SEISMIC RISK ANALYSIS FOR A METROPOLITAN AREA

by
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SYNOPSIS

A method to analyse the seismic effects over a system extended in space is developed. The system is characterized by the description in space of its dynamic and resisting properties. The seismic activity is controlled by three parameters β, λ, M_1, and by the spacial distribution of earthquakes. A penalizing function is used to convert the vector of performances into a scalar loss. The probability distribution of losses is generate using simulation. Comparisons between single site and system spread in space are analysed.

INTRODUCTION

The study of seimic risk at a site has been pursued since the late sixties, but only recently consideration of the seismic risk of systems spread in space became apparent[1,2]. The consequences of an earthquake in large systems (simultaneous failure or the non operation of two systems causes more loss than the sum of two separate failures), requires this kind of analysis: the total effect of an earthquake on a region or country cannot be expressed as a sum of individual losses. Rather, it is a nonlinear multiple of this sum, due to the combination of factors. The space problem becomes more complex when dealing with lifeline systems.

Due to uncertainty in determining the parameter values, the probability method is the sole capable of analysing comprehensibly the problem. Two main philosophies have been considered in this respect: on studying economical problems, determination of insurance rates or in optimization one should care for the mean value estimators of the parameters while on studying codes safety of populations, catastrophic insurance reserves or decision-making, an extreme value estimation is more convenient. Fig 1 presents the overall problem of seismic risk analysis in its generalized version[II]. Each box represents a single event and the string of events shows the interrelationships involved. The main parameters of this description are as follows (the quantification of those parameters referred in items (i) through (iv) has been done in great detail and is available in the literature; information is lacking on the parameters covering items (v) through (viii) but further details will be given in this paper): (i) generation of the earthquake process considered as as a stochastic point process random in time, space and magnitude; (ii) propagation of the seismic action from focus and fault to the site or sites according to the formula $y = b_1 e^{b_2 m} [f(R)]^{-b_3}$ in which y is the maximum acceleration, velocity or displacement, m is the magnitude, f(R) is a function of the focal distance R and b_1, b_2, b_3 are empirical constants, and taking into account the changes in the predominant period of ground motion with distance and magnitude; (iii) soil influence considered through a simple parameter ω_{soil}, Fig 2; (iv) structural response of a one degree of freedom system, ω_{st}, ξ_{st}, under the action of a stationary Gaussian stochastic process having a white noise power spectral density $S(\omega)$ filtered by the propagation and soil; (v) performance in terms of response through the damage

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II The word risk, as used in this paper, is related to those random variables which take into account all probable earthquakes to occur during a period of time; not just a single earthquake.

ratio function, DRF; (vi) individual loss, ILF; (vii) direct loss to the metropolitan area, GLF; (viii) global consequences to the metropolitan area, GCF. The quantification of all these parameters is enhanced with uncertainty some of which has a wide range of variation.

ENVIRONMENTAL RISK (ER)

Based on items (ii) through (iv) response spectra was developed for given m and R, Fig 3. Considering the randomness in time, space and magnitude, a distribution of extreme value for acceleration or response spectra could be obtained. For low risks ($<10^{-3}$) the final distribution that takes into account both the randomness of intensity and the randomness of response, has to be computed using the distribution of intensity and response. It is no longer valid to consider only the distribution of intensity and the mean value of maximum response[3].

Let's look at the extreme value distribution of maximum acceleration felt at a point or over a space element. The second case differs from the first one in the way we define R: an equivalent R_{eq} is the minimum distance between the earthquake epicenter $F(u^1,u^2)$ and the area element $M(u^1,u^2)$

$$R_{eq} = \mathrm{Min}\,[\,\mathrm{Dist}\,(F(u^1,u^2) - M(u^1,u^2))\,]$$

where u^1,u^2 are the space coordinates. Fig 4 shows the extreme value distribution of acceleration obtained for the case of earthquake generation line paralel to a metropolitan line of variable length. The following values characterize the system: $\lambda=4$/year, $\beta=2$, t=1 year, $m_0=3.5$, $m_1=8.1$, $b_1=1230$, $b_2=0.8$, $b_3=2$, f(R)=R+25, $\ell=100$ km, d=20 km. As shown in Fig 4, there is a substantial difference between seismic risk at a point and over a line.

DAMAGE RATIO FUNCTION (DRF)

Data has shown that there is a correlation between the degree of damage or damage ratio (cost of repair to replacement cost at the time of the earthquake) and the intensity of shaking. This observation led to the development of the concept of damage probability matrix[4], which is a global measure of damage and includes the environmental impact and the socio-economic standards of the region. It would seem more convenient to separate, at the beginning of the study of losses, the degree of damage from the costs resulting from such damages. Based on reliability theory and extending the concept of step function describing costs associated to limit states, to a continuous description, we have defined a damage ratio function, DRF, as, Fig 5

$$DRF = \begin{cases} 0 & \text{if} \quad Z<Yd \\ \left(\dfrac{Z-Yd}{C-Yd}\right)^{\alpha} & \text{if} \quad Yd\le Z\le C \\ 1 & \text{if} \quad Z>C \end{cases}$$

where Z is the response of the building, Yd is the yield displacement, C is the collapse point and α is a parameter characterizing the type of building system. In order to take into account the dispersion of the building resistance, Yd and C are considered as random variables, RV. Using the standard techniques of transformation of RV, the probability density function, pdf, of DRF is obtained as, Fig 5,

$$f_{DRF}(dr) = \int_E \int_F f_{Z,Y,C}\left(E, \frac{E-dr^{1/\alpha}F}{1-dr^{1/\alpha}}, F\right) \left|\frac{1}{\alpha}\, dr^{1/\alpha-1}\left(\frac{E-F}{(1-dr^{1/\alpha})^2}\right)\right| dE\,dF \qquad 1)$$

where the pdf of Z,Yd,C in most cases, is expressed as the product of the density of Z and the joint density of Yd and C, with correlation coefficient ρ. Fig 6 shows the influence of ρ on the pdf of DRF given Z for Z=1.25,..., 5.75, when Yd and C have bivariate normal densities with mean values 2 and 5 respectively and coefficient of variation $V_{Yd}=V_C=0.2$ and Z has an extreme type I distribution with mean value 1 and V_Z=0.2. Fig 7 shows the pdf of DRF for some combinations of values mentioned before. The influence of V_Z is also analysed.

While the DRF is a measure of structural response or an index of the level of damage suffered by the building during an earthquake, the individual loss function, ILF, is the translation of those damages into losses. It may include (i) direct losses such as cost of repair and replacement, fire, (ii) indirect losses such as losses caused to other buildings, decrease in productivity, operational losses and (iii) losses to people, Fig 8, and it may be obtained directly from the definition of DRF.

ANALYSIS OF LOSSES IN A METROPOLITAN AREA FOR A GIVEN EARTHQUAKE

In the previous discussion, the existence of buildings spread over a metropolitan area was not considered. The interaction between environmental risk and the space distribution of population and property is most important in the characterization of global losses, due to earthquake action. In the location of disaster relief facilities, predictions of the total amount of damage to be expected in large metropolitan areas, planning, etc. require the development of a method to analyse the seismic risk in an element of area. The knowledge of seismic intensity at a point is not sufficient. The analysis of risk in a given time interval T, for a metropolitan area cannot make use of the environmental curves for a site and sum the individual losses over the entire area because the RV that measure the individual risk are not independent. To find the solution of this problem we should look at the dependence among the different ILF for a given earthquake (m and R) and compute the global loss function, GLF, as

$$GLF_{m,R} = \int_{area} d(\,ILF(u^1,u^2)\;Morph(u^1,u^2)\,) \qquad 2)$$

where Morph (u^1,u^2) is a scalar quantity that represents the building characteristics for the area coordinate $r(u^1,u^2)$. If the ILF are essentially governed by shaking intensity, then the correlation coefficient ρ_{ILF} for two different buildings is approximately equal to one. In other cases is smaller then one. At the lower bound, $\rho_{ILF}=0$, the problem of spacial dependence no longer has any meaning. Making use of the probability laws that regulate the mean and variance of the sum of dependent RV, in the case of a line of length ℓ, one gets

$$E[GLF_{m,R}] = \int_0^{\ell} Morph(u)\; E[ILF(u)]\; du \qquad 3)$$

$$\sigma^2[GLF_{m,R}] = \int_0^{\ell} (Morph(u))^2\, \sigma^2[ILF(u)]\; du + 2\rho \int_{u=0}^{\ell}\int_{v=u}^{\ell} Morph(u) Morph(v)\; [ILF(u)]\; [ILF(v)]\; du\; dv \qquad 4)$$

To obtain the global consequence function, GCF, we should penalize the GLF according to the total number of loss of life and/or according to the degree of extension of damage. The aversion function which relates the number of fatalities with the effects on the society might be used.

ANALYSIS OF RISK

To compute the total amount of losses in an interval T, one should sum the individual contribution occurring during the interval

$$GRF = \sum_{t_i=0}^{T} GLF\,(m_i,R_i,t_i)\; C \quad e^{-\gamma t_i} \qquad \text{or} \qquad GRF(t) = \sum_{i=1}^{N(t)} GLF_i\; C \quad e^{-\gamma T_i} \qquad 5)$$

where $GLF(m_i, R_i, t_i)$ is the GLF for a given area and for an earthquake m_i and R_i occuring at time t_i, C is the cost per unit of time and γ is the discount factor. It is assumed that after each event the metropolitan area is rebuilt to its original condition. Making use of the moment generating functions,one can compute the expected values of GRF

$$E[GRF^{(j)}] = \sum_{t_i=0}^{T} E[(GLF(m_i,R_i,t_i)\, C\,)^j\, e^{-j\gamma t_i}] \quad j=1,2 \qquad 6)$$

The pdf of GRF approach the Gaussian distribution when the number of terms in 5) increases. Assuming that the GLF can be reduced to a simple RV with known pdf, occuring at arrival times $T_1,\ldots T_n$ of a Poisson process N(t) with mean value λ_N, the process 5) has

$$E[GRF(t)] = \lambda_N E[GLF]\, C\, \frac{1-e^{-\gamma t}}{\gamma} \qquad 7) \qquad \sigma^2[GRF(t)] = \lambda_N E[GLF^2]\, C^2 \frac{1-e^{-2\gamma t}}{2\gamma} \qquad 8)$$

To illustrate this method, the following example of a metropolitan line paralel to the fault line is given. The values used in this simulation are hypothetical and do not represent a real case. Earthquakes with the characteristics presented before were generated uniformly for a 100 km line. The metropolitan line with 20 km has uniform distribution of construction: T_{st}=1 s; ξ_{st}=0.05; E[Yd]=2; E[C]=8; $V_{Yd}=V_C$=0.2; α=1; ρ=0; ILF=DRF; γ=0.08; C=0.5/km; population=50inhab/km; T_{soil}=0.4sec. For these values computations show that V_Z varies from 0.27 to 0.32. Using the value 0.3 and a type I distribution to represent Z, it was assumed that

$$E[DRF] = \frac{1}{2\pi}\left(\arctan g(m_Z - 5.0) + \frac{\pi}{2}\right) \qquad 9) \qquad \sigma^2[DRF] = 0.05\left(\cos\left(\frac{2\pi}{5} m_Z - 2\pi\right) + 1\right) \qquad 10)$$

Fig 9 compares the values obtained using the integration procedure,1) with 9) and 10). It also shows the mean value of casualties if loss of life is considered (when DRF=1). Fig 10 shows the variation of E[ILF] ,σ^2[ILF] and mean loss of life along the metropolitan line. Fig 11 shows the distribution of E[GRF] obtained from the simulation of 16 members of the family of earthquakes, based on 6). From the mean value and variance of GLF per year,using 7) and 8), the mean value and the variance of GRF for the period of 50 years, are respectively 53.12 and 30.70, V_{GRF}=0.10 while in the simulation E[GRF]= 52.172, σ^2[GRF]=87.486 and V_{GRF}=0.179.

CONCLUDING REMARKS

The results in the present simulation suggest: (1)risk for a metropolitan area must be defined in terms of global quantity,ie, total losses; not a single parameter has been identified to represent this risk and the entire pdf should be generated, (2) moderated earthquakes contribute in large part to the total losses, (3) the pdf of GLF shows a concentration corresponding to the moderate events and a spike due to the large ones, (4) for small λ and short interval of time, the normal approach cannot be used to generate the pdf of GRF, (5) the GRF has a wide dispersion that tends to decrease when the metropolitan area spreads, (6) based on the pdf of material and human losses, application of the present methodology can be carried on to the study of optimization,insurance, etc., Fig 12.

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2. Whitman, R.V., C.A. Cornell and G. Taleb-Agha, 1975: "Analysis of Earthquake Risk for Lifeline Systems", Proc. U.S. National Conference on Earthquake Engineering
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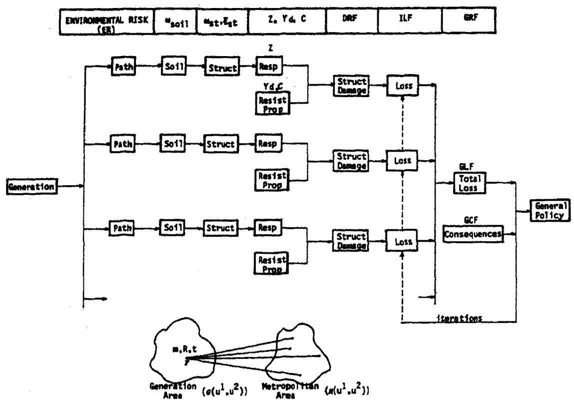


Figure 1. General sketch of the seismic risk event

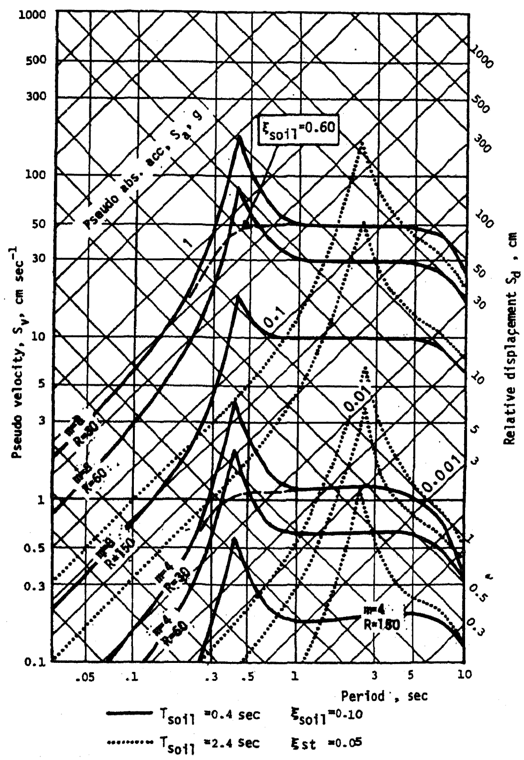


Figure 3. Response spectra for different combinations of T_{soil}, m and R

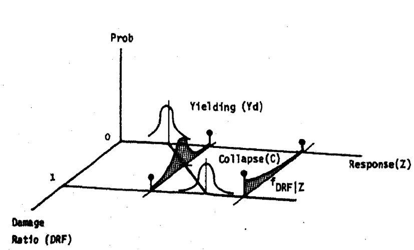
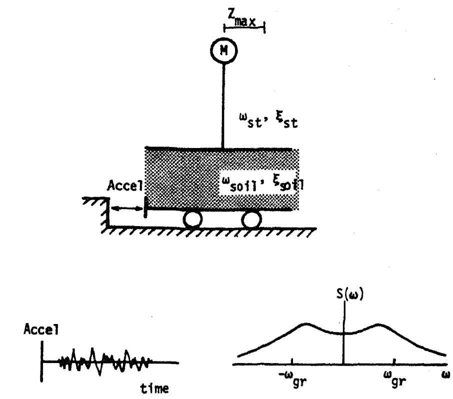


Figure 5. Probability of different levels of damage



Temporal description

Spectral description

INPUT ACCELERATION

Figure 2. Model for soil and structure

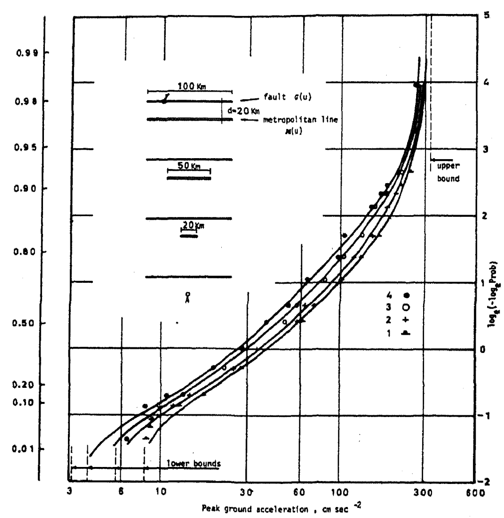


Figure 4. Environmental risk over a line segment

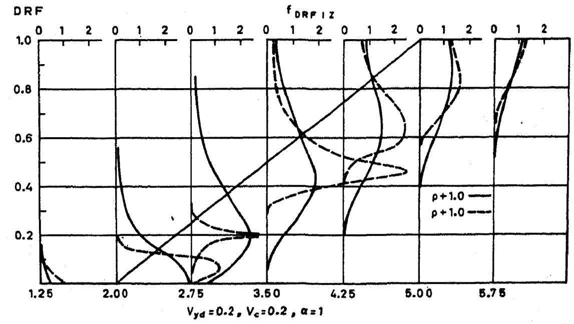


Figure 6. Influence of ρ

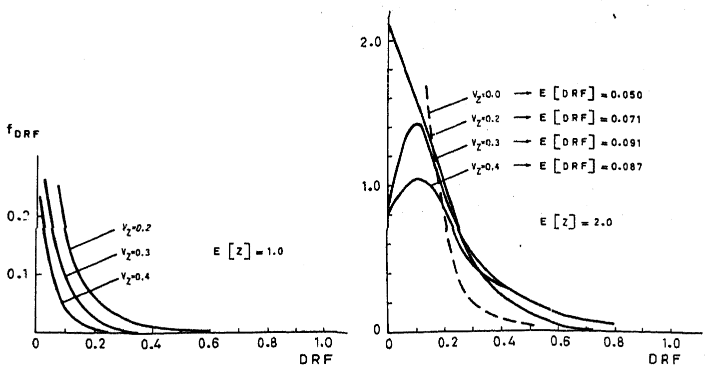


Figure 7. Probability density function of DRF; $V_{Y_d} = 0.2$; $V_C = 0.2$; $\rho = 0$; $\alpha = 1$

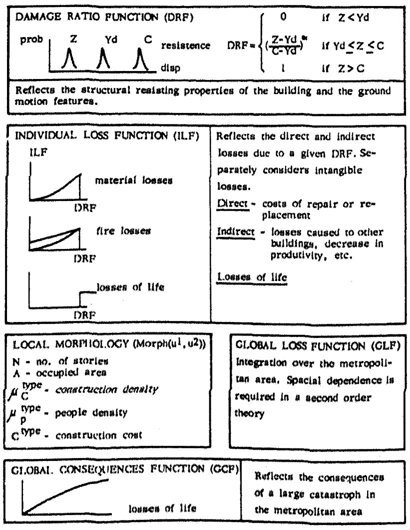


Figure 8. Sketch of the entire process to compute losses for a given earthquake

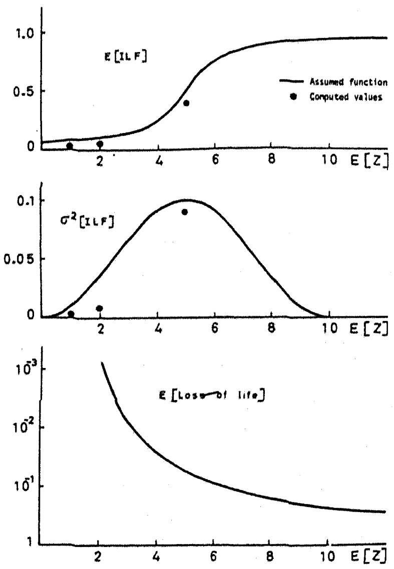


Figure 9. Functions used in simulation to compute ILF and mean loss of life

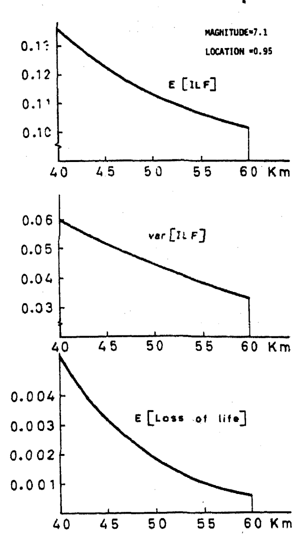


Figure 10. Variations of ILF and mean loss of life along the metropolitan line

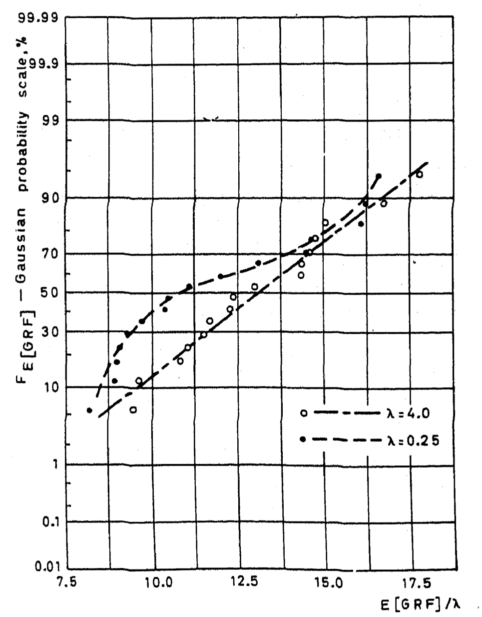


Figure 11. Probability distribution function of E [GRF]/λ

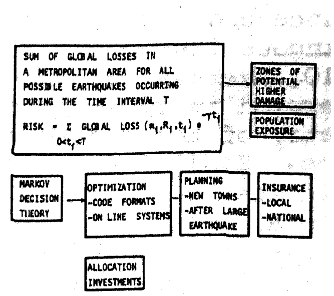


Figure 12. Some possible applications of the present theory

DISCUSSION

S.A. Anagnostopoulos (U.S.A.)

Can you please mention the sources on which the third curve of figure 10 is based ? Is it based on actual data ?

Author's Closure

With regard to the question of Mr. Anagnostopoulos, we wish to state that the third curve of Figure 10 is not based on any real data, but was obtained considering the following assumptions: (i) life loss occur whenever structural response crosses the collapse threshold i.e., when DRF = 1. For the values indicated in page 4, characterizing the structural system, the third curve of Figure 9 can also be taken from curve 16, Fig. 4.7 of Borges and Castanheta[5]; (ii) the structural response is computed uisng the concept of response spectrum for a given M and R.

It is well known that assumption (i) does not describe completely the full reality, because life loss occur at other levels of damage. An extension of assumption (i), worked out in relation to material losses, will be easy to include in the analysis. However, the paper wants to draw the attention to the difference between the continuous formulation of ILF and the step function formulation used for life losses (for more details see Oliveira[6]). Another important parameter, not discussed above, controlling the life loss is the percentage of people in the building when the earthquake occurs. One can consider during weekdays 3 different states for the system; working hours, home, commuting hours; and during weekends 2 states: in and out of town. A multinominal RV could be used to define the state of the system.

Past recent earthquakes, such as the Guatemala (Feb 4,76) and Romania (March 9, 77) earthquakes, are good examples to gather data for calibration of Figure 10. The variations of the pdf of losses along the metropolitan area are good indications of overall influences of earthquake action and of special dependence.

Four more Figures present a few more results of the simulation procedure. Figure a) shows the dependence between the variance and the mean value of GLF (for a given earthquake). Continuous line corresponds to the solution obtained with eqs. 9 & 10, whereas the dots corresponds to the simulation. The dispersion in relation to the line depend largely upon the geometry of the system and is a measure of the spatial dependence. Similar conclusions can be drawn from Figure b). It should also be emphasized that the more frequent events causing total costs below 1, do not inflict loss of life. Figure c) shows that, even though correlation between maximum acceleration felt anywhere in the metropolitan line and total cost, is not very good specially for large accelerations, it was found that this simple parameter is the best indicator of total losses. Figure d) shows the variation of the expected value and the variance of GRF in function of the life time of existance of the metropolitan area, for a member of the simulation.

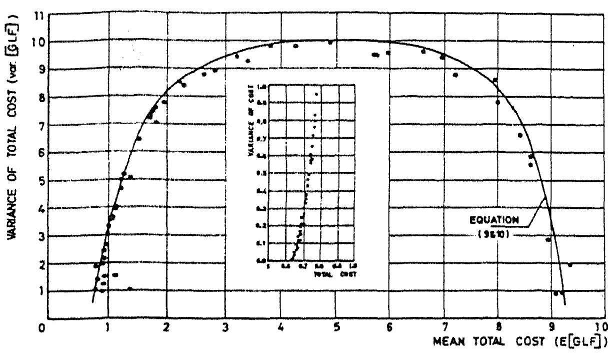


Figure a. Correlation between variance and mean total cost.

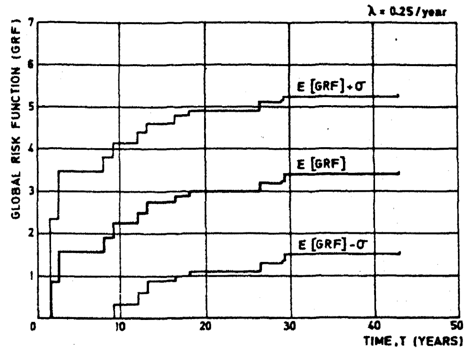


Figure d. The GRF as function of elapsed time,T.

5. Borges, J.F: and M. Castanheta, 1971: "Structural Safety", 2nd edition, Laboratorio Nacional de Engenharia Civil, Lisboa.

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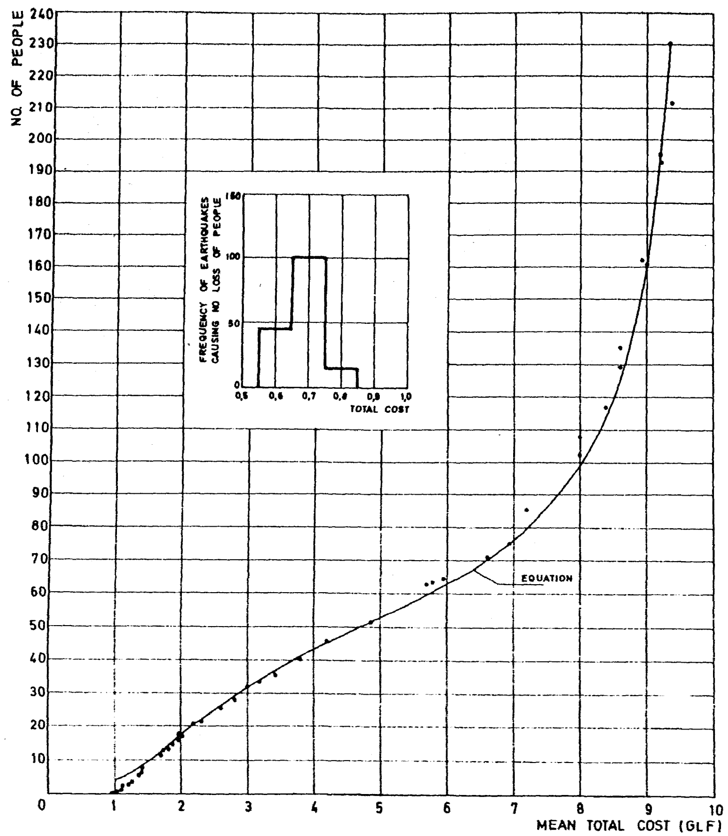

Figure b. Correlation between loss of people and mean total cost

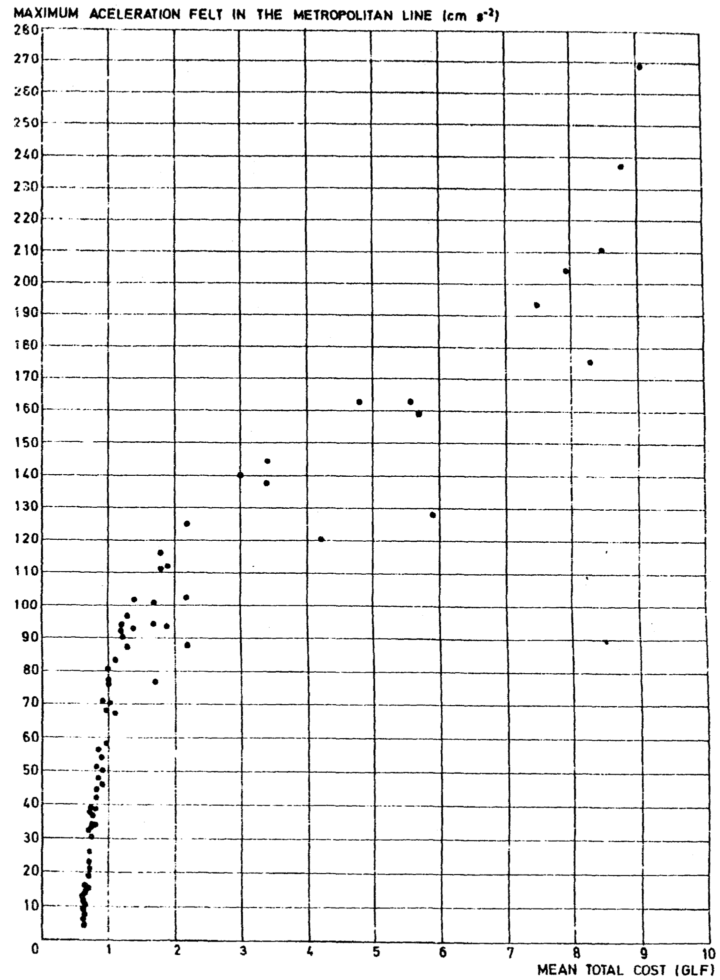

Figure C. Correlation between maximum acceleration and mean total cost.