ULTIMATE RESISTANCE OF MULTISTORY REINFORCED CONCRETE BUILDINGS WITH CANTILEVER SHEAR WALLS

by

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SYNOPSIS

This paper aims to make clear the ultimate resistance of reinforced concrete structures with cantilever shear walls. For this purpose, a structure is assumed to be a system (Fig.1) composed of cantilever shear walls, adjacent beams connected to shear walls, and frame elements. The ultimate moment-shear interaction (Fig.12) of a story of such structures is clarified theoretically by means of superposing the ultimate strengths of resisting elements. Calculated moment-shear interactions are compared with test results (Fig.13).

INTRODUCTION

Multistory reinforced concrete buildings with cantilever shear walls infilled in rigid frames are reasonable resisting systems against earthquake load. It is important to evaluate the ultimate resistance of multistory structures with cantilever shear walls for aseismic design. Several researches [2][3][4][5] have been carried out to make clear the elasto-plastic behaivors of such structures. However, an analytical method with respect to the evaluation of the ultimate resistance of such structures is not established in consideration of collapse modes of them.

The authors [1] presented the yield polyhedron of shear walls subjected to axial force, bending moment and shear force. In this paper, the ultimate moment-shear interaction of the multistory structures with cantilever shear walls is developed theoretically on the basis of the superposition of the ultimate strengths of resisting elements in structures. Calculated moment-shear interactions of such structures are compared with experimental results [2][3][4].

RESISTING MECHANISMS

Classification of Resisting Elements

In order to examine the ultimate resistance of multistory structures with cantilever shear walls, these structures are considered to be a system composed of cantilever shear walls, adjacent beams connected to shear walls, and frame elements, as shown in Fig. 1. Equilibrium equations for these structures are expressed [6], as follows (see Fig. 2):

$$Q_{oi} = Q_{wi} + Q_{fi} , \quad M_{oi} = M_{wi} + M_{gi} + M_{fi} \tag{1}$$

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where Q_{oi} : external shear force, M_{oi} : external moment $(= \sum_{j=i}^{n} Q_{oj} h_j)$,

M_{wi}, M_{gi}, M_{fi} : flexural resistances caused by shear walls, adjacent beams and frame elements,

$$M_{wi} = \sum_{j=i}^{n} Q_{wj} h_j , \quad M_{gi} = \sum_{j=i}^{n} m_{gj} , \quad M_{fi} = \sum_{j=i}^{n} Q_{fj} h_j \tag{2}$$

Then, the following are assumed to represent characteristics of resisting elements.

(1) Load-deformation relations of these elements are assumed to be rigid, perfectly plastic.
(2) Adjacent beams and frame elements fail in only flexural yielding.
(3) The variation of axial forces and deformations in resisting elements and N-Δ effects are not taken into account.
(4) Shear walls and columns are fixed on their foundations.

Characteristics of Resisting Elements

(Shear Walls) : A unit shear wall in a story of cantilever shear walls infilled in multistory frames is subjected to axial force, bending moment and shear force at the upper and lower boundaries. This shear wall may be idealized into a truss model [1], which is composed of column elements and brace elements hinged at the upper and lower boundaries, as shown in Fig. 3. Through this idealization, N_{wi}-M_{wi}-Q_{wi} yield polyhedron of shear walls of the i th story is clarified analitically [1], as shown in Fig. 4.

The axial forces (N_{wi}) acting on shear walls are assumed to be restricted as $N_{wb} < N_{wi} \leq N_{wa}$ (see Fig.4). In such a case, M_{wi}-Q_{wi} interaction of shear walls under constant axial forces is obtained from N_{wi}-M_{wi}-Q_{wi} interaction, as shown in Fig. 5. Then, six segments of M_{wi}-Q_{wi} interaction correspond to collapse mechanisms and their compatible conditions of shear walls in Fig. 5. These collapse mechanisms can be considered to be classified into two types, i.e., flexural yield type and shear failure type. Yield moment (M_{wyi}) and ultimate shear strength (Q_{wui}) of shear walls, which are represented in Fig. 5, are obtained [1], as follows:

$$M_{wyi} = (0.5 N_{wi} - {}_tN_{cyi} - {}_tN_{byi} \sin \alpha) \ell_w \tag{3}$$
$$Q_{wui} = ({}_cN_{byi} - {}_tN_{byi}) \cos \alpha \tag{4}$$

where ${}_tN_{cyi} = - a_c \, {}_s\sigma_y$ (: tensile yield force of column elements),

${}_tN_{byi} = - 2 p_w \ell_w t_w \sin \alpha \, {}_s\sigma_y$, ${}_cN_{byi} = B_e t_w f_c$ (: tensile and compressive yield forces of brace elements),

${}_s\sigma_y$: yield stress of reinforcing steel,
a_c : cross sectional area of longitudinal reinforcements in column,
p_w : reinforcement ratio in wall,
f_c : compressive strength of concrete,
B_e : effective width of concrete brace elements [1].

Then, the collapse modes of structures, in which the shear wall of the i th story fails in flexural yielding or shear, can be shown in Fig. 6.

(Adjacent Beams) : Adjacent beams connected to shear walls have two types, i.e., one is in the same plane as shear walls, the other orthogonal to them, as shown in Fig. 1. The states of load and deformation of these adjacent beams are illustrated in Fig. 7. It is assumed that yield hinges form at both ends of adjacent beams. Therefore, bending moment versus rotation angle relations of these adjacent beams are represented in Fig. 8. The bending moment is defined at the centroid axis of shear walls. Yield moment (m_{gyi}) regarding two types of adjacent beams is assumed to be expressed as $m_{gyi} = (m_{gpi})_y + (m_{goi})_y$, where $(m_{gpi})_y, (m_{goi})_y$: yield moments caused by adjacent beams in the same plane as shear walls and orthogonal to them.

If the axial deformation of shear walls is neglected, relations between rotation angles (ϕ_{gpi}, ϕ_{goi}) and flexural deformation (θ'_{wi}) of shear walls are expressed as $\phi_{gpi} = \ell_w \theta'_{wi} / 2 \ell_p$, $\phi_{goi} = \ell_w \theta'_{wi} / 2 \ell_o$.

Therefore, from the assumption (1), in order to produce the plastic deformation of adjacent beams, shear walls which are lower than these adjacent beams have to fail in flexural yielding (see Figs. 5 and 6).

(Frame Elements) : Frame elements of the i th story are represented by a resisting element composed of columns and beams, as shown in Fig. 9. It is assumed that plastic hinges at beams and columns adjacent to the upper and lower beam-column joints form enough to sway this element (see Fig. 9(b)). Load-deformation relations of this element are represented in Fig. 10. Then, yield shear force (Q_{cyi}) of this element is expressed as $Q_{cyi} = ({}_uM_{cyi} + {}_dM_{cyi}) / h_i$, where ${}_uM_{cyi}, {}_dM_{cyi}$: yield moments at each beam-column joint. Therefore, yield shear force (Q_{fyi}) of frame elements in the i th story is obtained as $Q_{fyi} = \Sigma Q_{cyi}$.

The following two cases are necessary in order to produce the plastic deformation of frame elements of the i th story. One case is that shear walls of the i th story fail in either flexure or shear. The other case is that shear walls which are lower than the i th story yield flexurally (see Figs. 5 and 6).

ULTIMATE RESISTANCE AND COLLAPSE MODES

Moment-Shear Interaction of Adjacent Beams and Frame Elements

Collapse modes with plastic flexural rotation (θ_i) and/or story sway (R_i) of cantilever shear walls at the i th story are taken into account to evaluate the moment-shear interaction of adjacent beams and frame elements between the i th and the n th story (see Figs. 5 and 6). That is, this moment-shear interaction can be obtained as vectors ($\pm (M_{gyi} + M'_{fyi})$, 0) and vectors ($\pm M''_{fyi}$, $\pm Q_{fyi}$)

$$\text{where} \quad M_{gyi} = \sum_{j=i}^{n} m_{gyj} , \quad M'_{fyi} = \sum_{j=i+i}^{n} Q_{fyj} h_j , \quad M''_{fyi} = Q_{fyi} h_i , \qquad (5)$$

as shown in Fig. 11.

Moment-Shear Interaction of Structures at the i th Story

The moment-shear interaction of structures in M_{oi}-Q_{oi} plane of $M_{oi}>0$, $Q_{oi}>0$ can be obtained in consideration of collapse modes shown in Fig. 6 of structures in which shear walls of the i th story fail in flexure or shear, as shown in Fig. 12. Then, yield moment (M_{oyi}) and ultimate shear force (Q_{oui}) are obtained, as follows:

$$M_{oyi} = M_{wyi} + M_{gyi} + M_{fyi}\ , \quad Q_{oui} = Q_{wui} + Q_{fyi} \tag{6}$$

where $M_{fyi} = M'_{fyi} + M''_{fyi} = \sum_{j=i}^{n} Q_{fyj}\, h_j$

Similarly, the moment-shear interaction of structures in the whole region of M_{oi}-Q_{oi} plane is obtained, as shown in Fig. 12. Then, this moment-shear interaction can be interpreted also by the superposition principle [7], i.e., this is obtained as an enveloped polygon by superposing the ultimate moment-shear interaction of each resisting element in structures (see Figs. 5, 11 and 12).

A shear span (h_{oi}) at the i th story is defined as $h_{oi} = \sum_{j=i}^{n} Q_{oj}\, h_j / Q_{oi}$. If the structures are subjected to lateral loads with the following two regions of the shear span (h_{oi}), the structures fail at the i th story with two collapse modes shown in Fig. 6, i.e., with shear walls of flexural yield type and shear failure type.

$M_{oyi}/Q_{oui} \leq h_{oi} \leq M_{oyi}/Q_{fyi}$: flexural yield type,

$h_i - M_{oyi}/Q_{oui} \leq h_{oi} \leq M_{oyi}/Q_{oui}$: shear failure type.

Yield shear force $(Q_{oyi})_{flex.}$ of structures with shear walls of flexural yield type is obtained as $(Q_{oyi})_{flex.} = M_{oyi} / h_{oi}$.

Moment-Shear Interaction of the Whole System of Structures

The yield moment (M_{oyi}) and ultimate shear force (Q_{oui}) at each story are transformed into yield moment (M_{byi}) and ultimate shear force (Q_{bui}) at the base story according to the distribution of lateral loads, respectively. Then, the moment-shear interaction of the whole system of structures is obtained as the minimum of M_{bi}-Q_{bi} interactions of every story.

COMPARISON OF EXPERIMENTAL RESULTS WITH CALCULATED VALUES

Calculated values are compared with experimental results of structures with cantilever shear walls of flexural yield type and shear failure type. Then, yield moments of beams and columns in structures are calculated by the analytical method [8]. The effective width of a concrete brace element in the truss model is assumed to be $0.2\ \ell_w \cos\alpha$ [1]. The tensile ultimate strength of reinforcing steel is neglected in the analysis in the foregoing chapters, but in this chapter it is considered in the calculation. However, the compressive ultimate strength of steel is neglected.

The summary of tests [2][3][4] is presented in Table 1. Experimental results are plotted in M_b-Q_b planes, as shown in Fig. 13. Since the ultimate strength of reinforcing steel is not described in the original documents [3][4], its assumed value is applied in the calculations.

The failure modes of shear walls in the structures are classified into the flexural yield mode and the shear failure mode by the broken line in Fig. 13(a). Similarly, the broken line in Fig. 13(b) represents the boundary of the following two failure modes of them. In the first mode, shear walls may fail in shear after the yielding of reinforcements of a tensile column element. In the other mode, these may not fail in shear. The test values of the ultimate moments of the structures with shear walls of flexural yield type are somewhat higher than the calculated yield moments, but coincide well with the calculated ultimate moments. The test values of the ultimate shear capacities of the structures with shear walls of shear failure type coincide well with the calculated ultimate shear capacities.

CONCLUDING REMARKS

The resisting elements in reinforced concrete multistory structures with cantilever shear walls are classified into cantilever shear walls, adjacent beams and frame elements. The moment-shear interaction of the shear walls under constant axial forces is obtained through the idealization of the shear walls into the truss model (Fig. 3). Then, the ultimate moment -shear interaction (Fig. 12) of the structures is clarified theoretically by summing up the moment-shear interactions of shear walls, adjacent beams and frame elements in consideration of collapse modes of structures (Fig.6). The calculated moment-shear interactions are compared with the experimental results [2][3][4] of the structures subjected to lateral loads (Fig. 13). It is shown that the coincidence between the calculated values and the test results is reasonable. Therefore, this analytical method may be useful for the aseismic design of the multistory buildings with cantilever shear walls.

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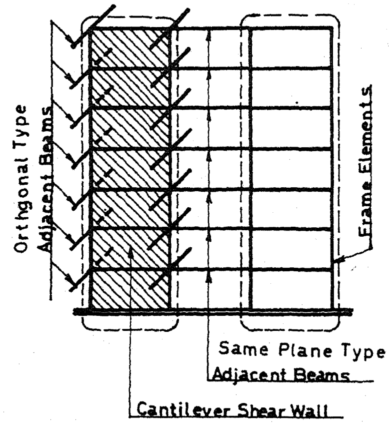


Fig.1 RESISTING ELEMENTS IN STRUCTURES

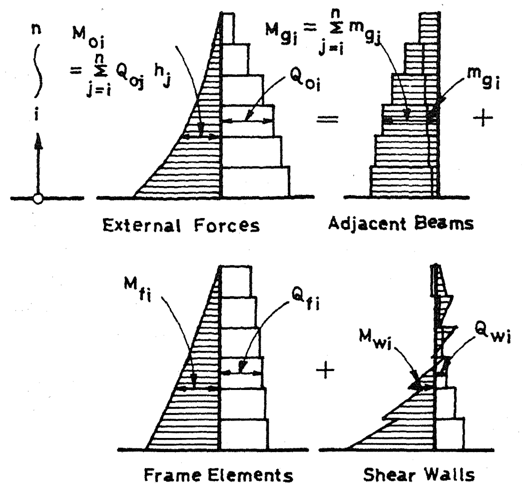


Fig.2 EQUILIBLIUM CONDITIONS OF STRUCTURES

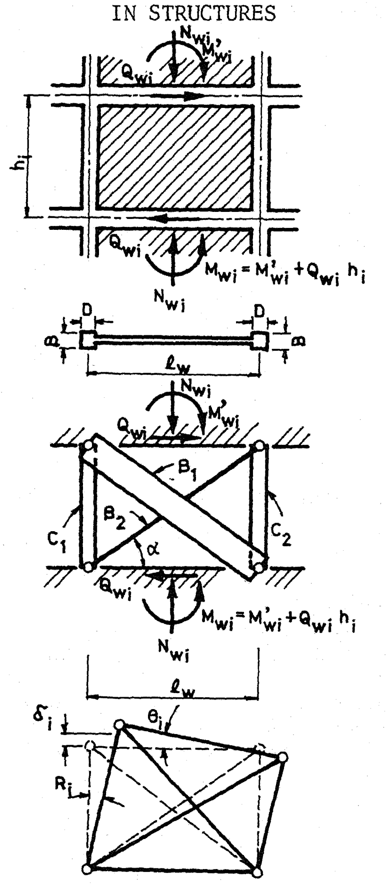


Fig.3 IDEALIZATION OF SHEAR WALLS

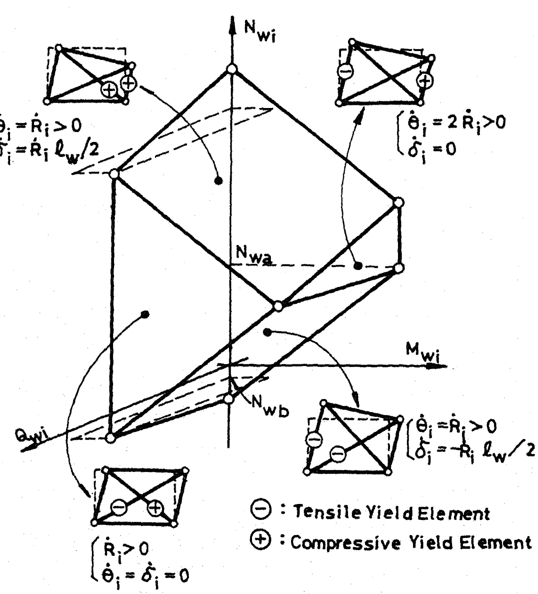


Fig.4 N_{wi}-M_{wi}-Q_{wi} Yield POLYHEDRON OF SHEAR WALLS

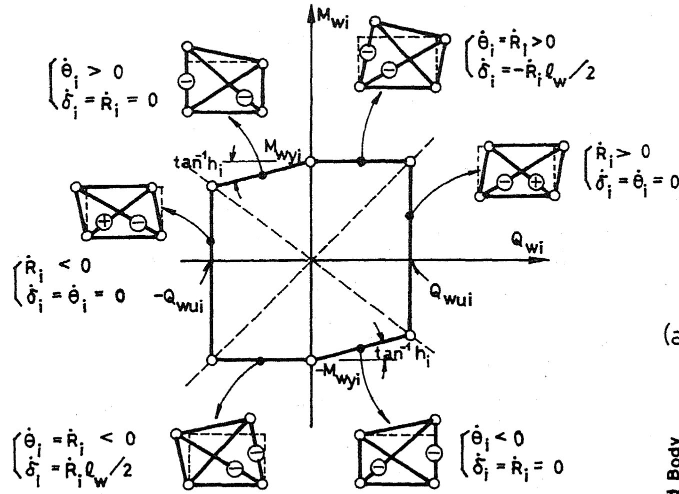


Fig.5 M_{wi}-Q_{wi} INTERACTION OF SHEAR WALLS

⊖ : Tensile Yield Element
⊕ : Compressive Yield Element
● : Plastic Yield Hinge

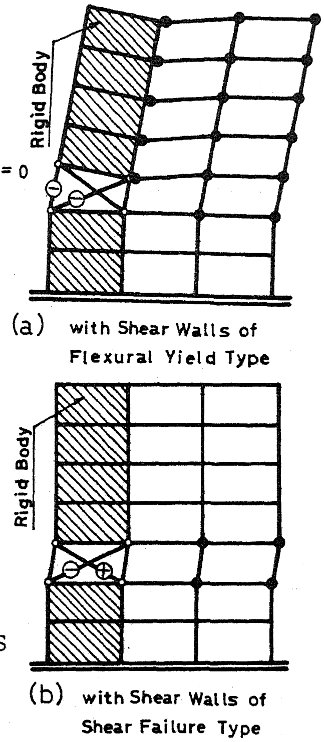


Fig.6 COLLAPSE MODES OF STRUCTURES

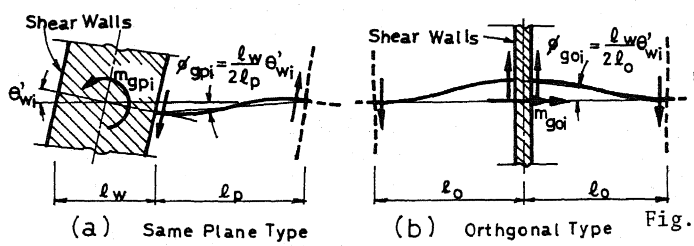


Fig.7 LOAD AND DEFORMATION OF ADJACENT BEAMS

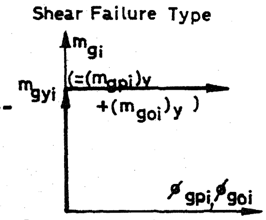


Fig.8 MOMENT-ROTATION RELATIONS OF ADJACENT BEAMS

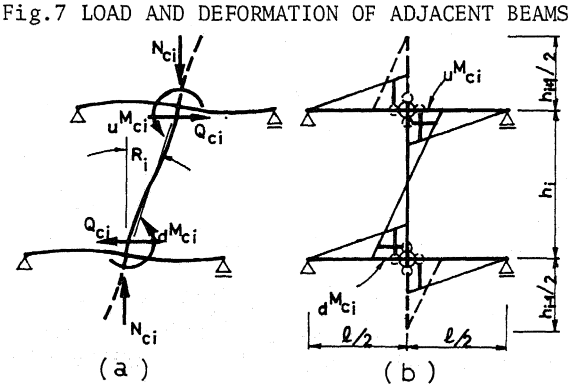


Fig.9 MECHANISMS OF FRAME ELEMENTS

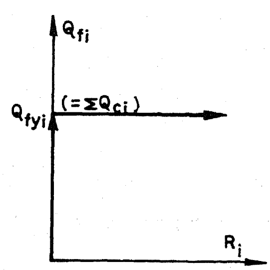


Fig.10 LOAD-DEFORMATION RELATIONS OF FRAME ELEMENTS

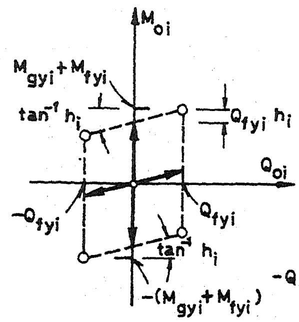


Fig.11 MOMENT-SHEAR INTERACTION OF ADJACENT BEAMS AND FRAME ELEMENTS

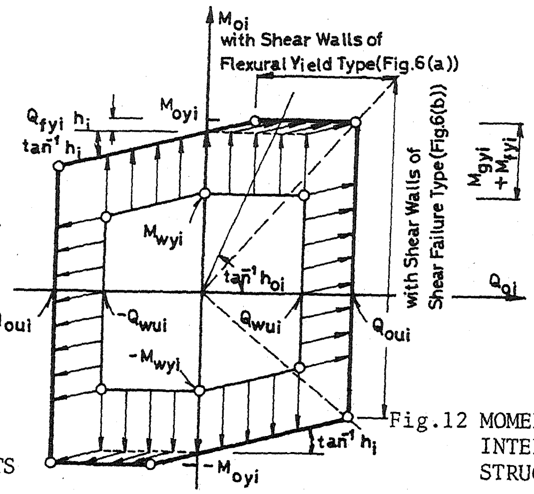
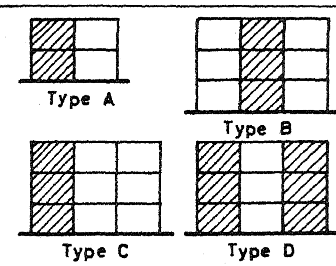


Fig.12 MOMENT-SHEAR INTERACTION OF STRUCTURES

Table 1 SUMMARY OF TESTS [2][3][4]

Ref.	Speci. No.	Struc. Type	Total Story	No. of Loading Points	Failing Story
[2]	1-4	A	1	1*	1 st
	5-12	A	2	2	1 st
[3]	13	B	2	1*	2 nd
	14,15	B,C	3	1*	1 st
[4]	16-19	D	1,2,4,8	1*	1 st



* Loaded at the top of structures.

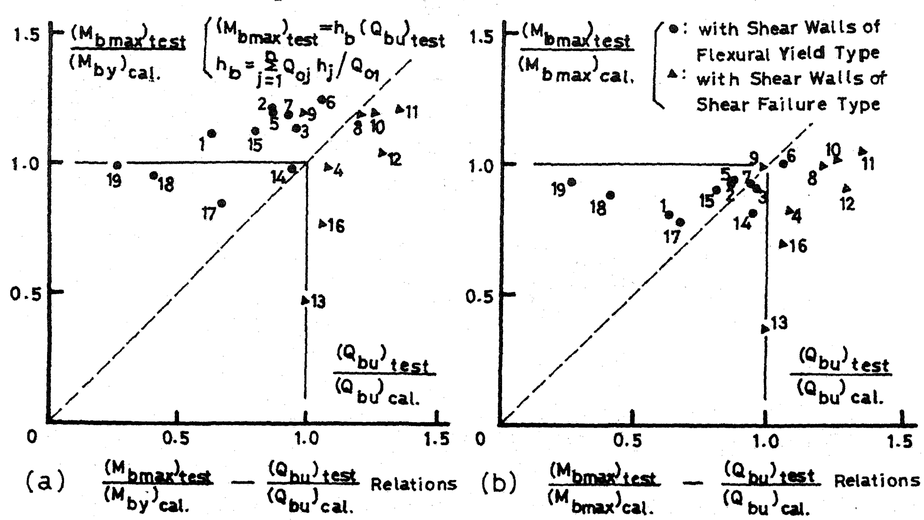


Fig.13 COMPARISON OF TEST RESULTS [2][3][4] WITH CALCULATED VALUES