MULTICOMPONENT SEISMIC DESIGN

BY

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SUMMARY

Some recent developments related to the definition of the simultaneous variation in various responses of a structure subjected to multicomponents of earthquake, are presented in conjunction with the response spectrum method of analysis. Any response in several modes of vibration under multicomponents of earthquake can be represented by the response in a small number of equivalent modes. The response values which are expected to occur simultaneously to cause the extreme probable effect lie on an interaction ellipsoid. Approximate methods suitable for practical design applications are also presented.

INTRODUCTION

The earthquake motion at any point can be resolved into three orthogonal directions, two horizontal and one vertical. Penzien and Watabe[1] have shown that two horizontal components, which are approximately radial and tangential with respect to the epicenter, are uncorrelated. Further, assuming that the two horizontal components have equal intensities, Rosenblueth and Contreras[2] have concluded that one can take zero correlation between motions along any pair of orthogonal horizontal directions. This assumption introduces a slight conservatism in the design. The vertical component of the earthquake motion always has some correlation with the horizontal components. This correlation may or may not be significant depending upon whether the different components excite different or the same modes, and whether the corresponding modal frequencies are close or apart.

Because of the wave motion a building footing is also subjected to three rotational components of motion. These rotational components have a strong correlation with the horizontal translational components, and the rotational components about the two orthogonal horizongal axea aremutally correlated[3]. The structures supported on multiple supports, such as bridges can be considered to be subjected to multiple "components." These components would have variable degrees of correlations depending upon the situation.

In the response spectrum method, one calculates the maximum probalbe response in various modes of vibrations due to each component of earthquake. Conceptually, these maximum probable responses can be considered to be a constant factor times the respective standard deviations. Proceeding on this basis one can show that the combined maximum probable value of any response R is given by

$$R_e^2 = A_{imjn} \, R_{im} \, R_{jn} \tag{1}$$

where R_{im} is the maximum probable response in the m^{th} mode of vibration due

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to the i^{th} component of earthquake as given by the response spectrum analysis; the A_{imjn} array signifies the correlation between the responses in mth and n^{th} modes due to i^{th} and j^{th} components, respectively. (In Eq. 1, and later in this paper, repeated sub or superscripts denote summation). In many practical applications, the effect of correlation between the earthquake components is ignored, (sometimes incorrectly,) in which case Eq. 1 is modified to be

$$R_e^2 = E_{mn} R_{im} R_{in} \qquad (2)$$

where the array E_{mn} represents the correlation between the responses in m^{th} and n^{th} modes and is a function of the two modal frequencis and respective dampings[4]. If the modal frequencies are far apart, the correlation is practically negligible, in which case Eq. 2 becomes

$$R_e^2 = R_{im} R_{im} \qquad (3)$$

Equation 3 represents the well known SSRS (Square Root of the Sum of the Squares) combination. Equations 1 to 3 are in succession, most general to most simple. For generality, in the present discussion, Eq. 1 will be used.

Equation 1 yields the maximum probable value of any response. Often the design of a structural element is based upon more than one response; for example, a column subjected to axial force P and bending moment M; or a metal element subjected to stresses $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}$. Heuristically, the maximum probable values of these responses would not occur at the same time. However, in most conventional designs it is implicitly assumed that the maximum probable response do occur simultaneously. This assumption introduces error on the safe side, which may be significant. Various aspects of this problem are discussed in subsequent sections.

SIMULTANEOUS VARIATION IN RESPONSES

A probabilistic treatment of the problem is given by Rosenblueth and Contreras[2]. Let us assume that there are N responses of interest R^r. For the earthquake responses alone, the square roots of the covariances of the vetor $R^r(t)$ define an ellipsoid in the N - Cartesian space R^r, with center at the origin. Assuming that the ground accelerograms are Gaussian processes, responses associated with any fixed exceedance probability are proportional to the corresponding standard deviations. Surfaces joining points of equal exceedance probability are then geometrically similar concentric ellipsoids. The ellipsoid of interest is of course, the one, whose axes have the values calculated from Eq. 1. From the application point of view, points on this ellipsoid represent the values of R^r which are expected to occur simultaneously with the same probability of exceedance.

Rosenblueth and Contreras[2] did not give an explicit equation for the ellipsoid under discussion. Using a quasi-mechanistic approach Gupta and Chu[5] and Gupta and Singh[6] arrived at the following equation for the ellipsoid

$$H^{rs} R^r R^s = 1 \qquad (4)$$

where the array H^{rs} is inverse of an array G^{rs}, which is given by

$$G^{rs} = A_{imjn} R^r_{im} R^s_{jn} \tag{5}$$

Parametrically, the equation of the ellipsoid can be represented in terms of the equivalent modal responses, [5,7]

$$R^r = K_\alpha R^r_\alpha \tag{6}$$

where

$$K_\alpha K_\alpha = 1 \tag{7}$$

The equivalent modal responses R^r are calculted from the following equation

$$R^r_\alpha R^s_\alpha = G^{rs} \tag{8}$$

where G^{rs} is given by Eq. 5. In the equivalent modal responses, the subscript α represents the equivalent mode number. The total number of equivalent modal responses is equal to N, the number of responses R^r under consideration. Sample equivalent modal responses for vaious applications are given in references 5,7,8 and 9.

Equation 4 represents an ellipsoid with its center at the origin, which is the case when only earthquake responses are being considered. In most cases, there will be responses R^r_o present due to other (static) loads, shifting the center of the ellipsoid to R^r_o; the modified Eq. 4 becomes

$$H^{rs} (R^r - R^r{}_o) (R^s - R^s{}_o) = 1 \tag{9}$$

Similarly, the modified Eq. 6 is

$$R^r = R^r{}_o + K_\alpha R^r_\alpha \tag{10}$$

In effect the ellipsoid given by Eq. 9 or 10 represents the seismic loading combined with other loads. For a safe design, the capacity diagram for the structural element should completely envelop this ellipsoid [2,5,6]. Fig. 1 illustates this criterion for a reinforced concrete column designed for axial force P and bending moment M. The dash line represents the loading diagram in the conventional design, when it is assumed that the maximum probable values of P and M occur simultaneously, in which case, the column capacity will have to be significantly higher. It can be shown that the conventional method can overestimate the design stress by a factor of upto $\sqrt{N}$, where N is the number of reponses R^r. For example, in case of a steel column[8] designed for the axial force P and biaxial bending moments M_x and M_y, the combined stress can be overestimated by a factor of upto $\sqrt{3}$ or 1.73. The factor of $\sqrt{N}$ is maximum possible; in most cases it will be between 1 and $\sqrt{N}$.

APPROXIMATE METHOD

Equations 9 and 10 can be quite cumbersome to use in actural design application. Approximate methods have been developed to overcome this problem.[2,10] In Eq. 10, the vector K_α is replaced by a constant vector C_α, thus giving the approxiamte value of the responses.

$$R_a^r = R^r_o + C_\alpha R_\alpha^r \tag{11}$$

Rosenblueth and Contreras[2] have evaluated the values of $C_\alpha$by applying the condition that maximum possible errors on safe and unsafe sides, respectively, are same. Gupta[10], on the other hand, has calculated C_α's by allowing the error on safe side only, and further by keeping it minimum. By permutating on the $\pm$ sign and on the sequence of the equivalent modes Eq. 11 gives $2^N N!$ sets of R_a^r values. The polyhedron obtained by joining the various R_a^r points in the R^r space will completely envelop the ellipsoid given by Eq. 10 in case of reference 10, and will closely intersect the ellipsoid in case of reference 2. In both cases, the ployhedron will be a close approximation to the ellipsoid.

A further simplification is possible by introducting slightly greater approximation. By makeing C_α's same for $\alpha \geqslant 2$, the number of points in the R^r space can be reduced to $2^N.N$, which is a substantial reduction.[2] It is noted that in the real life, it may not be necessary to consider all points. Most or many may be discarded simply by inspection.

CONCLUDING REMARKS

The simultaneous variation in various responses of interest (R^r) due to multicomponent earthquake and other loads is represented by an ellipsoid in the response space. Probabilistically, this ellipsoid also represents the surface of constant probability of exceedance for various responses. A safe design requires that the capacity diagram of the structural element completely envelop this ellipsoid.

The loading ellipsoid can be replaced by a polyhedron using approximate methods,thus simplifying the design process. For N responses of interest, as many as $2^N N!$ points may have to be calculated, although in practice many may be discarded just by inspection. A further approximation reduces the number of polyhedron points to $2^N.N$.

The conventional method, in which the maximum probable values of various responses are assumed to occur simultaneously, may overestimate the design stresses by a factor of upto $\sqrt{N}$, where N is the number of responses R^r.

The data required to calculate the points on the ellipsoid consist of the modal responses R_{im}^r obtained from the response spectrum analysis, and the array A_{imjn} representing the correlation between the modal responses under various components. In most cases at present, the array A_{imjn} is not known, or has to be approximated; e.g.,the effect of earthquake component correlation is often ignored. Further research is needed to evaluate the impact of such approximations.

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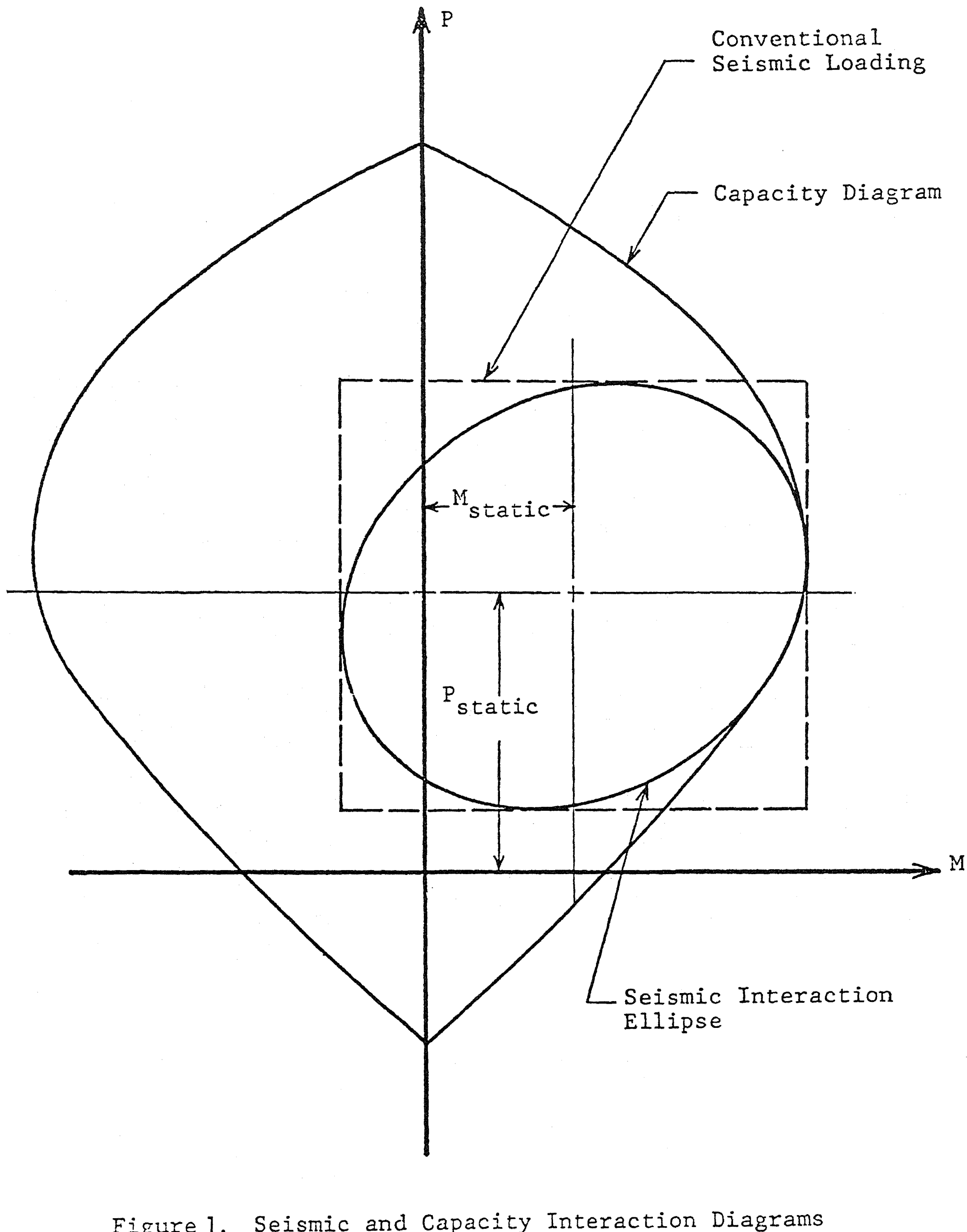

Figure 1. Seismic and Capacity Interaction Diagrams for a Reinforced Concrete Column