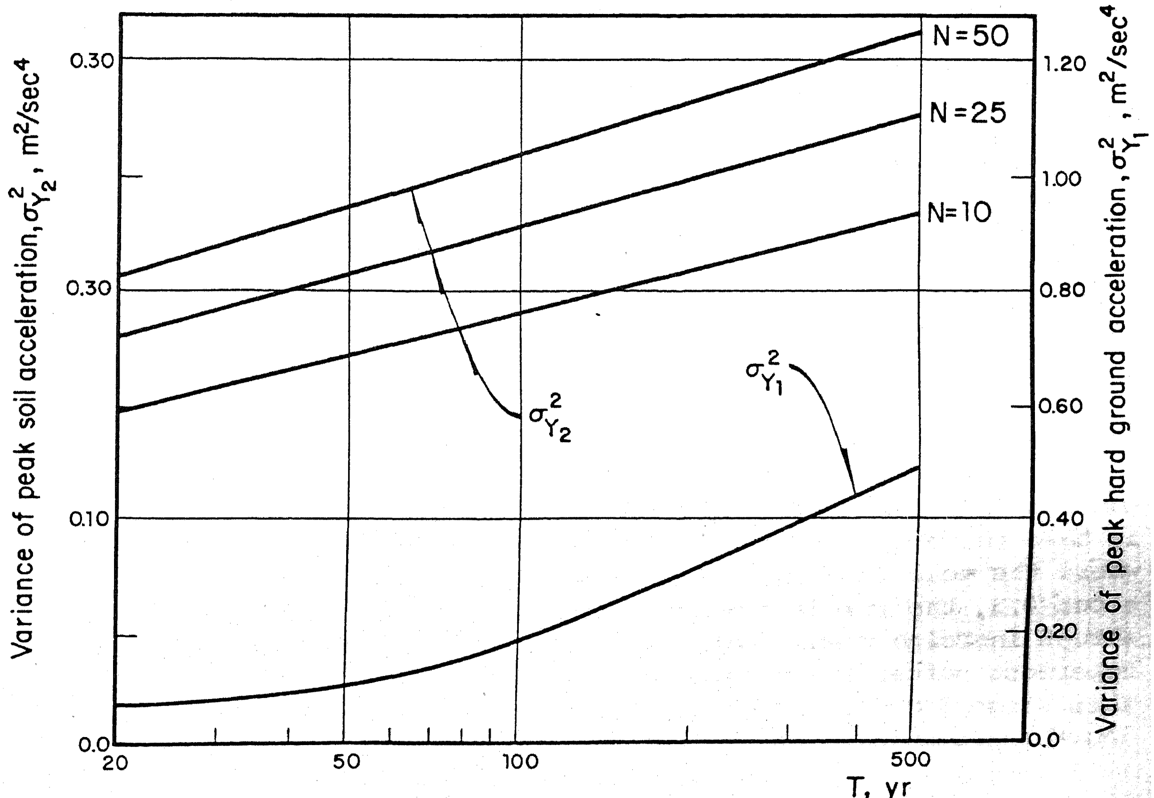

Fig 6. Variance of peak acceleration values vs recurrence period on hard ground ($\sigma^2_{Y_1}$) and sediments ($\sigma^2_{Y_2}$)

DISCUSSION

H. Tiedemann (West Germany)

Do you consider in your analysis special factor like resonance properties of the "Fondo Del Lago" system of Mexico city or depth of subsoil layers ?

J.B. Berrill (Australia)

Can you give guidelines for the soils of sites at which your model is applicable, preferably by giving values of impedance constant and possible restrictions on the regularity of boundaries between one soil zone and another ?

Author's Closure

Concerning the question by Mr. H. Tiedemann, detailed information can be found in reference 4. The profile corresponds to a site located in the Nonoalco-Tlaltelolco residential section, in the old lakebed area of Mexico City, and has a total thickness of 80 m, of which the uppermost 40 m consist of compressible clays (Formacion Arcillosa Superior e Inferior) and the remnant of compact alluvial layers (Depositos Profundos). Depending on the earthquake excitation intensity, the fundamental resonance period of this site has been found to vary between approximately 1.7 and 3.0 sec.

The point raised by Mr. J.B. Berrill is quite pertinent, since the availability of simple quantitative criteria should clarify the range of applicability of the proposed model. I have considered a number of sites characterized by relatively regular, horizontal near surface stratigraphy, located both in Mexico and Japan, for which several free-field strong ground motion records are available, as well as the profiles of shear wave velocity c and standard penetration index N. My temptative conclusion is that the model can be used with some confidence in cases where the mean impedance ratio $\gamma_2 c_2 / \gamma_1 c_1$ is less than about 0.3, the standard penetration resistance exhibits an abrupt increase when reaching the harder soil, and the uppermost softer layers are not less than 7-8 m thick. At this stage I can offer no quantitative guideline concerning the geometric characteristics of the boundaries of the soil system introduced in the analysis, since the model has been tested only with one-dimensional profiles.

I believe, however, that in the case of two-dimensional systems the previous restriction on the value of impedance ratio could be considerably released if localized resonance or focusing effects of seismic waves can be anticipated on the basis of purely geometrical considerations.

A fundamental requirement of the model is, in fact, that the response at a point on the ground surface be approximately narrow band.