ON SEISMIC WAVE PROPAGATION IN A CONTINUOUS RANDOM MEDIUM

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SYNOPSIS

This paper investigates statistically the effect of the irregular geological properties of the transmission medium on the seismic wave propagation from a focus of the earthquake to an observation point. The amplitude and phase characteristics of the average surface displacements to a point source buried in a random semi-infinite medium are evaluated in the case that the density of a medium is only considered as a random variable. As a result, it is pointed out that the scattering effect on the seismic wave propagation is mainly governed with the relationship between the size of the random inhomogeneity and the length of wave motion in traveling.

INTRODUCTION

The seismic wave motion radiated from a source undergoes the multiple reflection and refraction during propagation through the irregular stratified medium and boundary so as to attain the complicated wave shape. The objective of this paper is to investigate statistically the effect of the transmission of the wave motion through the medium involving irregular geological properties on the characteristics of earthquake ground motion. The problem of seismic wave motion in a random medium is of considerable importance in earthquake engineering, because it is very difficult for us to evaluate the refractive index describing the actual distribution of geological characteristics between a focus of the earthquake and a receiving station. If the elastic constants, density and boundary shape of such a medium vary randomly in a space, the equation of wave motion seems to be a kind of nonlinear equation, and its closed form solution may not be obtained through an analytical procedure without employing such the perturbation method as applied by J. B. Keller and P. R. Beaudet to a wave propagation problem. In this paper, Beaudet's method is utilized in order to study the fundamental characteristics of the seismic wave propagation in a random medium supposing that the average wave is denoted by the inverse Fourier transform of the spatial parts of the wave motion equation.

WAVE MOTION IN A RANDOM MEDIUM

Let us consider the nonstochastic linear operator L to a homogeneous medium and the perturbing linear operator L_i which represents the effect of the inhomogeneity in a random medium, then a wave motion $U(X)$ in a weakly random medium satisfies the following equation on the assumption that the effect of the perturbing linear operator L_i is slight:

$$[\, L - \sum_{i=1}^{n} \varepsilon^{i} L_{i} + O(\varepsilon^{n+1}) \,] U(x) = 0 \tag{1}$$

where ε is a measure of the deviation of the medium from homogeneity. A random medium is to be replaced with a mathematical model which consists of

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an ensemble of sample media together with its probability distribution. The formulation for the average wave motion is derived from expectation over the ensemble when the terms higher than $O(\varepsilon^3)$ are omitted. Then we have

$$L<U>-\varepsilon<L_1><U>-\varepsilon^2[<L_1\bar{L}^{-1}L_1>-<L_1>\bar{L}^{-1}<L_1>+<L_2>]<U>=0 \tag{2}$$

Moreover, substituting the Green's function $G(X, X')$ for the operator L^{-1} into eq.(2), the equation of average wave motion $<U(X)>$ may be expressed as

$$L(x)<U(x)>-\varepsilon<L_1(x)><U(x)>+\varepsilon^2[<L_1(x)>\int G(x,x')<L_1(x')><U(x')>dx' \\ -<L_1(x)\int G(x,x')L_1(x')<U(x')>dx'>+<L_2(x)><U(x)>]=0 \tag{3}$$

by expanding in powers of the fluctuations in random variables which describe the inhomogeneity of a medium. If the elastic constants λ, μ and the density ρ of a medium vary randomly in a space coordinate, the time harmonic wave motion $U(X)\propto e^{iwt}$ satisfies the following equation of wave motion.

$$(\lambda+\mu)\nabla(\nabla\cdot U)+\mu\nabla^2\cdot U+\nabla\lambda(\nabla\cdot U) \\ +\nabla\mu\times(\nabla\times U)+2(\nabla\mu\cdot\nabla)U+\omega^2\rho U+\rho\bar{F}=0 \tag{4}$$

in which $\nabla(A)$, $\nabla^2(A)$ and $\bar{F}$ denote the gradient, the Laplacian operators of A and the Fourier transform of a body force F, respectively. Because of the assumption that the elastic constants and the density contained in the equation of wave motion deviate slightly from their means $E\{\lambda, \mu, \rho\}=\{\lambda_0, \mu_0, \rho_0\}$, λ, μ and ρ are expressed as follows:

$$\lambda(x)=\lambda_0+\varepsilon\lambda_1(x) \quad , \quad \mu(x)=\mu_0+\varepsilon\mu_1(x) \quad , \quad \rho(x)=\rho_0+\varepsilon\rho_1(x) \tag{5}$$

where $\lambda_1(X)$, $\mu_1(X)$ and $\rho_1(X)$ are random fluctuations from their means, respectively. Then, from eqs.(4) and (5), the linear operator L, L_1 and L_2 are given by

$$L=(\lambda_0+\mu_0)\nabla(\nabla\cdot\ )+\mu_0\nabla^2(\ )+\omega^2\rho_0(\ ) \\ L_1=(\lambda_1+\mu_1)\nabla(\nabla\cdot\ )+\mu_1\nabla^2(\ )+\nabla\lambda_1(\nabla\cdot\ ) \\ +\nabla\mu_1\times(\nabla\times\ )+2(\nabla\mu_1\cdot\nabla)(\ )+\omega^2\rho_1(\ ) \\ L_2=0 \tag{6}$$

To evaluate the effective wave number in a random medium, the average wave motion $<U(X)>$ is defined by

$$<U(x)> = \int \frac{dk}{(2\pi)^3}\bar{U}(k)e^{ik\cdot x} \tag{7}$$

where $\bar{U}(k)$ denotes the Fourier transform of $U(k)$. With the aid of eqs.(3) and (7), the following equation is

$$L(x)<U(x)>-\varepsilon^2<L_1(x)\int G(x,x')L_1(x')<U(x')>dx'> \\ =\int\frac{dk}{(2\pi)^3}[\{\mu_0k^2-\omega^2\rho_0+\varepsilon^2D_1\}\bar{U}(k)+\{(\lambda_0+\mu_0)k^2+\varepsilon^2D_2\}(\hat{k}\cdot\bar{U}(k)\hat{k})e^{ik\cdot x}=0 \tag{8}$$

Analyzing eq.(8) by the two components of $\bar{U}(k)$ parallel and perpendicular to the propagation vector k, then the effective wave numbers for longitudinal and transverse waves are obtained by

$$k'^2_c = k^2_c - \frac{\varepsilon^2}{\lambda_0+2\mu_0}[D_1(k_c) + D_2(k_c)] \\ k'^2_s = k^2_s - \frac{\varepsilon^2}{\mu_0}D_1(k_s) \tag{9}$$

where $D_1(\ )$ and $D_2(\ )$ are random functions involving the integrals of the products of the Green's function $G(X, X')$ and the correlation functions of λ, μ and ρ. Thus, the effect of random inhomogeneity on a wave propagation can be evaluated by the effective wave number k_c' and k_s'. Next, of particular interest in the study of earthquake ground motion is a source mechanism at the foci of earthquake. Several mathematical models for the focal mechanism have been suggested to simulate the principle actions of the earthquake. If a point source in x-direction is considered as a simplified focal model, we have

$$\rho F = \rho(X,Y,Z) = (L,0,0)e^{i\omega t}\delta(x)\delta(y)\delta(z-H) \tag{10}$$

in which X, Y, Z and L denote each components of body force in cartesian coordinate and magnitude of source. When wave motions radiated from a point source propagate in a random medium, the nondimensional average surface displacements in the vertical, radial and cross-radial direction are evaluated in terms of the effective wave numbers k_c' and k_s' as follows:

$$E\{\frac{U_z(0)b\mu_0}{Le^{i\omega t}}\} = -\frac{a_0}{2\pi}\cos\phi\int_0^\infty \frac{\zeta^2}{F(\zeta)}[(2\zeta^2-g_2)e^{-\alpha_1 a_0 h}-2\alpha_1\alpha_2 e^{-\alpha_2 a_0 h}]J_1(\zeta a_0 r)d\zeta$$

$$E\{\frac{U_r(0)b\mu_0}{Le^{i\omega t}}\} = -\frac{1}{2\pi}\cos\phi\int_0^\infty \frac{\alpha_2}{F(\zeta)}[2\zeta^2 e^{-\alpha_1 a_0 h}-(2\zeta^2-g_2)e^{-\alpha_2 a_0 h}]\frac{\partial J_1(\zeta a_0 r)}{\partial r}d\zeta + \frac{1}{2\pi r}\cos\phi\int_0^\infty \frac{1}{\alpha_2}e^{-\alpha_2 a_0 h}J_1(\zeta a_0 r)d\zeta \tag{11}$$

$$E\{\frac{U_t(0)b\mu_0}{Le^{i\omega t}}\} = \frac{1}{2\pi r}\sin\phi\int_0^\infty \frac{\alpha_2}{F(\zeta)}[2\zeta^2 e^{-\alpha_1 a_0 h}-(2\zeta^2-g_2)e^{-\alpha_2 a_0 h}]J_1(\zeta a_0 r)d\zeta - \frac{1}{2\pi}\sin\phi\int_0^\infty \frac{1}{\alpha_2}e^{-\alpha_2 a_0 h}\frac{\partial J_1(\zeta a_0 r)}{\partial r}d\zeta$$

where, $F(\zeta)=(2\zeta^2-g_2)^2-4\zeta^2\alpha_1\alpha_2$, $\alpha_1=\sqrt{\zeta^2-n^2 g_1}$, $\alpha_2=\sqrt{\zeta^2-g_2}$

The independent and dependent variables in the nondimensional surface displacements are related to the following physical parameters as follows:

$$V_c=\sqrt{\frac{\lambda_0+2\mu_0}{\rho_0}},\quad V_s=\sqrt{\frac{\mu_0}{\rho_0}},\quad k_c=\frac{\omega}{V_c},\quad k_s=\frac{\omega}{V_s}$$

$$(\frac{V_s}{V_c})^2=(\frac{k_c}{k_s})^2=\frac{1-2\nu}{2(1-\nu)}=n^2,\quad \nu:\text{poisson ratio} \tag{12}$$

$$a=k_s b,\quad H=bh,\quad R=br,\quad k=k_s\zeta$$

and

$$(\frac{k_c'}{k_c})^2=g_1(\varepsilon,\lambda_0,\mu_0,\rho_0;a_0),\quad (\frac{k_s'}{k_s})^2=g_2(\varepsilon,\lambda_0,\mu_0,\rho_0;a_0) \tag{13}$$

where a_0, h, r and b denote nondimensional frequency, depth of a focus, epicentral distance and the reference of length, respectively.

NUMERICAL RESULTS

In order to show to what extent the random inhomogeneity affects on the characteristics of the earthquake ground motion, the nondimensional surface displacements of a semi-infinite medium are evaluated with the variation of the parameter $\varepsilon^2\langle\rho_1^2\rangle/\rho_0^2$. The statistical properties of the fluctuation of a wave field can be characterized by the correlation functions of $\lambda(X)$, $\mu(X)$ and $\rho(X)$. Here, only the density $\rho(X)$ is considered as a random variable and the corresponding correlation function is assumed as

$$R_{\rho\rho}(r)=E\{\rho(x)\rho(x')\}=\langle\rho_1^2\rangle e^{-\frac{r}{a}},\quad r=|r|=|x-x'| \tag{14}$$

where $<\rho_1^2>$ is the mean square of $\rho_1(X)$ and $a(=b\delta)$ is the correlation length of the inhomogeneity. Setting poisson ratio $\nu=1/4$ and substituting eq.(14) in eq.(9), then the effective wave numbers are obtained as follows:

$$g_1(\varepsilon,\lambda_0,\mu_0,\rho_0;a_0)=1-\varepsilon^2\frac{<\rho_1^2>}{\rho_0^2}[\frac{1}{sn}(-1+\frac{1}{n^2})\cot^{-1}(\frac{1}{sn}-i)+\frac{1}{1+(\frac{1}{sn}-i)^2}+\frac{i}{sn}(1-\frac{1}{n})]$$

$$g_2(\varepsilon,\lambda_0,\mu_0,\rho_0;a_0)=1-\varepsilon^2\frac{<\rho_1^2>}{2\rho_0^2}[\frac{1}{sn}(-1+n^2)\cot^{-1}(\frac{1}{sn}-i)+\frac{2}{1+(\frac{1}{sn}-i)^2}+\frac{i}{sn}(n-1)] \quad ,s=a_0\delta \tag{15}$$

In calculation of the average surface displacements, the parameters $\{\varepsilon^2<\rho_1^2>/\rho_0^2\}$ are taken to be $\{5\times10^{-4},\ 4.5\times10^{-3},\ 1.25\times10^{-2}\}$. Fig.1 shows the real and imaginary parts of the effective wave numbers g_1 and g_2. In the small values of $a_0\delta$, $Im(g_1)$ is considerably large compared to $Im(g_2)$, whereas $Im(g_1)$ is of same order as $Im(g_2)$ in the large values of $a_0\delta$. And the real parts $Re(g_1)$ and $Re(g_2)$ are almost flat along the abscissa $a_0\delta$. As the refractive index of the density $\varepsilon\rho_1/\rho_0$ fluctuates over the ensemble of realization of the medium, the values of $Im(g_1)$ and $Im(g_2)$ have physical significance related to the scattering effect of wave motion. Figs.2(a)~2(c) indicate three components of the amplitude characteristics of the average surface displacements and those of the surface displacements of the nonstochastic elastic medium are shown as a dotted line. When those waves are propagated in a medium with random inhomogeneity, the amplitude characteristics of the average surface displacement seem to decrease as if the damping characteristics exist in a propagation meidum. It may be understood that this apparent damping phenomenon is derived from the scattering of wave motion in the randomly distributed density. Figs.3(a)~3(c) show the dependence of the phase characteristics corresponding to Figs.2(a)~2(c) with respect to the wave traveling distance and wave number. The wave motion varies according to the spatial variation of the refractive index of a random medium, and the scattering of the wave motion grows up with the increase of path length.

To make the scattering effect clear, it is necessary to explain the dependence between the fluctuation of the wave field and the variation of the refractive index. To find such dependence theoretically, it is convenient to introduce the wave parameter

$$\Lambda=\frac{4r}{s\delta} \tag{16}$$

defined as the ratio of the size of the first Fresnel zone to the scale of the inhomogeneity. The scattering depends essentially on the value of the wave parameter Λ. To explain the amplitude and phase fluctuation due to the random inhomogeneity, the following two cases are considered, which correspond to two limiting values of the wave parameter; $\Lambda\ll1$ and $\Lambda\gg1$. In Fig.4, the fluctuations of the average surface displacements $f_i(i=z,\ r,\ t)$ are presented with respect to the nondimensional correlation length δ. In this figure, the fluctuation grows up rapidly with the increase of wave number and path length of wave motion for $\delta>1$. Since the dimension of the first Fresnel zone in the case $(\Lambda\ll1)$ is small in comparison with the scale of the random inhomogeneity, the deviation of the refractive index from its mean value has the same signs within the medium. Therefore, some of wave scattered by the different elements of the medium arrive at the observation point in phase. Thus, the scattering effect is emphasized with decreasing the average surface displacements for large value of a_0r. For $\delta<1$, the wave length being large compared with the scale of the inhomogeneity ($\Lambda \gg 1$), the deviation of the refractive index from its mean value has the different signs at the different points of the random medium. Therefore, all of the elementary scattered wave do not arrive in phase at the observation point. As they are partially interfered, the fluctuation grows up slowly and the damping effect is not so much emphasized in the amplitude characteristics of the average surface displacement for small value of a_0r.

CONCLUSIONS

The formulation and numerical solution of the seismic wave propagation problem in an elastic continuous random medium are facilitated by applying the perturbation technique in order to analyze statistically the influence of geological effect on the seismic ground motions. The analysis and illustrative example contribute qualitative information which suggests the following general conclusions:

1) the geological irregularity should have influence on the seismic ground motion statistically,
2) the main influence seems to be the scattering of seismic waves,
3) the scattering waves due to random geological inhomogeneity induce the damping effect on the amplitude characteristics of the average surface displacements,
4) this apparent damping effect depends upon the correlation length of random inhomogeneity, wave number and travelling distance, and
5) the scattering effect can be distinctly explained by virtue of two cases corresponding to the limiting values of the wave parameter.

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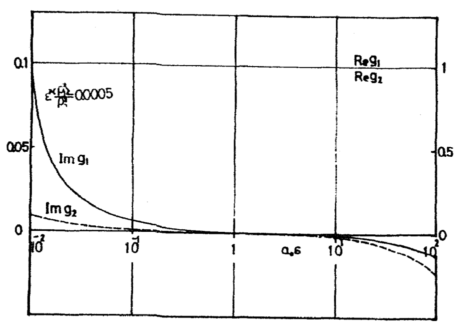
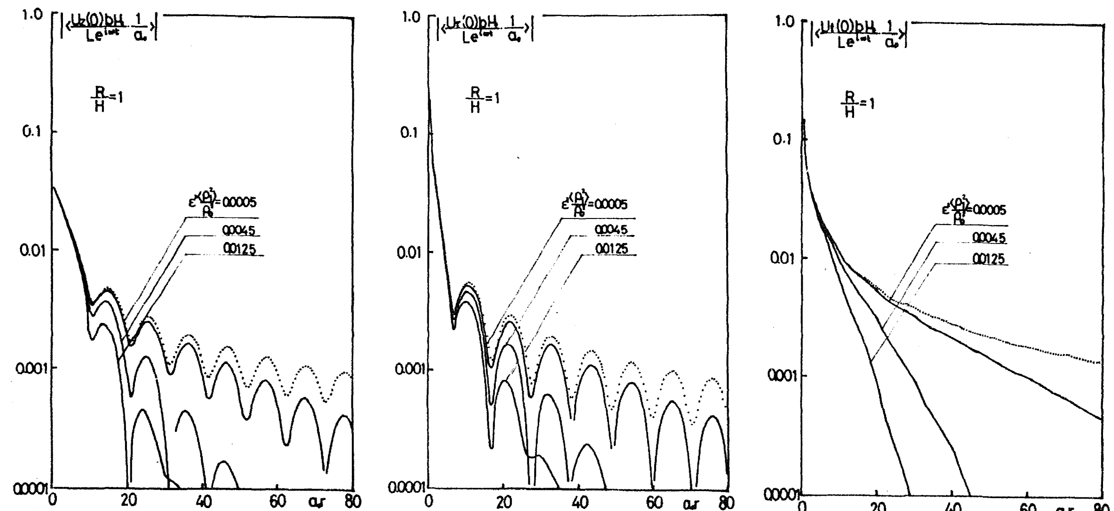


Fig.1 Effective wave number of a random medium



(a) vertical direction (b) radial direction (c) cross-radial

Fig.2 Amplitude characteristics of average surface displacement

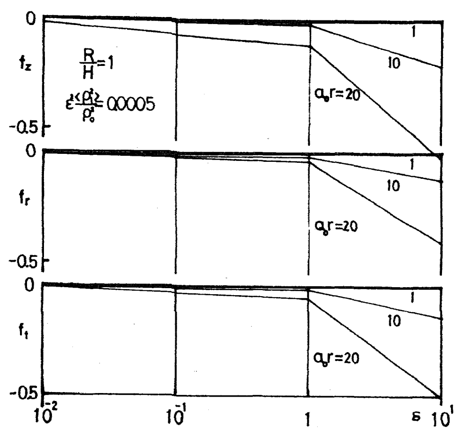
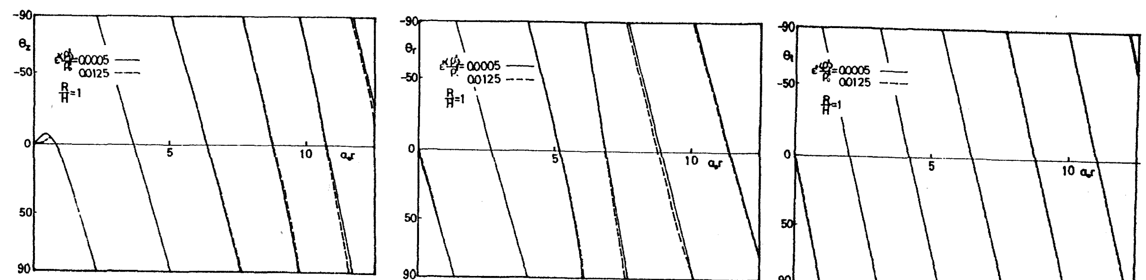


Fig.4 Fluctuations of amplitude characteristics



(a) vertical direction (b) radial direction (c) cross-radial direction

Fig.3 Phase characteristics of average surface displacement