SHORT PERIOD SURFACE WAVES IN A LAYERED MEDIUM

by

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SYNOPSIS

In this paper we investigate some properties of surfaces waves at relatively short periods and small epicentral distances by means of existing seismological techniques. We assume a single layered model of the earth and calculate the Love wave displacement spectra that would be produced by shallow earthquake sources located within the layer. The interference between the higher modes is shown to be a significant factor in producing complex ground motions. The amplification of the waves between the interface and the free surface is shown to have a highly anomalous behavior due to the presence of the higher modes.

INTRODUCTION

A majority of the studies on site effects have so far been carried out under the assumption that all of the energy released by the source travel as spherical body waves through bedrock and propagate vertically thereafter as plane waves through the layers. While this may be approximately true for deep focus earthquakes, there is strong evidence that in shallow earthquakes a large percentage of the energy is trapped in the layer and propagate horizontally as surface waves. The amplitudes of the surface waves decay less rapidly than those of body waves with epicentral distance and frequency.

Thus it appears that in strong ground motion studies the contribution of the surface waves must be carefully assessed. Although a large body of seismological literature exists on the subject, most of the available results are applicable at long periods (> 10 sec.) and large epicentral distances (> 100 km). We are currently involved in a systematic study of surface waves at shorter periods (.1-10 sec.) and smaller epicentral distances (10-100 km). In this note we present a number of results based on our preliminary investigation of Love waves propagating in a single layered two dimensional medium.

THEORY

The theory of Love wave propagation is well established and can be found in standard seismological literature (see, e.g. [1]). We quote only the relevant results.

The Love wave phase velocity c_n in a single layered medium of layer thickness H is the real root of the equation,

$$\eta_{1n}H = n\pi + \tan^{-1}(\mu_2\eta_{2n}/\mu_1\eta_{1n}),\quad n = 0,1,2,\ldots \tag{1}$$

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where n is the mode number,

$$\eta_{1n} = \omega(\beta_1^{-2}-c_n^{-2})^{1/2}, \quad \eta_{2n} = \omega(c_n^{-2}-\beta_2^{-2})^{1/2},$$

ω is the circular frequency, μ_1, β_1 are the shear modulus and the shear wave velocity in the layer and μ_2, β_2 are those in the half space. It can be shown that $\beta_1 \leq c_n \leq \beta_2$ and that the higher modes appear at the cut off frequencies,

$$\omega = \omega_n = n\pi\beta_2/H\{(\beta_2/\beta_1)^2-1\}^{\frac{1}{2}}, \; n = 1,2,... \tag{2}$$

Taking the x-axis along the horizontal and y-axis vertically downward from the free surface, the displacement spectrum at any point within the layer may be expressed in the form,

$$u(x,y,\omega) = \sum_{n=0}^{N(\omega)} U_n(y,\omega)\exp(i\omega x/c_n) \tag{3}$$

where $N(\omega)$ is the number of modes present at frequency ω, and the mode-shape functions $U_n(y,\omega)$ with surface amplitude S_n are given by

$$U_n(y,\omega) = S_n\cos(\eta_{1n}y), \; 0 \leq y \leq H$$

$$= S_n\cos(\eta_{1n}H) \; \exp\{-\eta_{2n}(y-H)\}, \; y \geq H \tag{4}$$

In order to calculate S_n we introduce a strike-slip dislocation of (small) length $\delta\ell$ and dip angle θ located at depth $h(< H)$ on the y axis. We assume that the dislocation has a time dependence $f(t)$ so that the source (displacement) spectrum is $F(\omega)$, the Fourier transform of $f(t)$. It can be shown that for $x > 0$,

$$S_r = 2\delta\ell F(\omega) \; (P_n\sin\theta + i \; Q_n\cos\theta)/R_n \tag{5}$$

where,

$$P_n = \eta_{1n}\cos(\eta_{1n}h), \; Q_n = \eta_{1n}^2 c_n\sin(\eta_{1n}h)/\omega$$

$$R_n = 2\eta_{1n}H + \{1-(\beta_1/\beta_2)^2\} \sin(2\eta_{1n}H)/\{(\beta_1/c_n)^2-(\beta_1/\beta_2)^2\}$$

The amplification of the surface waves between the interface and the free surface can be calculated from (3) through the relation

$$A_s(x,\omega) = |u(x,o,\omega)/u(x,H,\omega)| \tag{6}$$

It may be recalled, for comparison purposes that for plane shear waves with incident angle ϕ the amplification is given by

$$A_B(\omega) = 1/\cos(\nu_1 H) \tag{7}$$

where $\nu_1 = \omega\{\beta_1^{-2}-\beta_2^{-2}\sin^2\phi\}^{1/2}$. Thus resonance occurs at the frequencies,

$$\omega = \Omega_n = (2n+1)\pi\beta_2/2H\{(\beta_1/\beta_2)^2-\sin^2\phi\}^{1/2}, \quad n = 0,1,2,\ldots \qquad (8)$$

NUMERICAL RESULTS

The numerical results are presented for a model in which $\beta_1 = 1.0$ km/sec., $\beta_2/\beta_1 = 1.3$, $\mu_2/\mu_1 = 3.3$, and $H = 1.3$ km. The solution of (1) for these assumed values of the site parameter is plotted in Fig. 1. It can be seen that the phase velocity is multiple valued for $\omega > 2.2$ rad/sec., and that the number of modes increases to 22 at $\omega = 60$ rad/sec.

In order to calculate the spectral amplitudes we assume that the slip function is a step of magnitude D in time. Then $F(\omega) = iD/\omega$ for $\omega \neq 0$. We further assume that the source is either located near the interface ($h = 1$ km) or near the free surface ($h = .25$ km). The normalized spectral surface amplitudes $|u(x,o,\omega)\beta_1/D\delta\ell|$ are plotted against frequency in Fig. 2, for two epicentral distances ($x = 8$ km, 15 km) in the case of the interface source ($h = 1$ km). The only common feature in the two curves is the large amplitude near $\omega = 1.2$ rad/sec., which is caused by the fundamental mode alone. The secondary peaks at higher frequencies are more important in earthquake engineering studies. The spectral amplitude at $x = 8$ km for the surface source ($h = .25$ km), exhibit more prominent secondary peaks (solid curve, Fig. 2) than that for the interface source ($h = 1$ km). The complexity of the spectra at high frequencies is caused by strong interference between the higher modes. Although the plots have been terminated at $\omega = 20$ rad/sec. similar pattern is repeated at higher frequencies.

In order to further examine the influence of the site parameters on the surface motion the amplification factors $A_s(x,\omega)$ for $x = 8$ km and 15 km are plotted in Fig. 3 for a source depth of 1 km. The natural frequencies of the layer are also identified in the figure. Comparison of the amplification curves indicates that the surface wave motion at two stations with identical site properties may be considerably different. The peak amplitudes appear to be unrelated to the natural-frequencies of the layer.

We note one additional feature of the spectra. The ω^{-1} decay introduced in the source function is not sufficient to cause significant reductions in the amplitudes at high frequencies. Even with the introduction of damping ($Q \simeq 50$) and geometrical spreading ($\sim (\omega x)^{-1/2}$) factors the velocity and acceleration spectra calculated from the plots in Fig. 3 will have unreasonably large amplitudes at high frequencies. Thus the spectrum of the source function must decay more strongly with frequency, indicating the nexessity of including loss mechanisms at the source.

REFERENCE

1. Ewing, Jardetsky and Press, Elastic Waves in Layered Media, McGraw Hill, 1957.

ACKNOWLEDGMENT

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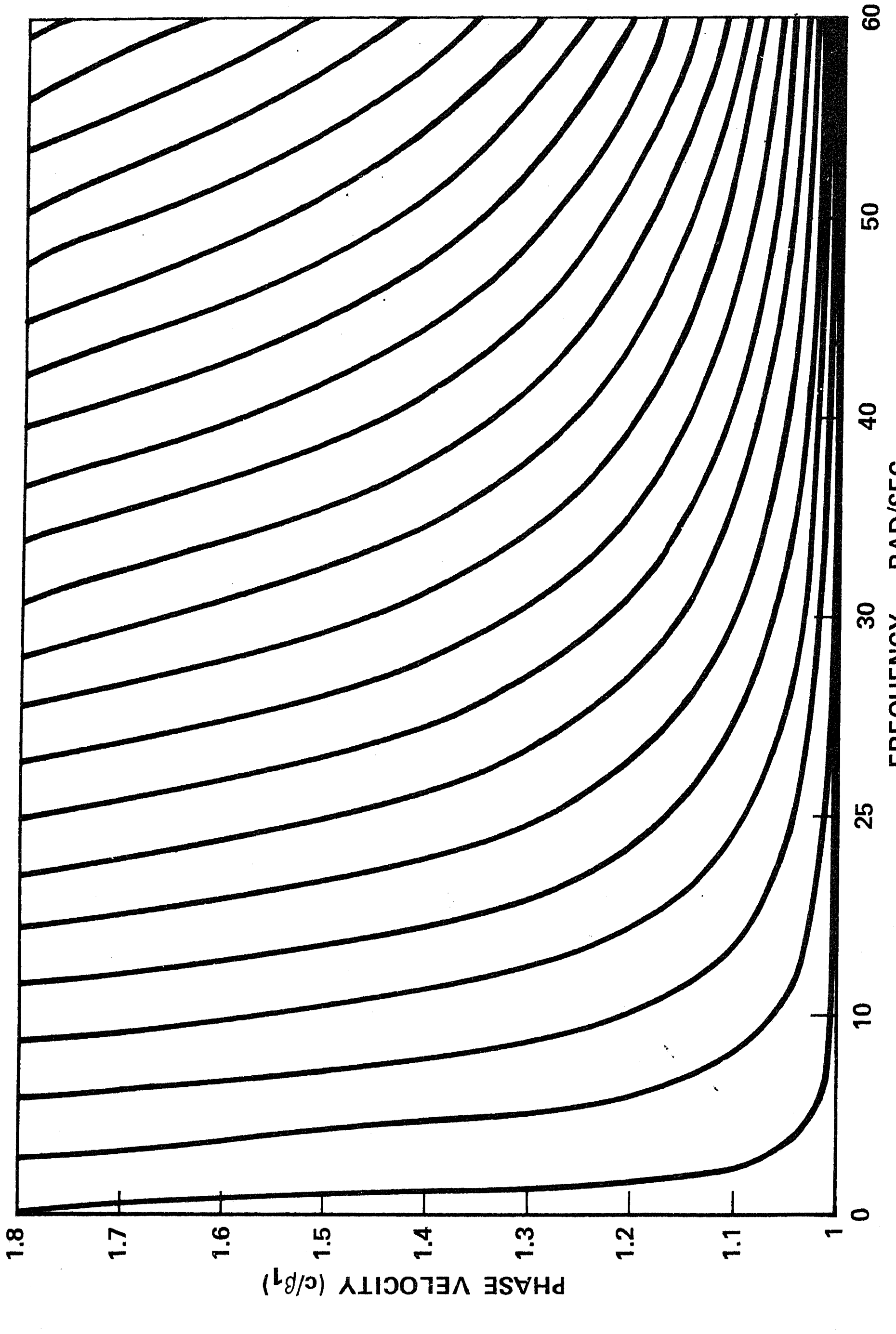

Figure 1. The Love Wave Phase Velocity Curves.

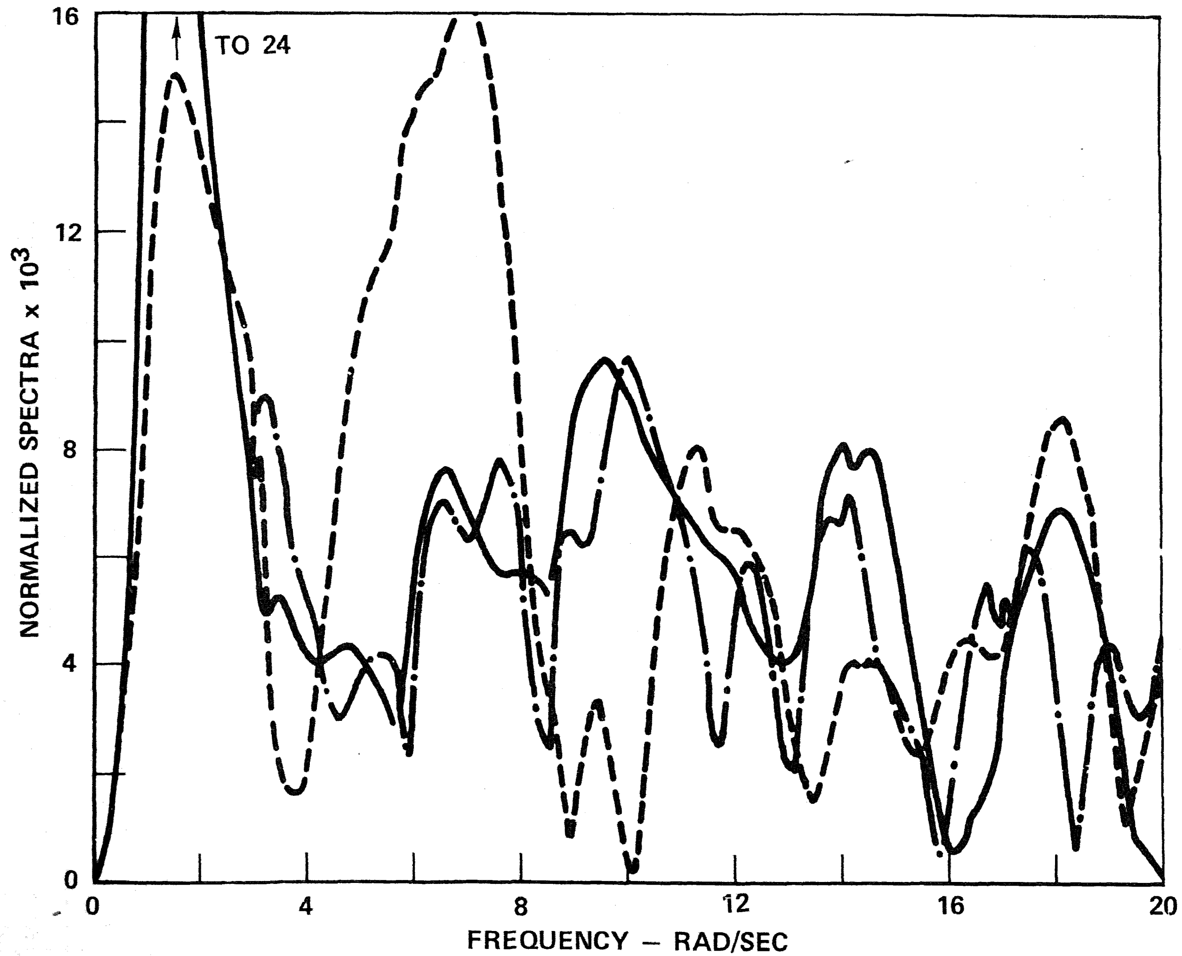

Figure 2. Normalized Displacements ; $|\beta_1 u(x,o,\omega)/D\delta\ell|$.
Solid — h = 1 Km, x = 8 Km, Dash–dots— h = 1 Km., x = 15 Km,
Dashes — h = .25 Km, x = 8 Km.

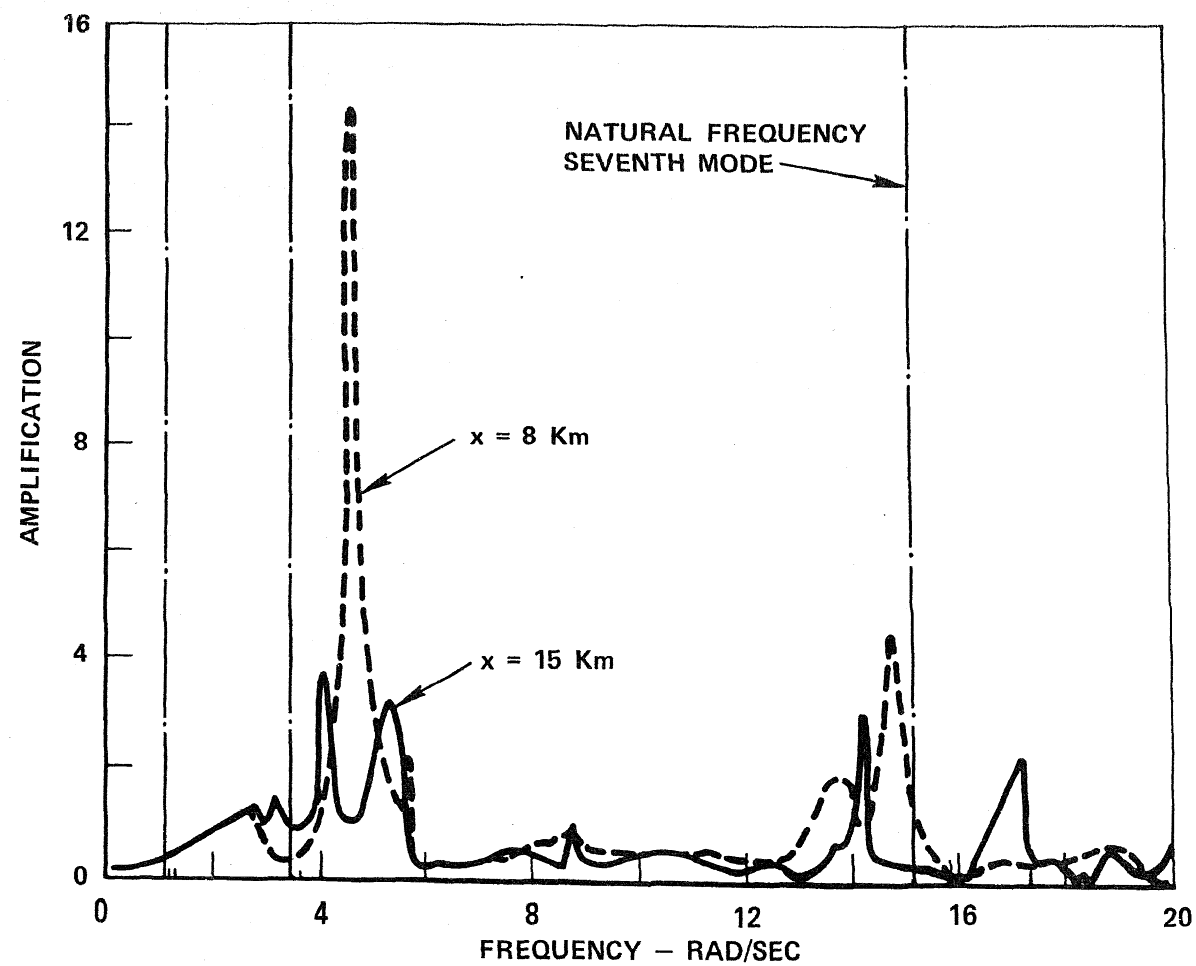

Figure 3. Amplification | u(x,o,ω)/u(x,H,ω) | for h = 1 Km. The vertical lines indicate natural frequencies at normal Incidence. The identified frequencies following these lines are those for incidence angles of 30° and 60° respectively.