EVALUATION OF DYNAMIC STIFFNESS OF EMBEDDED FOUNDATION
USING DYNAMIC REACTION OF SURFACE LAYER

by

Takanori HARADA[I] and Keizaburo KUBO[II]

SUMMARY

A new approximate method is presented for the evaluation of dynamic stiffness of embedded foundation. The dynamic stiffness by this method incorporates the freedom of coupled swaying and rocking motion of cylindrical rigid foundation and is applicable to the three embedment cases as follows; Case I, embedded in a viscoelastic surface layer with the foundation base resting on a much stiffer bedrock, Case II, embedded in a viscoelastic surface layer with the foundation base resting within the surface layer, and Case III, embedded in a viscoelastic half-space. Its capability is examined by comparing with the past field experiments.

INTRODUCTION

Although some mathematical models have been available for the dynamic stiffness of embedded foundation, there does not exist a rational model so far which is applicable to the various embedment conditions. Table 1 is a list of the current available models for dynamic stiffness of three typical embedded cylindrical rigid foundation. Models for embedded strip foundation are excluded because the three-dimensional configurations of foundation-subsoil system are of major practical importance for the introduction of soil-foundation interaction computations into engineering design, for instance, for the seismic design of nuclear power plants and long-span bridge foundation structures. The three cases shown in Table 1 represent most typical, simplified embedment conditions.

These models are usually treated by a Continuum Formulation Method(CFM) or a Finite Element Method(FEM). The FEM has been found to be a powerful tool. However, the computational cost in many three-dimensional cases often inhibits the analyses covering a wide range of design parameters. Therefore, efforts have been devoted for many years to obtain a closed form solution. Unfortunately, a rigorous solution has not been found because of the difficulty in mathematical treatment of the boundary conditions between foundation and surrounding soil. Hence, all the solutions by the CFM shown in Table 1 are derived in approximate fashion. The main assumptions for these solutions are as follows;

(1) the dynamic reactions acting on foundation side walls are evaluated by assuming an independent foundation-subsoil system, and
(2) the reactions acting on foundation base are obtained from the solution of the foundation resting on a viscoelastic half-space or stratum.

Regarding the assumption(1), various methods exist, and all the solutions by the CFM shown in Table 1 can be characterized by the method of evaluating the foundation side walls reactions.

In this paper, a new method is presented of evaluating the dynamic soil

I Graduate Student, Department of Civil Engineering, University of Tokyo.
II Professor, Institute of Industrial Science, University of Tokyo, Japan.

reactions acting on foundation side walls. The usefulness of the method is illustrated by constructing a closed form solution for the evaluation of dynamic stiffness of foundation which is applicable to the three embedment cases in Table 1 and capable of incorporating the freedom of coupled swaying and rocking mode of foundation. Since the solution developed in this paper is only approximate, its characteristics are compared with those of previous solutions listed in Table 1 and with the results observed during field tests.

THEORY

The dynamic stiffness of embedded cylindrical rigid foundation as shown in Fig. 1 can be obtained in a straightfoward manner if the dynamic soil reactions N's and R's acting on foundation side walls and base can be determined. Although rigorous expressions for N's and R's are difficult to find, approximate expression may be obtained under the two basic assumptions described in Introduction. In this section, more detail descriptions and the main features of the method of evaluating the reactions are presented. Note here that the following assumptions are made in the whole of the following theory. 1. The soil is of a homogeneous and isotropic linearly elastic medium with frequency independent material damping of hysteretic type. 2. The foundation is perfectly rigid and of cylindrical cross-sectional shape. 3. No separation is allowed between foundation and soil.

SOIL REACTIONS N's ACTING ON FOUNDATION SIDE WALLS: Following the assumption (1) stated in Introduction, consider here a hypothetical(hypo.) embedded rigid cylindrical foundation-subsoil system as shown in Fig. 2. The hypo. foundation undergoes a linear displacement with the depth of the surface layer and is fixed at the rigid bedrock. The soil column underneath the foundation base is assumed to be a part of the hypo. foundation and to experience the displacement pattern mentioned above. Furthermore, the following assumptions are made regarding the motions of the surface layer corresponding to the motions of the hypo. foundation. In investigating the horizontal reaction, the vertical displacement of the surface layer is assumed to be zero, whereas in considering the rocking reaction, the horizontal displacements of the surface layer are assumed to be zero. These assumptions permit the horizontal reaction to be evaluated independently of the rocking reaction.

By the adoption of the assumptions stated above, the dynamic soil reactions acting on the hypo. foundation side walls can be obtained in a closed form and their variation with the depth of surface layer were found similar to the displacement pattern of hypo. foundation[4]. This result suggests that the local dynamic stiffness of hypo. foundation, which is defined as the ratio of dynamic soil reaction of hypo. foundation at any depth to the corresponding displacement of the hypo. foundation, is almost constant with the depth of surface layer. Therefore, the dynamic soil reactions acting on the hypo. foundation side walls may be expressed by the dynamic stiffnesses defined by a unit length of hypo. foundation. Their derivations are presented in Ref.[4] in detail.

These stiffnesses are expressed for individual modes as follows;

$$\begin{aligned}
&\text{Rocking:} && k_\psi = G_s a^2 [\ s_{\psi 1}(\omega/\omega_g, \nu, 2D, a/H) + i s_{\psi 2}(\omega/\omega_g, \nu, 2D, a/H)\ ] \\
&\text{Horizontal:} && k_H = G_s \varepsilon\ [\ s_{u1}(\omega/\omega_g, \nu, 2D, a/H) + i s_{u2}(\omega/\omega_g, \nu, 2D, a/H)\ ]
\end{aligned} \quad (1)$$

In Eq.(1), G_s = shear modulus of the surface layer, s_{j1}, s_{j2} = the real and imaginary parts of the dimensionless dynamic stiffness in direction j, respectively. These stiffnesses s_{ij}'s depend on frequency ω normalized by the Fundamental Horizontal Natural Frequency(FHNF)ω_g of surface layer, Poisson's ratio ν, material damping D of soil, and the a/H ratio. The parameter, a/H, is the geometrical parameter which arises from the consideration of the effect of dynamic response of surface layer on the dynamic reactions. It should be noted here that the dynamic stiffness for plane strain case(Novak's solution)[1,8,9], which is derived by assuming an infinitely long rigid embedded foundation undergoing uniform motions, does not include the parameter a/H.

Figures 3 and 4 show the frequency variations of s_{ij}'s for several values of a/H and $2D$=0.01, ν=0.4. The functions s_{ij}'s are normalized by their corresponding static values. For comparison, the functions for plane strain case(the approximated values by Beredugo,et al.[1]) are also shown in Figs. 3 and 4. In the rocking reaction shown in Fig. 3, the Fundamental Vertical Natural Frequency(FVNF) which is defined as $\omega_p=\pi v_p/(2H)$, v_p=longitudinal wave velocity of surface layer, is assumed as $3.0\omega_g$. In the rocking case, the soil particle of the surface layer vibrates only in vertical direction due to the aforementioned assumption. Under this condition, the longitudinal wave propagates between the free surface and the rigid bedrock. Hence, the dynamic behavior of surface layer may be totally affected by the longitudinal wave. In the horizontal case, on the other hand, the shear wave becomes important for the dynamic response of surface layer as the surface soil particles are assumed to vibrate only horizontal directions.

From Figs. 3 and 4, it is found that the proposed solutions and the plane strain solutions are almost the same above the FVNF or the FHNF(ω_p or ω_g) of surface layer in individual modes. Below these frequencies, however, the proposed solutions lead to the larger stiffnesses and the smaller dampings compared with the plane strain solutions. This discrepancy between the two solutions is due to the effect of the dynamic response of surface layer.

By utilizing the dynamic stiffnesses expressed in Eq.(1), the dynamic soil reactions N's can be obtained in an ordinary manner. It should be noted here that these stiffnesses are derived by assuming the hypo. foundation-subsoil system as shown in Fig. 2 and that this situation is not true for the foundation-subsoil system of Fig. 1. However, some comparisons with other solutions and field experimental results conclusively indicate the potential capability of this approach as will be shown in the following sections.

SOIL REACTIONS R's ACTING ON FOUNDATION BASE: Under the aforementioned assumption(2), the dynamic stiffnesses of a flat foundation resting on the free surface of soil medium may be expressed as

$$k_R = Ga^3(\, c_{\psi 1} + ic_{\psi 2}\,) \quad , \quad k_S = Ga(\, c_{u1} + ic_{u2}\,) \tag{2}$$

where G = shear modulus of the soil on which the foundation base rests. From the study of Kausel,et al.[6] and Beredugo,et al.[1], the functions c_{ij}'s are given by the following forms:

$$c_{\psi 1} = \frac{8}{3(1-\nu)}\left(1 + \frac{a}{6H_1}\right) \quad , \quad c_{u1} = \frac{8}{2-\nu}\left(1 + \frac{a}{2H_1}\right)$$

$$c_{\psi 2} = 0.43a_0 \quad , \quad c_{u2} = \begin{cases} 2.70a_0 & \text{for } \nu = 0.0 \\ 3.15a_0 & \text{for } \nu = 0.5 \end{cases} \tag{3}$$

valid for $a/H_1 \leq 1$, $0 \leq a_0 \leq 2.0$.
where $a_0 = \omega a/v$ = dimensionless frequency, v = shear wave velocity of the soil on which the foundation base rests. Notice that the parameter, a/H_1, must be taken to be zero when the cases I and III in Table 1 are treated.

With the dynamic stiffnesses expressed in Eq.(2), the dynamic soil reactions R's can be obtained in a usual manner.

COMPARISON WITH OTHER SOLUTIONS

Since the dynamic stiffness of embedded foundation obtained by using the dynamic stiffnesses given by Eqs.(1) and (2) is only approximate, it is necessary to examine its acceptability by comparing the other solution, a finite element solution and experiment.

FREQUENCY VARIATION: For the case shown in Fig. 5, compared first are the results of the proposed solution and the results of the Novak's solution with those of the FEM obtained by Kausel,et al.[6]. Here, the Novak's solution can be obtained by utilizing the approximate dimensionless dynamic stiffnesses s_{ij}'s for plane strain case to the evaluation of foundation side walls reactions. The results of this comparison are shown in Figs. 6 and 7. Notice that the validity range of both the Novak model and the Proposed model is in frequency below $a_0=2.0(=0.64\pi)$ due to the approximation of the dimensionless dynamic stiffnesses s_{ij}'s and c_{ij}'s. In Figs. 6 and 7, however, they are applied beyond this range to somewhat larger frequency range($a_0=\pi$).

For swaying shown in Fig. 6, the curve for the stiffness coefficient k_{11} obtained by the proposed model very closely follows that by the FEM model in the range of frequency below $a_0=2.0=0.64\pi$. Above this frequency, while the curve by the FEM model exhibits a significant valley near $a_0=2.5=0.8\pi$ whose frequency corresponds to the third horizontal natural frequency of the surface layer. Because of this valley the proposed model overestimates k_{11} compared with the FEM model above $a_0=2.0=0.64\pi$. Here, remember that the proposed model is only valid below $a_0=2.0$. The stiffness k_{11} by the Novak model is found to be approximately the same as that by the proposed model above the FHNF, ω_g or $f_g=0.083$. Below this frequency, the Novak model somewhat underestimates k_{11} compared with the other two models.

The damping coefficient c_{11} by both the FEM model and the proposed model are almost the same except for the extremely low frequency range. The proposed model gives an infinitely large value of c_{11} as the frequency approaches zero, while the FEM model zero for the same condition. Although this discrepancy may be due to the difference of the definition for c_{11} and c_{22} between the proposed model and the FEM model, this is not important for the dynamic response of foundation having mass because the response in extremely low frequency range does not depend much on the damping coefficient but mostly on the stiffness coefficient and the foundation mass.

For rocking shown in Fig. 7, the frequency variation of k_{22} by the proposed model is very similar to that by the FEM model. The proposed model, however, predicts somewhat smaller value of k_{22} than the FEM model. This difference is minor for the computation of the natural frequency of foundation with mass which is proportional to the square root of the stiffness coefficient. The general behavior of k_{22} by the Novak model is very similar to that of k_{11} discussed in the swaying case. Similar behavior is also observed between c_{11}(Fig. 6) and c_{22}(Fig. 7). In this case, notice that the main difference between the Novak model and the proposed model can be observed below and above the FVNF denoted by ω_p or f_p=0.17.

From the comparisons shown in Figs. 6 and 7, it is apparent that the Novak model, the FEM model and the proposed model predict the similar values of stiffness and damping coefficients in relatively higher frequency range than the FHNF or the FVNF, while in lower frequency range the Novak model overestimates the damping coefficients compared with the other two models.

STATIC STIFFNESS: The next comparison is made in terms of the static stiffness. Figures 8 and 9 compare the static stiffnesses predicted by the proposed model with those by Johnson,et al.[5] using the FEM with lateral boundaries on roller supports. Note here that their result does not converge to the analytical solution for a flat foundation on a half-space and that it leads to stiffness which is probably an average of 10% too high in the vertical case[5]. The agreement in the general trend of the results is very good, but the stiffnesses from the FEM model are slightly larger.

COMPARISON WITH FIELD EXPERIMENTS

To examine the capability of the proposed model, the results of two carefully controlled field tests are compared with those predicted by the Novak model and the proposed model. As was discussed in the previous section, the differences between the two models should become apparent in lower frequency range than the FHNF where the Novak model underestimates stiffness and overestimates damping compared with the proposed model. The two cases are studied, one where the natural frequency ω_s of foundation is almost equal to or slightly below the FHNF ω_g, and the other where ω_s is above ω_g.

THE CASE OF $\omega_s \fallingdotseq \omega_g$: The field test conducted by Toki,et al.[12] used a cylindrical concrete block with a radius a=0.68m and a height l=2.0m. This model foundation was fully embedded into the surface layer of fine silty clay with an approximate thickness of H=2.0m, which is underlain by a more rigid sand-gravel layer. The model was constructed about two years before the test by directly pouring concrete into an excavated pit. A vibrator was attached to the top of the model and the dynamic response of the model was measured at several different force levels and for different exciting modes. By using the data reported by Toki,et al.[12] with some additional judgements, the shear wave velocities of the surface and the base layer were assumed as v_s=161.6m/sec and v=243.4m/sec, respectively. The predominant frequency of the surface layer was calculated as 20.2Hz(=161.4/4x2) which was found consistent with the frequency range of 18-23Hz determined by microtremor observation at the site.

Figure 10 shows the response curves for the top horizontal translation obtained from the experiment and the theories. The two experimental curves in Fig. 10 correspond to two different exciting force levels, the upper curve for the exciting force 400kg and the lower for 200kg. The theoretical curves with κ=1.0 correspond to the results from the Novak model and the proposed model. In this figure, the result from more rigorous model by the authors shown in Table 1 is also shown. Good agreement is found but agreement between the experimental and theoretical curves with κ=1.0 is found not satisfactory.

One of the reasons for the discrepancy observed in Fig. 10 may be the imperfect bonding between the foundation and the surrounding soil that might have possibly taken place during experiments. To approximately incorporate this effect, a reduction factor κ was introduced for the side walls stiffnesses given by Eq.(1). Similar reduction was also adopted in the Novak model. It is seen that the results by the proposed model with κ=0.4 well agree with the measured response curves while the Novak model is still incapable of representing the observed phenomena. This discrepancy between the results by the Novak model and the measured phenomena may be due to the fact that the Novak model does not include the effect of the dynamic response of surface layer on the dynamic stiffness of foundation as was discussed in the previous sections.

THE CASE OF $\omega_s \geqq 1.5\omega_g$*:* The test was performed by the Architectural Institute of Japan[10]. A concrete block(5x5x3.75m) was embedded into the surface layer of clay loam with an approximate thickness of 12.0m, which is underlained by a much stiffer clay layer. A vibrator was attached at the top of the model foundation and the dynamic response was measured for several embedment depth by filling sand into the gap between the model and the surrounding soil. This backfill sand was sufficiently compacted for each embedment depth. By using the data shown in Ref.[10], the shear wave velocity of the surface layer was assumed as v_s=192m/sec. Base layer underlying the surface layer was assumed to be rigid. In the theories an equivalent radius of the foundation was determined in such a manner that the cross-sectional area of the prototype equals to that of the mathematical model.

Figure 11 show the response curves for the top horizontal translation obtained from the experiment and the theories. The three experimental curves correspond to the three different embedment depth investigated. In the theories, the reduction factor κ=0.3 was used in each model. Good agreement is found among the experiment, the Novak model and the proposed model. It should be noted that the effect of the reduction factor κ on the dynamic response of foundation may be explained by the reduction of the shear modulus G_s of the surface layer. However, this explanation of the reduction of G_s leads to inconsistency of the predominant frequency of surface layer with that observed at the site.

CONCLUSIONS

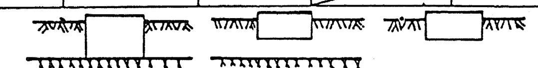
Examinations of Figs. 6 through 11 seem to indicate the potential capability of the proposed model using the dynamic stiffness of surface layer given by Eq.(1), and also indicate the need of introducing the reduction factor of κ=1/2-1/3 into the theoretical models based on the CFM which is essentially based on a wave propagation linear theory to obtain a consistent result with the field experimental data.

REFERENCES

(1)Beredugo,Y.O. and M.Novak(1972): Coupled horizontal and rocking vibration of embedded footings, Canadian Geotechnical Jour., Vol.9, 477-497. (2)Goto,H., K.Toki and S.Takada(1971): On the vibrational characteristics of foundation structures in the ground, (in Japanese), Disaster Prevention Research Institute Annuals, Kyoto University, No.14, A, 263-282. (3)Harada,T. and K.Kubo(1978): Dynamic (complex) stiffness and vibration of embedded cylindrical rigid foundation, Proc. of the 5th Japan Earthquake Engineering Symposium -1978, 401-408. (4)Harada,T., K.Kubo and T.Katayama(1979): Dynamic soil reactions (impedance functions) including the effect of dynamic response of surface stratum, SEISAN-KENKYU, Monthly Jour. of Institute of Industrial Science, University of Tokyo, Vol.31, No.9-11. (5)Johnson,G.R., P.Christiano and I.E.Howard (1974): Stiffness coefficients for embedded footings, Proc. of the American Society of Civil Engineers, Vol.101, No.GT8, 789-800. (6)Kausel,E. and J.M.Roesset(1975): Dynamic stiffness of circular foundations, Proc. of the American Society of Civil Engineers, Vol.101, No.EM6, 771-785. (7)Lysmer,J. and R.L.Kuhlemeyer (1969): Finite dynamic model for infinite media, Proc. of the American Society of Civil Engineers, Vol.95, No.EM4, 859-877. (8)Novak,M. and Y.O.Beredugo(1972): Vertical vibration of embedded footings, Proc. of the American Society of Civil Engineers, Vol.98, No.SM12, 1291-1310. (9)Novak,M. and K.Sachs(1973): Torsional and coupled vibrations of embedded footings, Earthquake Engineering and Structural Dynamics, Vol.2, 11-33. (10)Subcommittee on Dynamic Soil-Structure Interaction Problems, AIJ,(1976): Research report on experimental study on dynamic soil-reactor containment structure interaction problems, (in Japanese), AIJ. (11)Tajimi, H.(1969): Dynamic analysis of a structure embedded in an elastic stratum, Proc. of the 4th WCEE. (12)Toki, K. and A.Komatsu(1979): Seismic response of well foundation, (in Japanese), Proc. of the Japan Society of Civil Engineers, Vol.281, 29-40. (13)Urlich,C.M. and R.L.Kulemeyer(1973): Coupled rocking and lateral vibrations of embedded footings, Canadian Geotechnical Jour., Vol.10, 145-160.

Table 1 List of Past Works on Dynamic Stiffness of Embedded Cylindrical Rigid Foundation

Mathematical Model \ Surrounding Soil		Layered Medium with a Hard Base Layer			Half-Space Medium
		Vibrational Effect of Surface Layer			
		Considered		Not Considered	
		Case I*	Case II**	Case I and II	
CFM	One DOF	Tajimi (1969) Goto,et al.('71)	C	Novak, et al. (1972,1973)	Novak, et al. (1972,1973)
CFM	Two DOF	Harada,et al. (1978)	B	Novak, et al. (1972)	Novak, et al. (1972)
FEM	One DOF	D	E		Lysmer, et al. (1969)
FEM	Two DOF	F	Kausel,et al. (1975)		Urlich, et al. (1972)
NOTE	*Foundation Bottom on Stiffer underlying Layer		**Foundation Bottom within Surface Stratum		Embedded in Half-Space
NOTE	One DOF=One Degree of Freedom System (Rocking, Swaying, Vertical, or Torsional Vibration)				
NOTE	Two DOF=Two Degree of Freedom System (Coupled Swaying and Rocking Vibration)				
NOTE	Categories B to F are those for which no previous study' is available.				



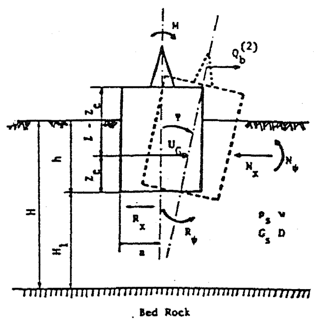


Fig.1 Mathematical Model of Embedded foundation

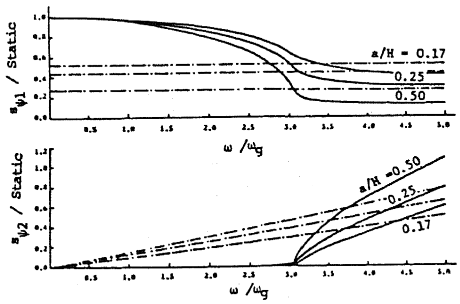


Fig. 3 Dimensionless Dynamic Stiffness for Rocking, Full Line for Proposed Solution and Dushed Line for Novak Solution

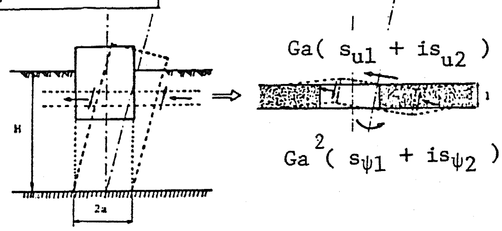


Fig.2 Hypothetical foundation for the Evaluation of foundation Side Walls Reactions

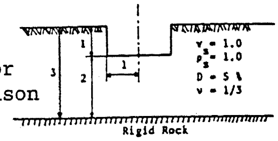


Fig. 5 Model used for Comparison

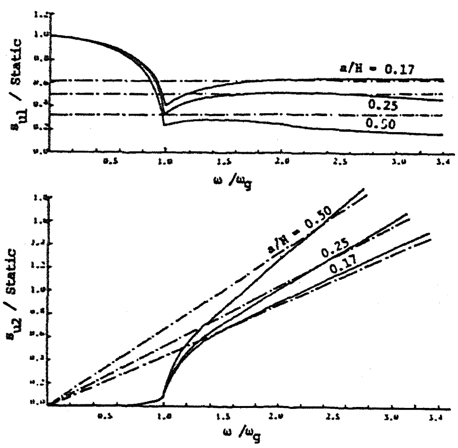


Fig. 4 Dimensionless Dynamic Stiffness for Swaying, Full Line for Proposed Solution and Dushed Line for Novak Solution

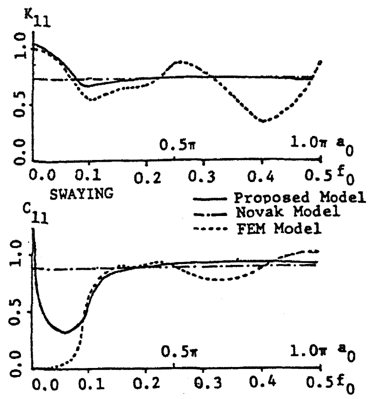


Fig. 6 Result of Comparison for Swaying Stiffness and Damping Coef.

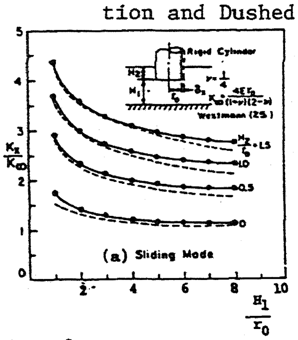


Fig. 8 Comparison in Static Stiffness for Swaying Mode

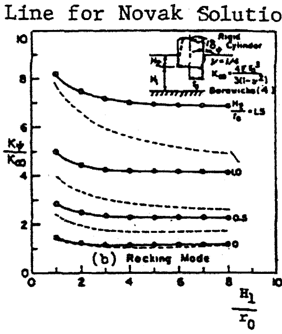


Fig. 9 Comparison in Static Stiffness for Rocking Mode

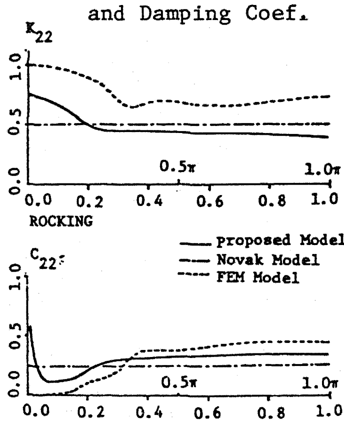


Fig. 7 Result of Comparison for Rocking Stiffness and Damping Coef.

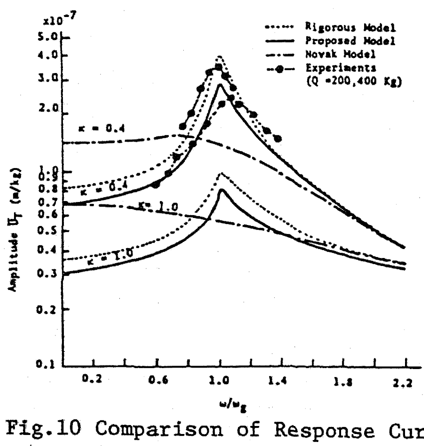


Fig. 10 Comparison of Response Curves in the Case of $\omega_s \fallingdotseq \omega_g$

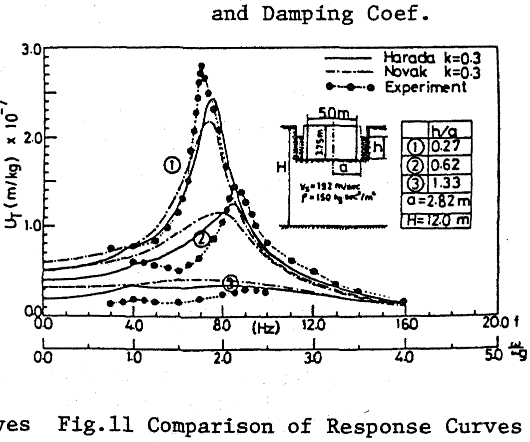


Fig. 11 Comparison of Response Curves in the Case of $\omega_s \geqq 1.5\omega_g$