PARAMETRIC ANALYSIS OF THE SHEAR RESISTANCE OF MASONRY BUILDINGS

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SUMMARY

The basic equations defining the coefficients which are functions of the parameters affecting the seismic resistance of masonry buildings are developed. These parameters are: the quality and quantity of the masonry walls, and the design of the building given in terms of the distribution of the walls in both orthogonal directions and the distribution of the vertical loading. In the concluding table the seismic resistances of characteristic stone-masonry and brick-masonry buildings, given in terms of Base Shear Ceofficients, are presented.

1.0 INTRODUCTION

Knowing the parameters of strength and deformability of each particular wall in the story of a building (from its H-δ diagram) and knowing the expected horizontal inertial force acting on the building (given in terms of a base shear coefficient - BSC), and its distribution onto particular walls, a calculation method for evaluating the safety factor of each particular wall and hence of the building as a whole can be developed. If the condition for the stability of a building can be expressed in the form:

$$\frac{1}{\Phi} \cdot n\ (p_1) \cdot n\ (p_2) \ \ldots\ n\ (p_i)\ \ldots\ n\ (p_n) = 1, \qquad \ldots (1)$$

it is possible to define the parameters of the shear resistance of the building. In the above equation Φ is a function of the seismic coefficient K and the safety factor V (which together represent the base shear coefficient at failure "VK"), and n (p_i) is the coefficient which is a function of the parameter p_i.

2.0 THE SHEAR RESISTANCE OF WALLS

The hypothesis that the maximum principal tensile stress defines the initiation of diagonal cracks in the wall has been confirmed experimentally. On the basis of this hypothesis, the following parameters define the corresponding shear stress in the wall τ_0 at failure: the vertical bearing stress "σ_0", the experimentally obtained tensile strength of the wall material "σ_n", and the geometrical properties of the wall - factor b, which is a function of the ratio h/d:

$$\tau_o = \frac{\sigma_n}{b} \cdot \sqrt{1 + \sigma_o / \sigma_n}\ ; \qquad \sigma_n / b = \tau_k \qquad \ldots (2)$$

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For all cases where St.Venant's principle is valid, b = 1,5. For wider walls with h/d < 1,5 and simultaneously for larger eccentricities in the bearing surfaces e/d > 2/12, $1 \leq b \leq 1,5$. Taking into account the same value of b = 1,5 for every wall in the story, we are on the conservative side by about 10%.

3.0 DISTRIBUTION OF THE HORIZONTAL SEISMIC FORCE

The distribution of the horizontal seismic force acting in the story H_{tot} onto the individual wall r is obtained assuming rigid floor diaphragm action. It is proportional to the stiffness of the wall K_r:

$$H_r = H_{tot} \cdot \frac{K_r}{\sum_{i=1}^{n} K_i} , \qquad \dots (3)$$

where:

$$\frac{1}{K_i} = \frac{\delta_i}{H_i} = \frac{1.2\ h_i}{G\ F_i^*}\left[1 + \frac{1}{1.2} \cdot \frac{G}{D}\left(\frac{h_i}{d_i}\right)^2\right] = \frac{1.2\ h_i}{G} \cdot \frac{\alpha_i}{F_i^*} \qquad \dots (4)$$

From Eq. (3) and (4) can be obtained:

$$\frac{H_r}{F_r^*} = \tau_r = \tau_o \cdot \frac{\frac{1}{\alpha_r}}{\sum_{i=1}^{n} \frac{F_i^*}{F_{tot}^*} \cdot \left(\frac{1}{\alpha_i}\right)} ; \qquad \tau_o = \frac{H_{tot}}{F_{tot}^*}$$

Denoting the ratio of the cross-sectional area of the walls standing in the direction of the seismic force to the total cross-sectional area of all the walls in the story by λ_h, and the ratio of the cross-sectional area of the walls standing perpendicularly by λ_p, the above equation can be written in the form:

$$\tau_r = \frac{\tau_o}{\lambda_h} \cdot \frac{1}{\frac{\alpha_r}{\alpha_h} + \frac{\lambda_p}{\lambda_h} \cdot \frac{\alpha_r}{\alpha_p}} = \frac{\tau_o}{\lambda_h} \cdot \omega \qquad \dots (5)$$

The value of ω varies between 0,75 and 1,05. With $\omega = 1$ we are on the safe side. If torsional effects are to be taken into account, then the coefficient ω must be multiplied by an estimated torsional factor. In Fig.1 the nondimensional H-δ diagrams are shown and also the calculation of the coefficient ω is presented, assuming there to be an equal number of walls in each group of walls with a ratio $h/d \leq 3$.

4.0 CALCULATION OF THE SAFETY FACTOR OF BUILDINGS

The safety factor of a building depends on the safety factor of one of the relevant walls. As the relevant walls are denoted those walls which define the failure mechanism of the building, according to their relatively greater stiffness and larger number.

$$V = \frac{①\left[\text{The resistance of the revelant wall}\right] \quad ③\left[\text{The cross - sectional area of the walls standing in the direction of the seismic force}\right]}{②\left[\text{The total inertial force acting in the story}\right] . ④\left[\text{The factor of increasing or decreasing of the load acting on the relevant wall}\right]}$$

Substituting for the expressions in the above equation with the parameters and symbols listed in the appendix, the following equation for V can be obtained:

$$V = \frac{① \left[\tau_k \sqrt{1 + \frac{\sigma_{or}}{\sigma_n}}\right] . \quad ③ \left[\phi \, F_{tot} \, \lambda_h\right]}{② \left[K \, \phi \, F_{tot} \, n \, \gamma \, h \left(1 + \frac{q}{\gamma \, h \, \phi}\right)\right] . \quad ④ \left[\omega\right]} \quad \ldots \quad (6)$$

The vertical bearing stress in the relevant wall σ_{or} can be expressed as follows:

$$\sigma_{or} = \gamma \, h \, n + \frac{q \, F_{tot} \, n \, f}{\phi \, F_{tot} \, \lambda_h} \quad \ldots \quad (7)$$

Substituting this expression into Eq. (6) we obtain:

$$V = \left(\frac{1}{K \, n}\right) \left(\frac{\tau_k}{\gamma \, h}\right) \left(\frac{\lambda_h}{\omega}\right) \left(\frac{1}{1 + \frac{q}{\gamma \, h \, \phi}}\right) \cdot \sqrt{1 + \frac{\gamma \, h \, n}{1.5 \, \tau_k} \left(1 + \frac{q \, f}{\gamma h \phi \, \lambda_h}\right)} \ldots \quad (8)$$

If Eq. (8) is squared and solved for the number of stories n, then the following expression is obtained:

$$n = \frac{1}{3 \, V^2 \, K^2} \, . \, n \, (\tau_k) . n \, (\phi) . \left\{ Z_h^2 \, . \, Z_v \left(1 + \sqrt{1 + \frac{9 \, V^2 \, K^2}{Z_h^2 \, Z_V}}\right)\right\} \quad \ldots \quad (9)$$

After normalization of the factor in the curly brackets containing the parameters Z_h and Z_v (the parameters of the structural layout of the building) to be equal to 1 for $Z_h = 0{,}5$ and $Z_v = 1$, the parametric equation for failure state can be obtained (see the appendix for definitions of symbols:

$$\frac{1}{\Phi} \cdot \frac{1}{n} \cdot n\ (\tau_k) \cdot n\ (\phi) \cdot n\ (Z_h\ Z_v) = 1 \qquad \ldots (10)$$

5.0 THE SEISMIC RESISTANCE OF MASONRY BUILDINGS

Knowing the coefficients of parameters in Eq.10, the base shear coefficient VK, which expresses the seismic resistance of the building, can be calculated. Because the factor VK appears in the function of the parameters of structural layout $n(Z_h\ Z_v)$ too, the exact value of VK must be obtained by iteration.

In Fig.2 the tabulated values of the parametric functions $n(\tau_k)$, $n(\phi)$, and $n(Z_h\ Z_v)$ are presented. From the tables the influence of the individual parameters can be seen clearly.

In Fig.3 the calculated values of VK, the base shear coefficient at failure, are presented for two different types of masonry buildings:

- stone-masonry buildings: extreme cases of structural layout have been chosen from a large number of analyzed buildings with a wall thickness of ± 50 to 60 cm. In the case of old stone-masonry buildings there is no great difference in resistance between buildings with good and poor structural layout, the essential element of their shear resistance is the quality of the walls, which can be improved by injecting them with cement grout.

- clay-brick masonry buildings: the structural layout of old buildings with walls 45 cm thick is considered adequate ($Z_h = 0{,}45$); a structural layout with a value $Z_h = 0{,}30$ is still acceptable, whereas the so-called "tunnel" type of construction cannot be accepted. The third example represents a modern building with walls 20 cm thick, constructed with a mortar of good quality (nominal compressive strength = 50 or 100 kp/cm^2).

From a comparison of the calculated values the following can be clearly seen:

1. In seismic regions with an earthquake intensity of degree IX according to the MSK-64 scale (realistic values of the BSC are ~ 0,30 g) the failure of existing stone-masonry buildings is to be expected.

2. The walls of stone-masonry buildings can be strengthened to an adequate degree of safety by the injecting of cement grout. Thus the revitalization of stone-masonry buildings is possible.

3. Clay-brick masonry buildings - if properly designed and constructed - can be built up to 5 stories high in regions with a high expected earthquake intensity.

LIST OF SYMBOLS AND FORMULAS NOT EXPLAINED IN THE TEXT:

h_i - height of the wall,

d_i - length of the wall,

G - shear modulus of the wall,

D - deformability modulus of the wall,

F_i^* - cross-sectional area of the wall,

F_{tot}^* - total cross-sectional area of walls in the story

q - weight of the floor structure (dead load + live load),

γ - volumetric weight of the wall,

n - number of stories,

φ - ratio of the cross-sectional area of the walls supporting the floor structures to the net-area of the story,

f - part of the floor structure area which is supported by the walls standing in the direction of the seismic force,

F_{tot} - net-area of the story,

λ_h - ratio of the cross-sectional area of the walls standing in the direction of the seismic force to the total cross-sectional area of walls in story,

λ_p - ratio of the cross-sectional area of the walls standing perpendicular to the direction of the seismic force to the total cross-sectional area of the walls in the story,

λ_v - ratio of the cross-sectional area of the walls supporting the floor structures and standing in the direction of the seismic force to the total cross-sectional area of the walls in the story,

ω - correction factor for the distribution of the total seismic force to individual walls.

$n(\tau_k) = \frac{\tau_k}{\gamma h}$ .... function of the quality of the walls,

$n(\varphi) = \frac{1}{1+\frac{q}{\gamma h \varphi}}$ .... function of the parameter of the ratio of the area of bearing walls to the net-area of the story,

$\frac{1}{n}$ .... function of the number of stories,

$Z_h = \frac{\lambda_h}{\omega}$ .... parameter of the distribution of the total seismic force onto the walls standing in the direction of the seismic force,

$Z_v = \frac{1+\frac{q}{\gamma h \varphi}\frac{f}{\lambda_v}}{1+\frac{q}{\gamma h \varphi}}$ .... parameter of the distribution of vertical load onto the walls standing in the direction of the seismic force,

$\Phi = \frac{12V^2K^2}{1+\sqrt{1+36V^2K^2}}$ .... function of seismic load, given with the seismic coefficient K and safety factor V (base shear coefficient at failure)

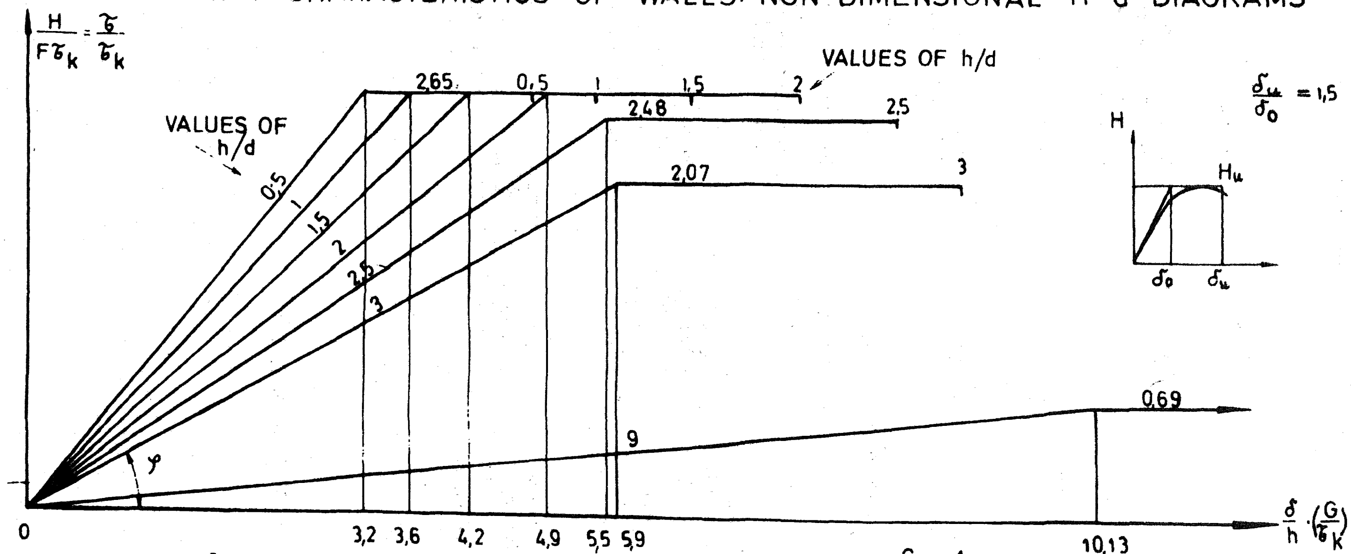
$VK = \Phi\sqrt{\frac{1}{6\Phi}+\frac{1}{4}}$

$n(Z_h Z_v) = \frac{Z_h^2 Z_v}{0.25} \cdot \frac{1+\sqrt{1+\frac{9V^2K^2}{Z_h^2 Z_v^2}}}{1+\sqrt{1+36V^2K^2}}$ ...... function of the structural layout of the building

APPENDIX

FIG. 1

DEFORMABILITY CHARACTERISTICS OF WALLS: NON-DIMENSIONAL H-δ DIAGRAMS



$$\frac{\delta}{H} = \frac{1}{K} = \frac{12 \cdot h}{G.F}\left[1 + \frac{1}{1,2} \cdot \frac{G}{D}\left(\frac{h}{d}\right)^{-2}\right] = \frac{1,2\,h}{G.F_i}(\alpha_i)$$

$$\tan \varphi = \frac{\tau}{\tau_k} \Big/ \left(\frac{\delta}{h} \cdot \frac{G}{\tau_k}\right) = \frac{H}{\delta}\left(\frac{h}{G.F}\right) = K\frac{h}{G.F} = \frac{1}{1,2\,\alpha_i}$$

$$\text{for } \lambda_h = 0,30, \quad \omega = \frac{1}{1,03 \cdot 0,82 + 0,70/0,30 \cdot 1,03/12,3} = 0,95$$

$$\frac{1}{\alpha_h} \cong \left[\frac{0,07}{2,3} + \frac{0,08}{1,9} + \frac{0,1}{1,6} + \frac{0,14}{1,3} + \frac{0,2}{1,14} + \frac{0,41}{1,03}\right] \cong 0,82$$

for $\frac{G}{D} = \frac{1}{6}$

h/d	α_i	tg φ	$\frac{\lambda_i}{\lambda_h}$
9	12,3	0,07	—
3	2,3	0,37	0,07
2,5	1,9	0,45	0,08
2	1,6	0,54	0,10
1,5	1,3	0,63	0,14
1	1,14	0,73	0,20
0,5	1,03	0,81	0,41
	Σ		1,00

FIG. 2 VALUES OF THE COEFFICIENTS OF THE PARAMETRIC FUNCTIONS

CONDITION OF STABILITY : $\frac{1}{\Phi} \cdot \frac{1}{n} \cdot n(\varphi) \cdot n(\tau_k) \cdot n(Z_h \cdot Z_v) = 1$

$$n(\varphi) = \frac{1}{1 + \frac{q}{\gamma h \varphi}} \quad ; \quad \gamma h = 5 Mp/m^2$$

q \ φ	0.06	0.08	0.10	0.15	0.20	0.30	0.40
0.400	.43	.50	.56	.65	.71	.79	.83
0.500	.38	.44	.50	.60	.67	.75	.80
0.600	.33	.40	.45	.56	.63	.71	.77
0.700	.30	.36	.42	.52	.58	.68	.74

$$n(\tau_k) = \frac{\tau_k}{\gamma h} \quad \quad \gamma h = 5 Mp/m^2$$

τ_k	1.0	1.5	2	5	10	15	20	25	Mp/m^2
$n(\tau_k)$	.20	.30	.40	1	2	3	4	5	

$$n(Z_h \cdot Z_v) = \frac{Z_h^2 \cdot Z_v}{0.25} \cdot \frac{1 + \sqrt{1 + \frac{9V^2 \cdot K^2}{Z_h^2 \cdot Z_v^2}}}{1 + \sqrt{1 + 36V^2K^2}}$$

$$Z_h = \frac{\lambda_h}{\omega} \qquad Z_v = \frac{1 + \frac{q}{\gamma h \varphi} \cdot \frac{f}{\lambda_v}}{1 + \frac{q}{\gamma h \varphi}}$$

Z_v \ Z_h	0.50	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.10
1	1.00	.83	.68	.54	.42	.31	.23	.15	.11
0.80	.85	.70	.58	.47	.37	.28	.21	.14	.10
0.60	.70	.60	.50	.41	.33	.25	.19	.13	.09
0.40	.57	.49	.42	.35	.28	.23	.17	.12	.08

FIG. 3 PARAMETRIC ANALYSIS OF THE SHEAR RESISTANCE OF STONE-MASONRY AND CLAY-BRICK MASONRY BUILDINGS (RESISTANCE IS GIVEN IN TERMS OF THE COEFFICIENT VK - BASE SHEAR COEFFICIENT AT FAILURE)

STONE - MASONRY BUILDINGS				
ANALYSIS OF STRUCTURAL LAYOUT				
PARAMETERS	GOOD LAYOUT		POOR LAYOUT	
φ	0.35		0.25	
$n(\varphi)$	0.83		0.77	
Z_h	0.45		0.35	
Z_v	0.83		0.80	
LABORATORY TESTS				
	BASIC	GROUT.	BASIC	GROUT.
τ_k [10^{-3} MPa]	20	80	15	80
γh [10^4 N/m^2]	6.7	6.7	6.7	6.7
$n(\tau_k)$	0.30	1.20	0.22	1.20
NUMBER OF STORIES	SHEAR RESISTANCE VK [in g]			
n = 1	0.20	0.56	0.12	0.41
n = 2	0.13	0.32	0.08	0.24
n = 3	0.10	0.24	0.06	0.18

CLAY - BRICK MASONRY BUILDINGS			
ANALYSIS OF STRUCTURAL LAYOUT			
PARAMETERS	OLD B'LDGS-GOOD LAYOUT, t=45cm	OLD B'LDGS-POOR LAYOUT, t=30cm	NEW BUILDINGS t = 20cm
φ	0.15	0.10	0.08
$n(\varphi)$	0.60	0.50	0.45
Z_h	0.45	0.30	0.40
Z_v	0.80	0.70	0.70
LABORATORY TESTS			
τ_k [10^{-3} MPa]	100	100	200
γh [10^4 N/m^2]	6.3	6.3	5.0
$n(\tau_k)$	1.60	1.60	4.0
NUMBER OF STORIES	SHEAR RESISTANCE VK [in g]		
n = 3	0.24	0.15	0.32
n = 4	0.20	0.13	0.26
n = 5	0.17	0.11	0.22