OPTIMUM RIGIDITY DISTRIBUTION OF ASEISMIC DESIGN FOR MULTI-STORY SHEAR FRAMES

By

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ABSTRACT

In this paper, the optimum aseismic designs of more than one hundred multi-story shear frames are obtained by using the full control (stress and deflection) method. The designs are all in compliance with the constraint conditions specified in the aseismic building code in China. On the basis of the analyses and syntheses of these optimum schemes, the authors recommend an approximate formula of the optimum rigidity distribution for structures of this kind. The parameters involved in the formula, as the functions of the intensity of the earthquake, the soil condition of the construction site, the total number of stories and the average floor weight per unit column of the frame, can be obtained from the numerical tables given in this paper.

GENERAL FORMULATION OF THE PROBLEM

As most of the theories and methods on optimum aseismic design are still immature and the process of computation with regard to most structures is highly cumbersome, many of the papers exploring the problem take the shear frame shown in Fig. 1 as their object of research at present. In the figure, I_K represents the average cross-sectional moment of inertia of the columns in the kth story while S_K stands for the story shear stiffness:

$$S_k = \frac{12EC_k}{l_k^3} I_k \qquad (1)$$

in which C_K is the number of columns in the kth story and E is the modulus of elasticity of the material used.

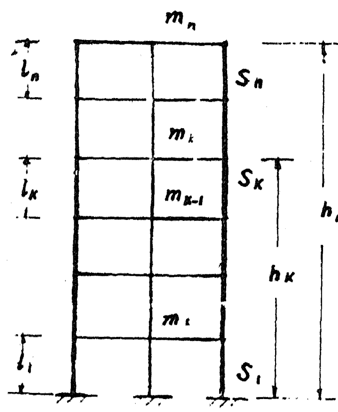

Fig. 1

The dimensions of beams and floors, which are hardly affected by the lateral vibrations of the structure, may be determined with respect to practical requirements and the optimum member design method and may thus be regarded as known quantities. Consequently the optimum aseismic design of multi-story shear frames mainly concerns the optimum design of the column cross sections in each story.

In order to provide stress-restraint conditions and reduce the number of design variables, it is assumed that steel columns with wide-flange sections are used and the empirical relationships between member properties are

$$\left.\begin{aligned} F_i &= aI_i^{1/2} \\ W_i &= bI_i^{3/4} \end{aligned}\right\} \qquad (2)$$

in which F and W are respectively the cross-sectional area and the section modulus of column, a and b, representing the statistical empirical parameters, take 0.8 and 0.78

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respectively in most of the relevant papers (such as [1]~[5]).

The average moments of inertia of the columns can then be used as design vector:

$$\{I_i\} = [I_1, I_2, \cdots\cdots I_n]^T \qquad (3)$$

and the function

$$\Phi = \sum_{i}^{n} C_i l_i I_i^{1/2} \qquad (4)$$

which is proportional to the total volume of the columns, can be used as objective function. If C_i and l_i do not vary with i the objective function can be replaced by:

$$\Phi = \sum_{i=1}^{n} I_i^{1/2} \qquad (5)$$

The purpose of the optimum design is to minimize the objective function subjected to all constraints.

According to the aseismic building code in China[6], the acceleration response spectrum generally equals

$$A(T) = 0.225 J \gamma g \alpha(T) \qquad (6)$$

in which γ is the structural effect factor ($\gamma = 0.35$, for the frame), g, gravitational acceleration, takes 981 cm/sec^2, J is the index number of the intensity of the earthquake (the values 1, 2 and 4 are adopted for intensities Ⅶ、Ⅷ、Ⅸ respectively) and $\alpha(T)$ is the earthquake response factor as shown in Fig. 2(a), in which the values of the abscissas c and d are listed in the following table.

Kind of base soil	c(sec)	d(sec)
rock or stiff soil(Ⅰ)	0.2	1.0
medium soil(Ⅱ)	0.3	1.5
soft soil(Ⅲ)	0.7	3.5

The velocity response spectrum $V(T)$ and displacement response spectrum $\Delta(T)$, corresponding to Eq. (6), are as below:

$$V(T) = 12.2953 J \beta(T) \qquad (7)$$

$$\Delta(T) = 1.9569 J T \beta(T) \qquad (8)$$

in which $\beta(T) = T\alpha(T)$, as shown in Fig. 2(b).

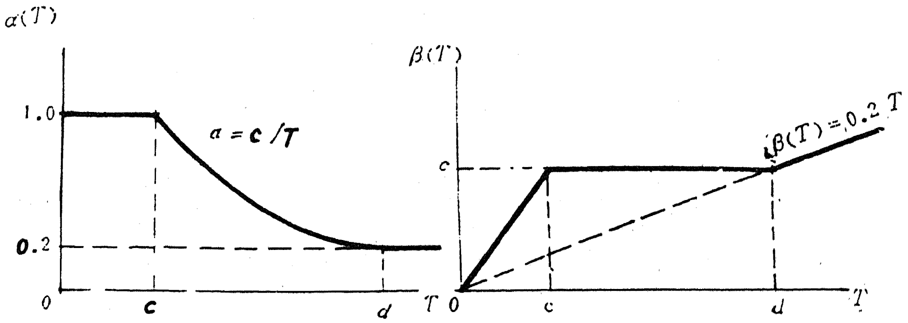

Fig. 2

OPTIMIZATION PROCEDURE

Combining the method suggested in [1] for improving the convergence rate of iterations with the results of our researches, the following optimization procedure is formed.

1. Initial scheme:

The initial stiffness distribution is given as

$$S_i^{(0)} = \mu_i^{(0)} S_1^{(0)} \tag{9}$$

in which

$$\mu_i^{(0)} = 1 - 0.65\left(\frac{i-1}{n-1}\right)^{1.5} \tag{10}$$

$$S_1^{(0)} = G\omega^{(0)2} \tag{11}$$

$$G = \frac{\sum_{i=1}^{n} m_i h_i^2}{\sum_{i=1}^{n} \mu_i^{(0)} l_i^2} \tag{12}$$

$$\omega^{(0)} = 25/n \qquad \text{(rad/sec)} \tag{13}$$

The average moment of inertia of columns in the ith story is

$$I_i^{(0)} = \frac{l_i^3}{12EC_i} S_i^{(0)} \tag{14}$$

When the optimum distributions for other frames with the same number of stories have been obtained, the one that is most similar can be chosen as the initial scheme for the given frame.

2. In accordance with the initial stiffness $\{S_i^0\}$, a modal analysis is performed to find the first five periods $T_j (j = 1, 2, 3, 4, 5)$ and the normalized mode vectors

$$\{a_i\}_j^{(0)} = [a_{1j}^{(0)}, a_{2j}^{(0)}, \cdots\cdots a_{nj}^{(0)}]^T$$

for the vibration of the frame.

Then, the maximum seismic relative deflection of the ith story in the response of the jth mode can be obtained by

$$\delta_{ij}^{(0)} = [a_{ij}^{(0)} - a_{(i-1)j}^{(0)}]\eta_j^{(0)}\Delta_j^{(0)} \tag{15}$$

where Δ_j is the value of the displacement spectrum when $T = T_j$ [sees Eq. 8], and η_j is the participation factor of that mode:

$$\eta_j^{(0)} = \frac{\sum_{i=1}^{n} m_i a_{ij}^{(0)}}{\sum_{i=1}^{n} m_i a_{ij}^{(0)2}} \tag{16}$$

Hence the estimated maximum relative deflection of the ith story is

$$\delta_i^{(0)} = \sqrt{\sum_{j=1}^{5} \delta_{ij}^{(0)2}} \tag{17}$$

The corresponding estimated maximum shear force is

$$Q_i^{(0)} = S_i^{(0)} \delta_i^{(0)} \tag{18}$$

3. Assuming that the shear forces remain unchanged (still satisfying Eq. 18), the full controlling conditions become

$$\delta_i = \frac{Q_i^{(0)}}{S_i} = \delta_{ai} \tag{19}$$

$$\sigma_i = \frac{N_i}{a I_i^{1/2}} + \frac{M_i^{(0)}}{b I_i^{3/4}} = \sigma_a \tag{20}$$

in which σ_a is the allowable stress, δ_{ai} is the allowable relative deflection of the ith story, and the bending moment and axial force are respectively

$$M_i^{(0)} = \frac{Q_i^{(0)} l_i}{2C_i} + N_i \delta_i^{(0)} \tag{21}$$

$$N_i = \frac{g}{C_i} \sum_{k=i}^{n} m_k \tag{22}$$

when the $P-\Delta$ effect is not considered, the last term on the right side of Eq. (21) ought to be omitted. From Eq. (19) it can be found that

$$I_i = I_i^{(0)} \delta_i^{(0)} / \delta_{ai} \tag{23}$$

and Eq. (20) is a cubic algebraic equation of the variable $I_i^{1/4}$. Since the left side of the equation decreases monotonically as I_i increases ($I_i > 0$) and on its right side is a constant, it must have one positive real root only. Let

$$q_i = \frac{M_i^{(0)}}{2b\sigma_a}; \qquad p_i = \frac{N_i}{3a\sigma_a}; \qquad D_i = q_i^2 - p_i^3 \tag{24}$$

When $D_i \geqslant 0$, let

$$A = \sqrt[3]{q_i + \sqrt{D_i}}; \quad B = p_i / A$$

thus, $\quad I_i^{1/4} = A + B \qquad (i = 1, 2, \cdots n)$ (25)

When $D_i \leqslant 0$, let

$$\theta = arctg(\sqrt{-D_i} / q_i)$$

thus, $\quad I_i^{1/4} = 2\sqrt{p_i} \cos(\theta/3)$

The larger one between the solutions obtained from Eq. (23) and (25) is taken as $I_i^{(1)}$

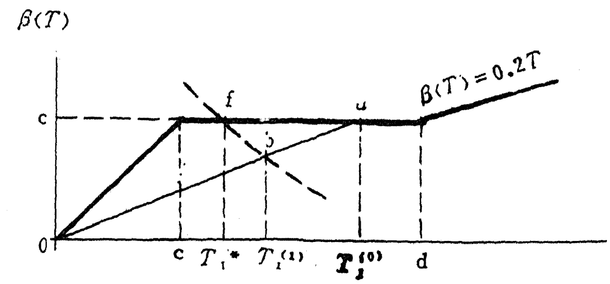

Fig. 3

4. When the vector $\{a_i\}_1^{(0)}$ is taken as an approximate mode shape, the corresponding fundamental period of this frame can be obtained by the energy method, this is

$$T_1^{(1)} = \sqrt{\xi}\, T_1^{(0)} \tag{26}$$

in which

$$\xi = \frac{\sum_{i=1}^{n} S_i^{(0)} \delta_{i1}^2}{\sum_{i=1}^{n} S_i^{(1)} \delta_{i1}^2} \tag{27}$$

It can be observed in Fig. 3 that $T_1^{(0)}$ is corresponding to the point a on the velocity spectrum, the vertical line passing $T_1^{(1)}$ intercepts oa at the point b and a hyperbola is drawn to pass b and intercepts the spectrum at the point f which is associated with the period T_1^*. Let

$$r = T_1^*/T_1^{(1)} \tag{28}$$

that is, the moments of inertia

$$I_i^* = I_i^{(1)}/r^2 \qquad (i = 1, 2, \cdots n) \tag{29}$$

can be taken as a new initial scheme for the next iteration.

5. Taking $\{I_i^*\}$ obtained from the preceding step as $\{I_i^{(0)}\}$ in the next iteration, the corresponding story shear stiffness $\{S_i^{(0)}\}$ can be calculated by Eq. (1).

Now, We can return to the second step in this optimization procedure and start the new iteration until the full controlling conditions for all stories are satisfied. The stress and deflection constraints are

$$|Y_i| = \left|\frac{\sigma_i - \sigma_a}{\sigma_a}\right| \leqslant \varepsilon_1 \qquad (i = 1, 2, \cdots n) \tag{30}$$

$$|Z_i| = \left|\frac{\delta_i - \delta_{ai}}{\delta_{ai}}\right| \leqslant \varepsilon_2 \qquad (i = 1, 2, \cdots n) \tag{31}$$

in which ε_1 and ε_2 are decided by the required precision.

The full controlling conditions are for each story to satisfy any of the following three requrements ① both of the constraint conditions (30) and (31) are satisfied, ②Eq. (30) is satisfied with $Z_i \leqslant 0$, ③ Eq. (31) is satisfied with $Y_i \leqslant 0$.

THE STATISTIC EMPIRICAL FORMULA OF OPTIMUM RIGIDITY DISTRIBUTION

1. The Objects of Computation:

In order to find out the optimum stiffness distribution of the frames under various conditions by means of statistical method, it is necessary to examine some regular frames. Considering the variation in story height is small, it can be assumed $l_i = 4$m. Since the mass of each story mainly concerns the mass of the floors and the equipments on the floors as well as the equivalent mass of the walls and columns in that stroy, the assumption, $m_i = m$, can be used for multi-story frames with a similar plane in each story. To reduce structural parameters it is assumed that the number of columns of each story is equal, namely $C_i = C$. Then the average moment of inertia of the columns of each story is directly proportional to the story stiffness, that is

$$S_i = \mu_i S_1, \quad I_i = \mu_i I_1 \tag{32}$$

the average floor weight shared by each column is:

$$w = mg/C \tag{33}$$

2. The Varying Scope of the Structural Parameters:

The number of total stories: n = 5, 8, 11, 14;
The average floor weight shared by each colomn: $w = 15$, 20, 25T;
The intensity of earthquake: Ⅶ, Ⅷ, Ⅸ;
Soil condition: rock or stiff soil, medium soil and soft soil are termed soil Ⅰ, Ⅱ and Ⅲ respectively.

3. Other Data:

$$E = 2.1\times10^{6}kg/cm^{2};\ \sigma_a = 1700kg/cm^{2};$$
$$\delta_a = l/400 = 1cm;\ a = 0.8;\ b = 0.78;\ \varepsilon_1 = \varepsilon_2 = 0.01;$$

4. The Statistical Formula:

The result of computation for the more than 100 multi-story frames has revealed the presence of high regularity: the top two or three stories are controlled by stiffness constraint while the others are controlled by strength constraint. A statistical analysis of the results obtained helped produce the approximate formula of the optimum rigidity distribution given below:

$$I_i = \left[\,1-(1-\mu)\left(\frac{2n}{n+i}\right)^{\lambda}\left(\frac{i-1}{n-1}\right)^{1.345}\right] I_1 \qquad (34)$$

in which I_1 = the moment of inertia of the bottom column; factor $\mu = I_n/I_1$. And the data of I_1, μ and index λ are shown in Table 1, 2 and 3 respectively. Once these parameters of a certain frame are determined, the approximate optimum rigidity distribution for the shear frame under design can be obtained directly by the formula. And the rigidity distribution thus obtained can serve as an appropriate preliminary design scheme for the multi-story frame.

Results of the analyses also show a more distinct influence of the $P-\Delta$ effect on the lower columns than on the upper ones, in view of which the total weight of all the frame columns is increased by 1~4%.

Table 1 The Moment of Inertia of Bottom Columns I_1(cm^4)

n	w	Ⅶ			Ⅷ			Ⅸ		
	(T)	Ⅰ	Ⅱ	Ⅲ	Ⅰ	Ⅱ	Ⅲ	Ⅰ	Ⅱ	Ⅲ
5	15	5380	5450	7530	7860	8060	17900	13100	15300	59000
	20	8830	8980	12600	12500	12800	29100	20500	24900	88000
	25	13000	13300	19700	18200	18700	42900	29100	37000	120000
8	15	11900	12100	14700	16500	16800	29400	25200	27500	92000
	20	20000	20200	25200	26800	27500	49100	41500	43800	154000
	25	30000	30300	39900	39500	40400	73600	59700	65600	230000
11	15	21100	21200	24500	28000	28600	45500	43000	45200	120000
	20	35600	35900	43900	46000	47400	76400	68900	72700	204000
	25	53800	54400	67700	68300	70400	115000	99900	105000	311000
14	15	32600	32700	36600	42200	42900	63000	63000	65800	153000
	20	55400	55800	66100	69900	71300	108000	101000	106000	260000
	25	84000	84700	102000	103000	106000	163000	147000	156000	397000

Table 2 Factor μ

n	w (T)	VII			VIII			IX		
		I	II	III	I	II	III	I	II	III
	15	0.170	0.179	0.268	0.237	0.260	0.316	0.304	0.346	0.298
5	20	0.138	0.148	0.232	0.200	0.224	0.277	0.267	0.310	0.269
	25	0.119	0.127	0.205	0.178	0.197	0.250	0.240	0.279	0.245
	15	0.086	0.090	0.140	0.124	0.139	0.188	0.168	0.196	0.181
8	20	0.069	0.074	0.120	0.105	0.118	0.161	0.146	0.172	0.158
	25	0.059	0.064	0.105	0.090	0.104	0.143	0.130	0.156	0.142
	15	0.052	0.056	0.091	0.079	0.091	0.129	0.112	0.136	0.130
11	20	0.042	0.046	0.075	0.066	0.077	0.111	0.097	0.118	0.112
	25	0.036	0.039	0.067	0.057	0.067	0.097	0.086	0.106	0.100
	15	0.036	0.039	0.064	0.056	0.064	0.094	0.080	0.099	0.100
14	20	0.029	0.032	0.054	0.046	0.054	0.081	0.070	0.086	0.085
	25	0.024	0.027	0.040	0.040	0.048	0.072	0.062	0.077	0.076

Table 3 Index λ

n	w (T)	VII			VIII			IX		
		I	II	III	I	II	III	I	II	III
	15	2.071	2.160	2.271	1.806	2.035	1.172	1.498	1.913	0.555
5	20	1.937	2.074	2.323	1.959	2.198	1.470	1.771	2.024	0.759
	25	1.915	2.017	2.194	1.929	2.120	1.538	1.906	2.043	0.590
	15	1.983	2.029	2.103	1.731	1.857	1.546	1.468	1.719	0.691
8	20	1.969	2.016	2.086	1.805	1.919	1.536	1.579	1.816	0.648
	25	1.968	2.015	2.027	1.803	1.937	1.505	1.630	1.814	0.518
	15	1.975	2.004	2.069	1.733	1.824	1.633	1.466	1.655	0.824
11	20	1.996	2.026	2.049	1.794	1.878	1.611	1.557	1.726	0.763
	25	2.013	2.035	2.017	1.814	1.901	1.556	1.589	1.745	0.668
	15	2.001	2.024	2.069	1.764	1.819	1.660	1.469	1.610	0.912
14	20	2.035	2.062	2.064	1.815	1.875	1.644	1.554	1.663	0.837
	25	2.047	2.070	1.965	1.855	1.911	1.595	1.587	1.699	0.798

NUMERACAL EXAMPLES

For a certain shear frame, $n=10$, $w=22T$, soil II. The values of I_1, μ and λ can be obtained from Tables 1, 2 and 3 by interpolation and are listed in Table 4 and following table respectively:

Intensity of earthquake	μ	λ
VII	0.052	2.025
VIII	0.086	1.900
IX	0.130	1.761

Substitute them into the following formula of the optimum relative rigidity ratio ($\mu_i = I_i/I_1$),

$$\mu_i = 1-(1-\mu)\left(\frac{2n}{n+i}\right)^{\lambda}\left(\frac{i-1}{n-1}\right)^{1.345} \qquad (35)$$

the results of hand calculation as shown in Table 4 can thus be obtained. For the purpose fo comparison the results obtained by using computer with the full control design method are also listed in the table. It is obvious that they are very close.

Table 4 Comparison between two kinds of calculated results

method of computation		hand results with the given formula			computer results with the F.C.D. method		
intensity		VII	VIII	IX	VII	VIII	IX
I_1 (cm^4)		37000	48700	74600	36100	47700	73500
the optimum stiffness distribution ratio	μ_1	1.000	1.000	1.000	1.000	1.000	1.000
	μ_2	0.861	0.874	0.889	0.843	0.861	0.877
	μ_3	0.700	0.725	0.754	0.696	0.725	0.749
	μ_4	0.555	0.590	0.628	0.563	0.601	0.632
	μ_5	0.430	0.470	0.519	0.444	0.487	0.526
	μ_6	0.325	0.367	0.416	0.336	0.380	0.424
	μ_7	0.236	0.278	0.328	0.241	0.284	0.331
	μ_8	0.162	0.203	0.252	0.161	0.198	0.245
	μ_9	0.102	0.140	0.187	0.093	0.125	0.179
	μ_{10}	0.052	0.086	0.130	0.050	0.083	0.128

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