A CONTRIBUTION TO THEORETICAL PREDICTION OF DYNAMIC STIFFNESS OF SURFACE FOUNDATIONS

H. Tajimi[I]

SUMMARY

The present paper describes a set of influence functions due to a concentrated load dynamically applied on an elastic soil layer in accordance with the semi-analytical finite element method. Also, using thus obtained functions, the computational procedure is presented to evaluate the foundation stiffness for the analysis of soil-structure interaction problems. Some discussions on the supporting bed of the layer system are made.

INTRODUCTION

To calculate the dynamic stiffness of a direct foundation resting on an elastic soil layer, the finite element approach has been effectively utilized by discretizing the soil layer into a number of horizontal thin layers and assuming linearly varying displacements across the thickness of each discrete layer, while the displacement functions with respect to the horizontal coordinates are of analytical forms. On performing this approach, it seems to be a basis to search a compact solution of wave propagation due to a harmonic point loading on an arbitrary location within the structure-soil interface. This paper presents a three-dimensional analysis in this course of the approach. The principal results herein presented are the influence functions due to a point load located at the origin of the coordinate system. The solutions are given by the mode superposition of the frequency-dependent normal modes of generalized Rayleigh waves as well as generalized Love waves in the N-layered system on the bed.

DISPLACEMENT FUNCTIONS FOR A HORIZONTAL POINT LOAD

When an elastic space is loaded by a harmonic horizontal force concentrated at the origin towards the x-direction, the wave motions are described by the following displacement functions in the cylindrical coordinates:

$$u_r = \bar{u}_r \cos\theta\, e^{i\omega t}, \quad u_\theta = \bar{u}_\theta \sin\theta\, e^{i\omega t}, \quad u_z = \bar{u}_z \cos\theta\, e^{i\omega t}$$

$$\begin{bmatrix} \bar{u}_r \\ \bar{u}_\theta \\ \bar{u}_z \end{bmatrix} = \frac{1}{2}\int_0^\infty \begin{bmatrix} J_2(\alpha r) - J_o(\alpha r) & J_2(\alpha r) + J_o(\alpha r) & 0 \\ J_2(\alpha r) + J_o(\alpha r) & J_2(\alpha r) - J_o(\alpha r) & 0 \\ 0 & 0 & 2J_1(\alpha r) \end{bmatrix} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \end{bmatrix} d\alpha \qquad (1)$$

The associated stresses working on a z-plane are represented in the similar fashion as

$$\sigma_{zr} = \bar{\sigma}_{zr} \cos\theta\, e^{i\omega t}, \quad \sigma_{z\theta} = \bar{\sigma}_{z\theta} \sin\theta\, e^{i\omega t}, \quad \sigma_{zz} = \bar{\sigma}_{zz} \cos\theta\, e^{i\omega t}$$

I Prof. of Structural Engineering, Nihon Univ., Tokyo, Japan

$$\begin{bmatrix} \bar{\sigma}_{zr} \\ \bar{\sigma}_{z\theta} \\ \bar{\sigma}_{zz} \end{bmatrix} = \frac{1}{2}\int_0^\infty \begin{bmatrix} J_2(\alpha r) - J_o(\alpha r) & J_2(\alpha r) + J_o(\alpha r) & 0 \\ J_2(\alpha r) + J_o(\alpha r) & J_2(\alpha r) - J_o(\alpha r) & 0 \\ 0 & 0 & 2J_1(\alpha r) \end{bmatrix} \begin{bmatrix} \tilde{\sigma}_1 \\ \tilde{\sigma}_2 \\ \tilde{\sigma}_3 \end{bmatrix} d\alpha \qquad (2)$$

Herein, the relation between $(\tilde{\sigma}_1, \tilde{\sigma}_2, \tilde{\sigma}_3)$ and $(\tilde{u}_1, \tilde{u}_2, \tilde{u}_3)$ are derived from the relation of $(\bar{\sigma}_{zr}, \bar{\sigma}_{z\theta}, \bar{\sigma}_{zz})$ and $(\bar{u}_r, \bar{u}_\theta, \bar{u}_z)$ in the following forms:

$$\tilde{\sigma}_1 = G(\frac{d\tilde{u}_1}{dz} - \alpha\tilde{u}_3), \quad \tilde{\sigma}_2 = G\frac{d\tilde{u}_2}{dz}, \quad \tilde{\sigma}_3 = \alpha\lambda\tilde{u}_1 + (\lambda+2G)\frac{d\tilde{u}_3}{dz} \qquad (3)$$

Then, the equation of wave motions in the three dimensional medium in the cylindrical coordinates will be reduced to

$$\alpha^2 G\tilde{u}_2 - G\frac{d^2\tilde{u}_2}{dz^2} - \rho\omega^2\tilde{u}_2 = 0 \qquad (4)$$

$$\begin{aligned} &\alpha^2(\lambda+2G)\tilde{u}_1 - G\frac{d^2\tilde{u}_1}{dz^2} + \alpha(\lambda+G)\frac{d\tilde{u}_3}{dz} - \rho\omega^2\tilde{u}_1 = 0 \\ &-\alpha(\lambda+G)\frac{d\tilde{u}_1}{dz} + \alpha^2 G\tilde{u}_3 - (\lambda+2G)\frac{d^2\tilde{u}_3}{dz^2} - \rho\omega^2\tilde{u}_3 = 0 \end{aligned} \qquad (5)$$

where λ = Lame's constant, G = shear modulus and ρ = density of material of the medium. λ and G take complex moduli to represent the hysteretic properties of the material. In the following, the time term $e^{i\omega t}$ will be omitted, if there is no confusion.

Instead of the homogeneous elastic space, consider a horizontal N-layered medium resting on a rigid bed, as shown in Fig. 1, in which the top surface and underlying interfaces are listed in order by the index 1 to N. Assuming the displacements varying across the thickness of each layer in accordance with the finite element analysis, Eqs. (4) and (5) are converted to the following forms:

$$(\alpha^2[A_s] + [G_s] - \omega^2[M])\{\tilde{u}_2\} = \{\tilde{\sigma}_2\} \qquad (6)$$

$$\begin{aligned} &(\alpha^2[A_p] + [G_s] - \omega^2[M])\{\tilde{u}_1\} - \alpha[B]^T\{\tilde{u}_3\} = \{\tilde{\sigma}_1\} \\ &-\alpha[B]\{\tilde{u}_1\} + (\alpha^2[A_s] + [G_p] - \omega^2[M])\{\tilde{u}_3\} = \{\tilde{\sigma}_3\} \end{aligned} \qquad (7)$$

where

$$\begin{aligned} &[A_s]^e = GH\frac{1}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad [G_s]^e = \frac{G}{H}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad [M]^e = \rho H\frac{1}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \\ &[A_p]^e = (\lambda+2G)\frac{H}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad [G_p]^e = \frac{\lambda+2G}{H}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad [B]^e = \begin{bmatrix} (\lambda-G)/2 & (\lambda+G)/2 \\ -(\lambda+G)/2 & (\lambda-G)/2 \end{bmatrix} \end{aligned} \qquad (8)$$

These equations are alternate expressions to those already obtained[1,2]. The displacement vector $\{\tilde{u}_1\}$, $\{\tilde{u}_2\}$ and $\{\tilde{u}_3\}$ and surface traction vectors $\{\tilde{\sigma}_1\}$, $\{\tilde{\sigma}_2\}$ and $\{\tilde{\sigma}_3\}$ are defined at the top surface and all the interfaces and are written in such a form as $\{\tilde{u}_1\}^T = [u_{11}, u_{21}, \ldots, u_{N1}]$. $[A_s]$, $[G_s]$,

... are matrices constructed by assemblage of the element matrices $[A_s]^e$, $[G_s]^e$, ... of discrete layers.

The surface tractions $\{\tilde{\sigma}_1\}$ $\{\tilde{\sigma}_2\}$ and $\{\tilde{\sigma}_3\}$ are determined as follows. As an example, consider a point load acting on the free surface. For this case, suppose firstly a circle of radius r_o on the free surface, the center of which is situated at the origin of the coordinates. When a uniform shearing stress p_x works in the x-direction within the circle, the stress conditions are given by

$$\sigma_{zx}\big|_{z=o} = \begin{cases} -p_x & r < r_o \\ 0 & r > r_o \end{cases} \tag{9}$$

When the cylindrical coordinates are considered, these conditions can be written in the integral forms of Bessel functions:

$$\bar{\sigma}_{zr}\big|_{z=o} = -\bar{\sigma}_{z\theta}\big|_{z=o} = -p_x r_o \int_0^\infty J_1(\alpha r_o)\, J_o(\alpha r)\, d\alpha \tag{10}$$

On the other hand, Eq. (2) becomes

$$\bar{\sigma}_{zr} + \bar{\sigma}_{z\theta} = \int_0^\infty (\tilde{\sigma}_1 + \tilde{\sigma}_2)\, J_2(\alpha r)\, d\alpha \tag{11}$$

Comparing Eqs. (10) and (11), one has

$$\tilde{\sigma}_{11} + \tilde{\sigma}_{12} = 0 \tag{12}$$

in which the first index 1 in the subscript indicates the free surface. Substituting Eq. (12) into Eq. (2) and paying attention to the sign of $\tilde{\sigma}_{11}$,

$$\bar{\sigma}_{zr}\big|_{z=o} = \int_0^\infty J_o(\alpha r)\, \tilde{\sigma}_{11}\, d\alpha \tag{13}$$

Comparing the above equation with Eq. (10), one has

$$\tilde{\sigma}_{11} = -p_x r_o\, J_1(\alpha r_o), \qquad \tilde{\sigma}_{12} = p_x r_o\, J_1(\alpha r_o) \tag{14}$$

In particular, the point loading is given by $r_o \to 0$, which leads to $J_1(\alpha r_o) \to \alpha r_o/2$. Then, denoting $p_x \pi r_o^2 = P_x$, one has

$$\tilde{\sigma}_{11} = -\frac{P_x}{2\pi}\,\alpha, \qquad \tilde{\sigma}_{12} = \frac{P_x}{2\pi}\,\alpha \tag{15}$$

Thus, the traction vectors are

$$\{\tilde{\sigma}_1\} = [\tilde{\sigma}_{11}, 0, \ldots, 0]^T, \qquad \{\tilde{\sigma}_2\} = [\tilde{\sigma}_{12}, 0, \ldots, 0]^T \tag{16}$$

In the following, the point loading on the top surface will be discussed. The external tractions working on arbitrary interfaces at depth will be treated in a similar manner. To solve Eq. (6), one uses the method of mode superposition analysis. The eigenvalue problem of Eq. (6) yields the eigenvalues β_k^2 and the corresponding eigenvectors $\{Y_k\}$, $(k = 1, 2, \ldots, N)$, for any values of ω^2. Then, the solution can be written in the form,

$$\{\tilde{u}_2\} = \sum_{k=1}^{N} \{Y_k\}\, q_{k\beta} \tag{17}$$

The generalized coordinates $q_{k\beta}$ are determined by inserting Eq. (17) into Eq. (6) introduced by the external traction of Eq. (16), premultiplying $\{Y_k\}^T$ and using the orthogonality condition of modes.

$$q_{k\beta} = \frac{P_x}{2\pi}\frac{Y_{1k}}{D_{k\beta}}\frac{\alpha}{\alpha^2 - \beta_k^2} \tag{18}$$

where

$$D_{k\beta} = \{Y_k\}^T[A_s]\{Y_k\} \tag{19}$$

Eqs. (7) can be solved in a similar manner. Writing them in the matrix form,

$$\left(\alpha^2\begin{bmatrix}[A_p] & \\ & [A_s]\end{bmatrix} + \alpha\begin{bmatrix} & -[B]^T \\ -[B] & \end{bmatrix} + \begin{bmatrix}[E_s] & \\ & [E_p]\end{bmatrix}\right)\begin{bmatrix}\{\tilde{u}_1\} \\ \{\tilde{u}_3\}\end{bmatrix} = \begin{bmatrix}\{\tilde{\sigma}_1\} \\ \{\tilde{\sigma}_3\}\end{bmatrix} \tag{20}$$

where

$$[E_s] = [G_s] - \omega^2[M], \quad [E_p] = [G_p] - \omega^2[M] \tag{21}$$

The loading term $\{\tilde{\sigma}_1\}$ is given in Eq. (16) with $\{\tilde{\sigma}_3\} = 0$. Furthermore, it can be converted to the form of the first order of α:

$$\left(\begin{bmatrix} & & [E_s] & \\ & & & [E_p] \\ [E_s] & & & -[B]^T \\ & [E_p] & -[B] & \end{bmatrix} - \alpha\begin{bmatrix}[E_s] & & & \\ & [E_p] & & \\ & & -[A_s] & \\ & & & -[A_p]\end{bmatrix}\right)\begin{bmatrix}\{\tilde{u}_1\} \\ \{\tilde{u}_3\} \\ \{\tilde{v}_1\} \\ \{\tilde{v}_3\}\end{bmatrix} = \begin{bmatrix}0 \\ 0 \\ \{\tilde{\sigma}_1\} \\ \{\tilde{\sigma}_3\}\end{bmatrix} \tag{22}$$

The eigenvalue problem of Eq. (22) yields the eigenvalue α_k, ($k = 1, 2, \ldots 2N$, $-\pi < \arg \alpha_k < 0$), and the corresponding eigenvectors $\{X_k\}$ and $\{Z_k\}$. In this case, $-\alpha_k$ are also the eigenvalues and the corresponding eigenvectors are $\{X_k\}$ and $-\{Z_k\}$, as readily examined by inserting them into Eq. (20) with vanishing of loading terms. In the application of the mode superposition, the solution of Eq. (20) are written in the form,

$$\begin{bmatrix}\{\tilde{u}_1\} \\ \{\tilde{u}_3\} \\ \{\tilde{v}_1\} \\ \{\tilde{v}_3\}\end{bmatrix} = \sum_{k=1}^{2N}\begin{bmatrix}\{X_k\} \\ \{Z_k\} \\ \alpha_k\{X_k\} \\ \alpha_k\{Z_k\}\end{bmatrix} q_{k\alpha} + \sum_{k=1}^{2N}\begin{bmatrix}\{X_k\} \\ -\{Z_k\} \\ -\alpha_k\{X_k\} \\ \alpha_k\{Z_k\}\end{bmatrix} q'_{k\alpha} \tag{23}$$

where $q_{k\alpha}$ and $q'_{k\alpha}$ are generalized coordinates and are determined by substituting Eq. (23) into Eq. (22), premultiplying by the eigenvector $[\{X_k\}^T, \{Z_k\}^T, \alpha_k\{X_k\}^T, \alpha_k\{Z_k\}^T]$ or $[\{X_k\}^T, -\{Z_k\}^T, -\alpha_k\{X_k\}^T, \alpha_k\{Z_k\}^T]$ and using the orthogonality properties of modes. Thus,

$$q_{k\alpha} = \frac{P_x}{2\pi}\frac{X_{1k}}{D_{k\alpha}}\frac{\alpha_k\alpha}{\alpha - \alpha_k}, \quad q'_{k\alpha} = \frac{P_x}{2\pi}\frac{X_{1k}}{D_{k\alpha}}\frac{-\alpha_k\alpha}{\alpha + \alpha_k} \tag{24}$$

where

$$D_{k\alpha} = \{X_k\}^T[E_s]\{X_k\} + \{Z_k\}^T[E_p]\{Z_k\} - \alpha_k^2\{X_k\}^T[A_p]\{X_k\} - \alpha_k^2\{Z_k\}^T[A_s]\{Z_k\} \tag{25}$$

Hence, from Eq. (23)

$$\{\tilde{u}_1\} = \sum_{k=1}^{2N} \frac{P_x}{2\pi} \frac{X_{1k}}{D_{k\alpha}} \{X_k\} \frac{2\alpha_k^2 \alpha}{\alpha^2 - \alpha_k^2}, \quad \{\tilde{u}_3\} = \sum_{k=1}^{2N} \frac{P_x}{2\pi} \frac{X_{1k}}{D_{k\alpha}} \{Z_k\} \frac{2\alpha_k \alpha^2}{\alpha^2 - \alpha_k^2} \tag{26}$$

Since $\{\tilde{u}_1\}$, $\{\tilde{u}_2\}$ and $\{\tilde{u}_3\}$ have been obtained as functions of α, the desired displacements $\{\bar{u}_r\}$, $\{\bar{u}_\theta\}$ and $\{\bar{u}_z\}$ are determined by performing the integration of Eq. (1). As a consequence, one finds

$$\{\bar{u}_r\} = \frac{P_x}{2\pi} \sum_{k=1}^{2N} \{X_k\} \frac{2\alpha_k^2 X_{1k}}{D_{k\alpha}} \Phi_1(\alpha_k r) + \frac{P_x}{2\pi} \sum_{k=1}^{N} \{Y_k\} \frac{Y_{1k}}{D_{k\beta}} \Phi_2(\beta_k r)$$

$$\{\bar{u}_\theta\} = \frac{P_x}{2\pi} \sum_{k=1}^{2N} \{X_k\} \frac{2\alpha_k^2 X_{1k}}{D_{k\alpha}} \Phi_2(\alpha_k r) + \frac{P_x}{2\pi} \sum_{k=1}^{N} \{Y_k\} \frac{Y_{1k}}{D_{k\beta}} \Phi_1(\beta_k r) \tag{27}$$

$$\{\bar{u}_z\} = \frac{P_x}{2\pi} \sum_{k=1}^{2N} \{Z_k\} \frac{2\alpha_k^2 X_{1k}}{D_{k\alpha}} \Phi_3(\alpha_k r)$$

In the derivation of the integration, the following formula[3] are applicable:

$$\int_0^\infty \frac{x^\nu J_\nu(x)}{x + z} dx = \frac{\pi z^\nu}{2 \cos \nu\pi} [\, \mathbf{H}_{-\nu}(z) - Y_{-\nu}(z)] \tag{28}$$

for $\quad |\arg z| < \pi, \quad -1/2 < R(\nu) < 3/2$

where $\mathbf{H}_{-\nu}(z)$ = Struve's function,

$Y_{-\nu}(z)$ = Bessel function of tne second kind.

Owing to multivalued function of $Y_\nu(z)$, the complex variable z cannot rotate across the barrier at $\theta = -\pi$ in the complex plane. Hence, when $-\pi < \arg z < 0$, one obtains the relation,

$$Y_1(z) - Y_1(-z) = 2Y_1(z) + 2i\, J_1(z) = 2i\, H_1^{(2)}(z)$$
$$Y_o(z) + Y_o(-z) = 2Y_o(z) + 2i\, J_o(z) = 2i\, H_o^{(2)}(z) \tag{29}$$

where $H_o^{(2)}(z)$, $H_1^{(2)}(z)$ = Hankel functions of the 2nd kind. Thus,

$$\Phi_1(z) = \frac{1}{2} \int_0^\infty (J_2(x) - J_o(x)) \frac{x}{x^2 - z^2} dx = -\frac{1}{z^2} - i\, \frac{\pi}{2} \frac{1}{z} H_1^{(2)}(z) + i\, \frac{\pi}{2} H_o^{(2)}(z)$$

$$\Phi_2(z) = \frac{1}{2} \int_0^\infty (J_2(x) + J_o(x)) \frac{x}{x^2 - z^2} dx = -\frac{1}{z^2} - i\, \frac{\pi}{2} \frac{1}{z} H_1^{(2)}(z) \tag{30}$$

$$\Phi_3(z) = \frac{1}{2} \int_0^\infty J_1(x) \frac{x^2}{x^2 - z^2} dx = -\, i\, \frac{\pi}{2} H_1^{(2)}(z)$$

If $0 < \arg z < \pi$, $H_\nu^{(2)}(z)$ will have to be replaced by $-H_\nu^{(1)}(z)$. According to the asymptotic representation of the Hankel function, when $\omega > 0$, $H_\nu^{(1)}(z)$ represents a wave travelling toward the origin, whereas $H_\nu^{(2)}(z)$ represents a wave travelling toward infinity. This is the reason why the condition of

$-\pi < \arg \alpha_k, \beta_k < 0$ is required.

DISPLACEMENT FUNCTIONS DUE TO A VERTICAL POINT LOAD

Because of axisymmetric motion with respect to the z-axis, the displacements and stresses will be described in the following forms:

$$\begin{bmatrix} u_r \\ u_z \end{bmatrix} = \int_0^\infty \begin{bmatrix} J_1(\alpha r) & \\ & J_o(\alpha r) \end{bmatrix} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_3 \end{bmatrix} d\alpha, \quad \begin{bmatrix} \sigma_{zr} \\ \sigma_{zz} \end{bmatrix} = \int_0^\infty \begin{bmatrix} J_1(\alpha r) & \\ & J_o(\alpha r) \end{bmatrix} \begin{bmatrix} \tilde{\sigma}_1 \\ \tilde{\sigma}_3 \end{bmatrix} d\alpha \tag{31}$$

where the displacement functions $\tilde{u}_1$ and $\tilde{u}_3$ will be easily proved to satisfy Eqs. (7). It follows that the displacement vectors $\{\tilde{u}_1\}$ and $\{\tilde{u}_3\}$ of the total system of horizontal layers have to satisfy Eq. (22). Hence, the solutions will be obtained in the same manner as the case of horizontal loading. When a vertical point load is applied to the top surface at the origin of the coordinates, the traction vectors are given by

$$\{\tilde{\sigma}_1\} = 0, \quad \{\tilde{\sigma}_3\}^T = [\tilde{\sigma}_{13}, 0, \ldots, 0], \quad \tilde{\sigma}_{13} = \frac{P_z}{2\pi}\alpha \tag{32}$$

Correponding to Eq. (24),

$$q_{k\alpha} = -\frac{P_z}{2\pi}\frac{Z_{1k}}{D_{k\alpha}}\frac{\alpha_k \alpha}{\alpha - \alpha_k}, \quad q'_{k\alpha} = -\frac{P_z}{2\pi}\frac{Z_{1k}}{D_{k\alpha}}\frac{\alpha_k \alpha}{\alpha + \alpha_k} \tag{33}$$

Finally,

$$\{u_r\} = -\frac{P_z}{2\pi}\sum_{k=1}^{2N}\{X_k\}\frac{2\alpha_k^2 Z_{1k}}{D_{k\alpha}}\Phi_3(\alpha_k r)$$

$$\{u_z\} = -\frac{P_z}{2\pi}\sum_{k=1}^{2N}\{Z_k\}\frac{2\alpha_k^2 Z_{1k}}{D_{k\alpha}}\Phi_4(\alpha_k r) \tag{34}$$

where

$$\Phi_4(z) = \int_0^\infty J_o(x)\frac{x}{x^2 - z^2}dx = -i\frac{\pi}{2}H_o^{(2)}(z) \tag{35}$$

EVALUATION OF SURFACE FOUNDATION STIFFNESS

For numerical calculation, it is more applicable to use the Cartesian coordinates rather than the cylindrical coordinates. From the transformation between both coordinates , one has

$$u_x = u_r \cos\theta - u_\theta \sin\theta = \bar{u}_r \cos^2\theta - \bar{u}_\theta \sin^2\theta = \frac{\bar{u}_r + \bar{u}_\theta}{2}\cos 2\theta + \frac{\bar{u}_r - \bar{u}_\theta}{2}$$

$$u_y = u_r \sin\theta + u_\theta \cos\theta = \frac{\bar{u}_r + \bar{u}_\theta}{2}\sin 2\theta$$

Hence, if a harmonic point loading (P_x, P_y, P_z) is applied at a location (x_a, y_a) on the surface $z = 0$, the surface displacement (u_x, u_y, u_z) at (x, y) are given by

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} V_1 \cos 2\theta + V_2 & V_1 \sin 2\theta & -V_3 \cos \theta \\ V_1 \sin 2\theta & -V_1 \cos 2\theta + V_2 & -V_3 \sin \theta \\ V_3 \cos \theta & V_3 \sin \theta & -V_4 \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} \tag{36}$$

where V_1 and V_2 are denoted for surface displacement of $(\bar{u}_r + \bar{u}_\theta)/2$ and $(\bar{u}_r - \bar{u}_\theta)/2$ due to a unit horizontal loading. V_3 and V_4 are remaining components of surface displacements. They are given in below:

$$V_1 = \frac{1}{2\pi} \sum_{k=1}^{2N} \frac{\alpha_k^2 X_{1k}^2}{D_{k\alpha}} F_1(\alpha_k r) + \frac{1}{4\pi} \sum_{k=1}^{N} \frac{Y_{1k}^2}{D_{k\beta}} F_1(\beta_k r)$$

$$V_2 = -\frac{1}{2\pi} \sum_{k=1}^{2N} \frac{\alpha_k^2 X_{1k}^2}{D_{k\alpha}} F_2(\alpha_k r) + \frac{1}{4\pi} \sum_{k=1}^{N} \frac{Y_{1k}^2}{D_{k\beta}} F_2(\beta_k r) \tag{37}$$

$$V_3 = \frac{1}{\pi} \sum_{k=1}^{2N} \frac{\alpha_k^2 X_{1k} Z_{1k}}{D_{k\alpha}} F_3(\alpha_k r), \qquad V_4 = \frac{1}{2\pi} \sum_{k=1}^{2N} \frac{\alpha_k^2 Z_{1k}^2}{D_{k\alpha}} F_2(\alpha_k r)$$

$$F_1(z) = -\frac{2}{z^2} - i\frac{\pi}{2}\frac{2}{z} H_1^{(2)}(z) + i\frac{\pi}{2} H_o^{(2)}(z) = -\frac{2}{z^2} - i\frac{\pi}{2} H_2^{(2)}(z)$$

$$F_2(z) = -i\frac{\pi}{2} H_o^{(2)}(z), \qquad F_3(z) = -i\frac{\pi}{2} H_1^{(2)}(z) \tag{38}$$

$$r = \sqrt{(x - x_a)^2 + (y - y_a)^2}, \qquad \theta = \tan^{-1}(y - y_a)/(x - x_a)$$

The following procedure to evaluate the foundation stiffness is normal in treating the base area subdivided in numerous meshes. The above equations are used to represent the relations of loads and displacements at any two nodes, which are located at center of mesh elements. As an exception, the displacement at the self-loaded node may be approximated by the displacement at a point with an appropriate distance from the node, as frequently used by a half of radius of circle, whose area is equal to that of the mesh element.

In the foregoing description, the rigid bed has been assumed. But, when the bed is considered as an elastic half space, there is no consistent approach to incorporate the elasticity of the bed, as far as the finite element approach is dealt with. Approximate methods to account for the radiation effects due to the elastic bed are available in placing the dashpot mat at the bottom of the soil layer[4] and otherwise in adding the equivalent external damping as usually employed in the calculation of soil amplification by a one-dimensional lumped-mass system analogy[5]. Fig. 3 shows a comparative illustration of horizontal stiffness and frequency diagrams for the different treatments in the bedding condition. The similar diagrams for rocking and torsional modes exhibit no significant valleys as found in the figure, even if the rigid bed is assumed.

The present approach can be applied to evaluate the foundation stiffness including the interaction effects of adjacent bases and even the foundation stiffness with embedment effects, because it is easily extended to the calculation for the load application on subsurfaces.

REFERENCES

1. Lysmer, J. and Drake, L.A., 1972, A Finite Element Method for Seismology, Methods in Computational Physics, Vol. 11, pp. 181-216, Academic Press.

2. Kausel, E., Roesset, J.M. and Waas, G., 1975, Dynamic Analysis of Footings on Layered Media, Journal of the Engineering Mechanics Division, ASCE, Vol. 101, EM5, pp. 679-693.

3. Watson, G.N., 1922, A Treatise on the Theory of Bessel Functions, p. 436 Cambridge Univ. Press.

4. Lysmer, J. and Kuhlemeyer, R.L., 1969, Finite Dynamic Model for Infinite Media, Journal of the Engineering Mechanics Division, ASCE, Vol. 95, EM5, pp. 859-877.

5. Roesset, J.M., 1970, Fundamentals of Soil Amplification, Seismic Design for Nuclear Power Plants (Hansen, R.J., editor), pp. 183-244, The M.I.T. Press.

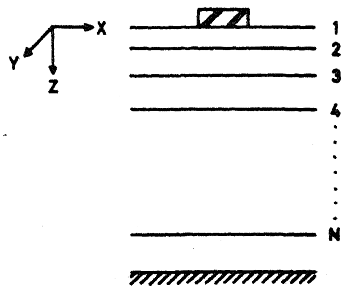

Fig. 1 Soil layered system

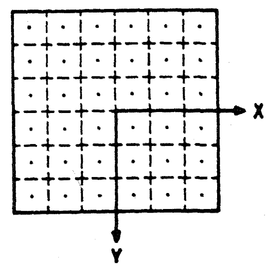

Fig. 2 Discrete model of base

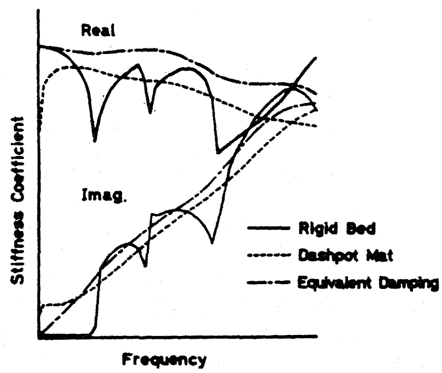

Fig. 3 Modification of horizontal stiffness and frequency curves due to different treatments for elastic bed