

# STRENGTH DETERIORATION OF H-SHAPED STEEL BEAMS UNDER RANDOM LOADINGS

by

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## SUMMARY

H-shaped steel beams subjected to cyclic bending loads fail by the excessive lateral and local buckling deformation, which causes the strength deterioration of the beams. A method of evaluating the strength deterioration of the beams subjected to random rotation amplitude is proposed, based on the data of constant rotation amplitude cyclic tests of the beams. To confirm the appropriateness of this method, cyclic tests on beams under random rotation amplitude were conducted. The strength deterioration of the beams predicted by the method was compared with the real strength deterioration in the experiments.

## INTRODUCTION

When H-shaped steel beams are subjected to cyclic and reversed loading with constant rotation amplitudes, moment-rotation hysteresis loops are stable within a certain critical rotation amplitude, while they become unstable beyond that critical rotation. However, the stable hysteresis loops are again obtained, if the rotation amplitudes are reduced to within the critical one, even after the beams are damaged to deteriorate the strength<sup>1)</sup>. If the strength deterioration of a beam subjected to random rotation amplitude can be predicted from the results of the constant rotation amplitude tests, it is thought to be one of clues to a prediction of a failure of beams during an earthquake. In this paper, how the strength of beams deteriorates due to various ratios of rotation amplitude in the unstable rotation amplitude region is examined on the basis of the constant rotation amplitude tests. By employing those experimental data with the above-mentioned characteristics of the restoring force, an evaluation method of the strength deterioration of the beams subjected to random rotation amplitude provided.

## STRENGTH DETERIORATION OF BEAMS ON CONSTANT ROTATION AMPLITUDE TESTS

In order to investigate the strength deterioration of the beams beyond the critical rotation amplitude, the cyclic tests of the beams which have sections of H-200x100x5.5x8 (JIS SS41) and length of 130cm (slenderness ratio with respect to weak axis is about 62) were carried out under constant rotation amplitudes. Three kinds of ratios of rotation amplitude\*, 1.0, 0.5 and 0, were adopted and the names of their specimens were DG-130-16, DG-130-20 and DG-130-17, respectively. The tests were done by using simple beams with central concentrated loads (See Fig.1). Reversed loads with five

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\*) Ratio of rotation amplitude means the ratio of the small absolute amplitude to the large absolute amplitude in a cycle.

cycles in each amplitude were repeatedly applied by the hydraulic actuator. The rotation amplitude was controlled so as to be a sinusoidal function of time and its frequency was 0.05Hz (See Table 1). The moment(M)-rotation( $\theta$ ) hysteresis loops obtained from the experiments are shown in Fig.2 to 4, where the inserted numerals beside loops denote the numbers of cycles. The moment and the rotation are divided by the full plastic moment,  $M_p$ , and  $\theta_p' = M_p/3EI_x$ , respectively, where  $I_x$  is moment of inertia with respect to strong axis. Based on these experimental results, the strength deterioration at each rotation amplitude was evaluated for the three ratios of rotation amplitude,  $\gamma$ , as indicated in Fig.5. The strength deterioration in the figure indicates the average strength deterioration,  $m_D$ , per half cycle to the full plastic moment,  $M_p$ . The abscissa indicates the larger rotation amplitudes in cycles. The solid lines in the figure show the strength deterioration on the larger side of amplitude, and the dashed lines show the strength deterioration on the smaller side. The figure shows that if the larger rotation amplitude is  $4\theta_p$ , the strength deterioration for the ratio of rotation amplitude 1 is 0.03 $M_p$  per half cycle on the larger rotation side. Therefore, the beam strength under constant rotation amplitude  $4\theta_p$  deteriorates by 10% of the full plastic moment with four cycles approximately. On the other hand, 10% strength deterioration of  $M_p$  is estimated after about nine cycles in case of the larger rotation amplitude  $4\theta_p$  and the amplitude ratio 0.5.

#### INTERPOLATION OF STRENGTH DETERIORATION

The strength deteriorations of beams were obtained as to only three kinds of ratios of rotation amplitude in the experiments. So the strength deteriorations for any ratio of rotation amplitude between 1.0 and 0 is calculated with the interpolation formulas which are derived on these experimental results.

1. In case an absolute turning rotation,  $\theta_t$ , is smaller than an absolute arriving rotation,  $\theta_a$  (See Fig.9-(a)). On the basis of experimental results of the solid lines as shown in Fig.5, the strength deterioration,  $m_D/M_p$ , per half cycle at the arriving rotation are summarized as follows:

- i)  $\theta_a \leq 2\theta_p$   $m_D/M_p = 0$  for any ratios of rotation amplitude
- ii)  $2\theta_p < \theta_a \leq 3\theta_p$   $m_D/M_p = 0.5(\%)$  for any ratios of rotation amplitude
- iii)  $\theta_a > 3\theta_p$   $m_D/M_p = a(\theta_a/\theta_p)^2 + b(\theta_a/\theta_p) + c$  (1)

$$\begin{aligned} \text{in which} \quad a &= 1.08\gamma^2 + 0.22\gamma + 0.12 \\ b &= -4.41\gamma^2 - 2.37\gamma - 0.83 \\ c &= 2.92\gamma^2 + 6.01\gamma + 1.55 \\ \gamma &: \text{ratio of rotation amplitude} \end{aligned} \quad (2)$$

Eqs.1 and 2 were determined as mentioned below. The strength deterioration,  $m_D/M_p$ , per half cycle is assumed to be a quadratic equation (1) with respect to the arriving rotation,  $\theta_a/\theta_p$ . Three sets of the coefficients,  $a$ ,  $b$  and  $c$  in Eq.1 for the ratios of rotation amplitude, 1.0, 0.5 and 0, were calculated from the least squares by using the respective experimental data. Each of the coefficients,  $a$ ,  $b$  and  $c$  for any ratio,  $\gamma$ , were also assumed to be quadratic equation (2) with respect to  $\gamma$ . The coefficients in Eq.2 were determined from each three values of  $a$ ,  $b$  and  $c$  for the ratio, 1.0, 0.5 and 0.

Fig.6 demonstrates the relation of above i)-iii).

2. In case an absolute turning rotation,  $\theta_t$ , is larger than an absolute arriving rotation,  $\theta_a$  (See Fig.9-(b)). On the basis of the experimental results of the dashed lines as shown in Fig.5, the strength deterioration,  $m_D/M_p$ , per half cycle at the arriving rotation are summarized as follows:

$$\begin{aligned} \text{i) } \theta_t \leq 2\theta_p & \quad m_D/M_p = 0 \quad \text{for any ratios of rotation amplitude} \\ \text{ii) } 2\theta_p < \theta_t \leq 3\theta_p & \quad m_D/M_p = 0.5 (\%) \quad \text{for any ratios of rotation amplitude} \\ \text{iii) } \theta_t > 3\theta_p & \quad m_D/M_p = \bar{a}(\theta_t/\theta_p)^2 + \bar{b}(\theta_t/\theta_p) + \bar{c} \quad (3) \end{aligned}$$

$$\begin{aligned} \text{in which} \quad \bar{a} &= 0.05 \gamma^2 + 1.13 \gamma + 0.25 \\ \bar{b} &= 4.01 \gamma^2 - 8.75 \gamma - 2.86 \\ \bar{c} &= -10.99 \gamma^2 + 13.07 \gamma + 8.40 \end{aligned} \quad (4)$$

Eqs.3 and 4 were derived by the same procedure as the case 1.

Fig. 7 shows the relation of the above i)-iii).

#### PREDICTION OF STRENGTH DETERIORATION OF BEAM UNDER RANDOM ROTATION AMPLITUDE

Fig.8-(a) shows two loading curves (peak number 1 to 8 and 1' to 8') of random rotation determined a priori for explanation. The maximum rotations of the first and the second loading curves are adjusted to  $4\theta_p$  and  $4.5\theta_p$ , respectively. Figs.8-(b) and -(c) are the moment-rotation relationship of the beam corresponding to the first and the second loading curves. It is observed from Figs.8-(b) and -(c) that the beam strength deteriorates at the positive maximum rotation 7 and 7' or at the negative maximum rotation 6 and 6'. Then, it is intended to calculate the amount of the strength deterioration of the beam at the positive or the negative maximum rotations.

The strength deteriorations are predicted on the hypotheses that 1) the strength deteriorations at the arriving rotation of every half cycle take the same value regardless of the past hysteresis if the magnitude and the ratio of the rotation amplitude of each half cycle are equal; 2) the total strength deterioration on the positive side in the hysteresis is a sum of the strength deterioration which takes place on the positive side of each half cycle, and the same may be said on the negative side. On these two hypotheses, the strength deterioration at the positive maximum rotation (peak number 7) in Fig.8-(b) is calculated by the sum of the strength deteriorations at the rotations of the peak numbers, 3, 5 and 7, of three half cycles, 2-3, 4-5 and 6-7. Similarly, in case of the positive maximum rotation (peak number 7') in Fig.8-(c), the total strength deterioration is obtained from the sum of the strength deteriorations in the half cycles, 2-3, 4-5, 6-7, 2'-3', 4'-5', and 6'-7'. The same procedure is applied for the calculation of strength deterioration on the negative side.

Half cycles in random cyclic bending are classified into four types shown in Fig. 9. The strength deterioration at the arriving rotation of the half cycle shown in Fig.9-(a) is obtained by substituting the rotation,  $\theta_a$ , and the ratio of rotation amplitude  $\gamma = \theta_t/\theta_a$  for Eqs.1 and 2. On the other hand, in case of Fig.9-(b), the strength deterioration is calculated from Eqs.3 and 4 for the rotation  $\theta_t$ , the ratio  $\gamma = \theta_a/\theta_t$ .

## STRENGTH DETERIORATION OF BEAMS UNDER EARTHQUAKE RESPONSE ROTATION AND RANDOM ROTATION

The cyclic tests on beams subjected to random rotation amplitude were conducted to confirm the appropriateness of the proposed method. Two kinds of random rotations were adopted: one was the response of rotation calculated for the ground acceleration of Hachinohe 1968 (hereafter referred to the response rotation); the other was the random rotation generated on a digital computer (hereafter referred to the random rotation). Fig.10 provides the time histories of these two rotations. The maximum rotation of the response rotation or the random rotation is adjusted to a multiple of  $\theta_p$ . In the tests, a beam specimen is loaded many times as the maximum rotation is changed variously. The name of specimen for the response rotation is DG-130-5, and the names for the random rotation are DG-130-9 and DG-130-10. The cyclic program in Table 2 indicates the maximum rotation amplitudes and the number of cycles. The material and the geometrical properties of the specimens were the same as those adopted in the constant rotation amplitude tests. Mean frequencies of the response rotation and the random rotation were 0.067Hz and 0.06Hz, respectively.

Fig.11 shows the hysteresis loops of the moment-rotation of the first loading and the second loading, where the maximum value of the rotation is set to  $4\theta_p$ , for the response rotation (DG-130-5). Fig.12 shows the hysteresis loops of the first and the second loadings with the maximum rotation,  $4.5\theta_p$ , for the random rotation (DG-130-9). The strength deteriorations of the beam specimens under the response rotation or the random rotation was calculated by the proposed method mentioned above and were compared with the real strength deteriorations which took place in the experiments.

### Specimen DG-130-5 under response rotation

For specimen DG-130-5, ten cyclic tests were done by setting the maximum rotation amplitude to  $1\theta_p$ ,  $2\theta_p$ ,  $2.5\theta_p$ ,  $3\theta_p$ ,  $3.5\theta_p$ ,  $3.5\theta_p$ ,  $4\theta_p$ ,  $4\theta_p$ ,  $3.5\theta_p$  and  $3\theta_p$ . In four cyclic tests underlined above, the total amount of the strength deterioration at the maximum rotation on the positive and the negative sides were calculated by the proposed method, respectively. The relationship between the calculative total strength deterioration,  $m_p/M_p$ , and the experimental load carrying capacities,  $M/M_p$ , at each maximum rotation on the positive side is described by a chain line in Fig.13. The similar relationship on the negative side is given by a chain line in Fig.14. Numerals in the figures denote the maximum values of the rotation amplitude where the strength deteriorations were calculated.

### Specimen DG-130-9 under random rotation

The strength deteriorations were calculated by the proposed method at the maximum values of the rotation amplitude of six cyclic tests, namely, the first ( $3.5\theta_p$ ,  $4\theta_p$ ,  $4.5\theta_p$ ) and the second ( $3.5\theta_p$ ,  $4\theta_p$ ,  $4.5\theta_p$ ) among 17 cyclic tests (See Table 2). The calculative total strength deterioration versus the experimental load carrying capacities on the positive and the negative sides during those tests is indicated by a fine solid line in Figs.13 and 14.

### Specimen DG-130-10 under random rotation

For specimen DG-130-10, 23 cyclic tests were done. The strength deteriorations were calculated at the six maximum rotation amplitudes of  $3.5\theta_p$ ,  $3.5\theta_p$ ,  $3.83\theta_p$ ,  $3.83\theta_p$ ,  $4.5\theta_p$ , and  $4.5\theta_p$  among them. The relationship between the

total strength deterioration and the load carrying capacities on the positive side is given by a dashed line in Fig.13, and the relation on the negative side by a dashed line in Fig.14.

Each bold solid line C-C in Figs.13 and 14 is given for the comparison of the strength deterioration calculated by the proposed method with that which took place in the experiments. That is to say, if the solid line, the dashed line and the chain line in the figures are parallel to the bold solid line C-C, the calculative values correspond to the experimental ones. It is observed from Figs.13 and 14 that the fine solid and the dashed lines for the specimens DG-130-9 and DG-130-10 are almost parallel to the bold solid line. However, for specimen DG-130-5, the chain lines declined less than the bold solid line. This implies the calculative strength deteriorations are greater than the experimental ones. It is probably because there exist many absolute rotations between the boundary rotations  $2\theta_p$  and  $3\theta_p$  in the response rotation for DG-130-5 than those of the random rotation for two other specimens. In the calculation, when the larger absolute rotation amplitude in a cycle exists between  $2\theta_p$  and  $3\theta_p$ , the strength deterioration at the arriving rotation is assigned to 0.005Mp per half cycle. Such a rule is uniformly applied for all the cycles larger rotation amplitude of which exists between  $2\theta_p$  and  $3\theta_p$ . However, the experiments showed that the strength deterioration of 0.005Mp took place in only the first few cycles and no deterioration was observed in the following cycles. Then, the deterioration might be overestimated in the calculation.

#### CONCLUSION

The cyclic bending tests of H-shaped steel beams under the constant rotation amplitude and the random rotation amplitude were carried out, and the strength deterioration of the beams in the region beyond the critical rotation amplitude was investigated. In conclusion, the strength deterioration of H-shaped steel beams under random rotation can be predicted by the following method: The hysteresis loops of the beam rotations are divided into half cycles, each of which is a part of the curve from the turning rotation to the arriving rotation. The strength deterioration per half cycle is estimated depending on the magnitude and the ratio of rotation amplitude, on the basis of the relationship between the strength deterioration and the rotation amplitude obtained from the constant rotation amplitude tests. Total strength deterioration of the beam in the hysteresis is evaluated by summing up the strength deterioration which occurs in each half cycle.

#### ACKNOWLEDGEMENT

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#### REFERENCE

1. Takanashi, K et al, "Failure of Steel Beams due to Lateral Buckling under Repeated Loads," Preliminary Report of IABSE Symposium, Lisbon 1973

Table 1 Summary of Cyclic Test

Specimen	Amplitude Ratio	Cyclic Program $\theta/\theta_p$ Ductility Ratio (Number of Cycles)
DG-130-16	1.0	1.0-2.0-2.5-3.0-3.5-4.0-4.5-5.0-5.5- (5) (5) (5) (5) (4) (4) (4) (3) (3) 6.0-3.0 (3) (5)
DG-130-17	0	2.0-3.0-4.0-4.5-5.0-5.5-6.0-6.5-7.0- (5) (5) (4) (4) (4) (4) (4) (4) (4) 7.5-8.0-8.5-9.0-9.5-10.0-5.0 (4) (4) (4) (4) (4) (4) (5)
DG-130-20	0.5	2.0-3.0-4.0-4.5-5.0-5.5-6.0-6.5-7.0- (5) (5) (5) (5) (5) (5) (5) (5) (5) 7.5-3.0 (5) (5)

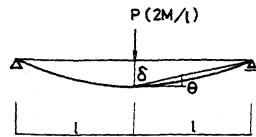
Table 2 Summary of Random Test

Specimen	Loading Condition	Cyclic Program $\theta/\theta_p$ Ductility Ratio (Number of Cycles)
DG-130-5	Earthquake Response <sup>1)</sup>	1.0-2.0-2.5-3.0-3.5-4.0-3.5-3.0 <sup>2)</sup> (1) (1) (1) (1) (2) (2) (1) (1)
DG-130-9	Random <sup>3)</sup>	1.0-2.0-2.5-3.0-3.5-4.0-4.5-2.0-2.5- (1) (1) (1) (1) (1) (1) (1) (1) (1) 3.0-3.5-4.0-4.5-2.5-3.0 <sup>2)</sup> (1) (1) (1) (1) (2) (2)
DG-130-10		1.0-2.0-2.5-3.0-3.5-3.83-4.5-2.0- (1) (2) (2) (2) (2) (2) (2) (2) 2.5-3.0-3.5-3.83 <sup>2)</sup> (2) (2) (2) (2)

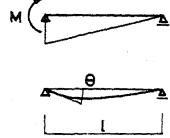
1) Response rotation calculated with ground acceleration of Hachinohe 1968

2) Ratio of maximum rotation to  $\theta_p$ 

3) Random rotation generated on a digital computer



(a) Test Beam



(b) Moment and Rotation of Beam

Fig.1

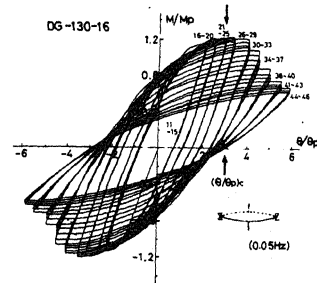


Fig.2 Hysteresis Loops, DG-130-16

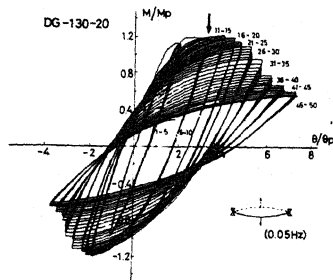


Fig.3 Hysteresis Loops, DG-130-20

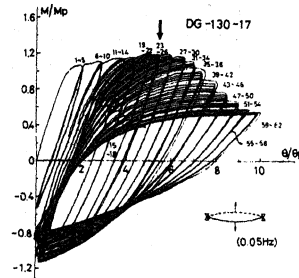


Fig.4 Hysteresis Loops, DG-130-17

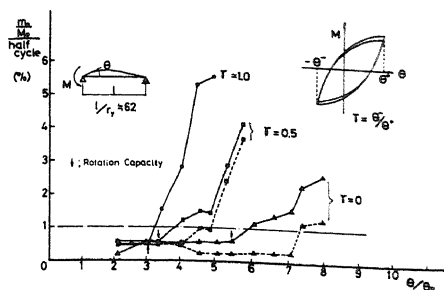


Fig.5 Strength Deterioration per a Half Cycle and Rotation Amplitude

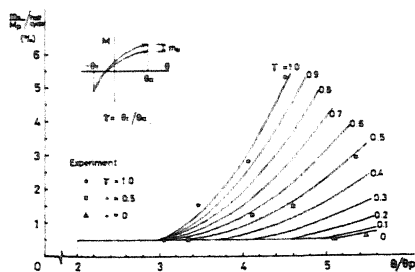


Fig.6 Interpolation of Strength Deterioration for  $\theta_a > \theta_t$

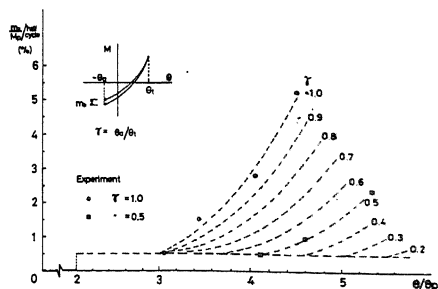
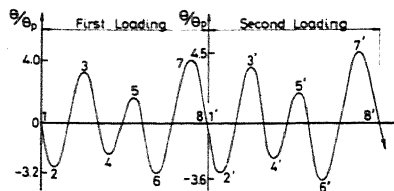
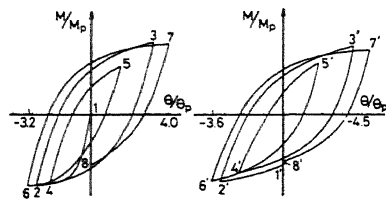


Fig.7 Interpolation of Strength Deterioration for  $\theta_t > \theta_a$

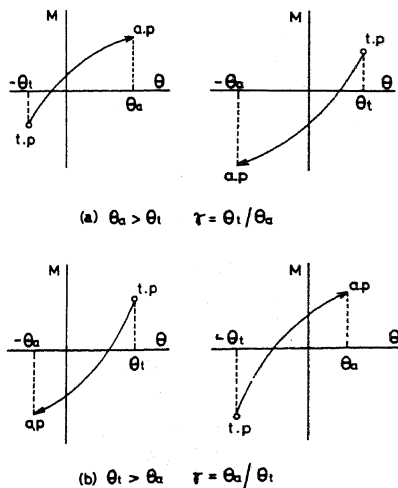


(a) Time Histories of Rotation Amplitude



(b) Hysteresis Curve for  $\theta_{max} = 4\theta_p$  (c) Hysteresis Curve for  $\theta_{max} = 4.5\theta_p$

Fig.8 Schematic Time Histories of Rotation and Hysteresis Curves



(a)  $\theta_a > \theta_t$   $T = \theta_t / \theta_a$

(b)  $\theta_t > \theta_a$   $T = \theta_a / \theta_t$

t.p : turning point a.p : arriving point

Fig.9 Half Cycle

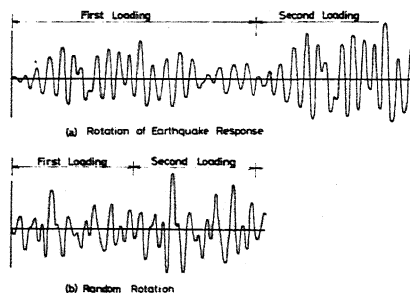
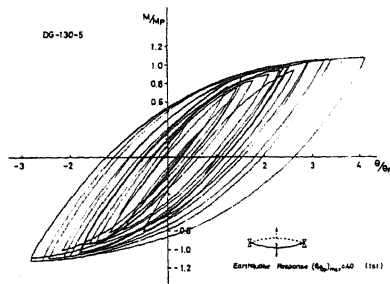
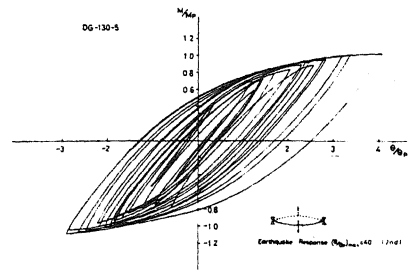


Fig.10 Hysteresis of Random Rotation

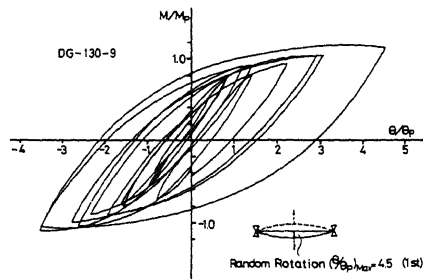


(a)  $\theta_{\max} = 4\theta_p$ , 1st

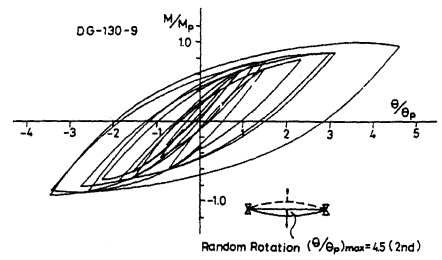


(b)  $\theta_{\max} = 4\theta_p$ , 2nd

Fig.11 Hysteresis Curve, Test DG-130-5



(a)  $\theta_{\max} = 4.5\theta_p$ , 1st



(b)  $\theta_{\max} = 4.5\theta_p$ , 2nd

Fig.12 Hysteresis Curve, Test DG-130-9

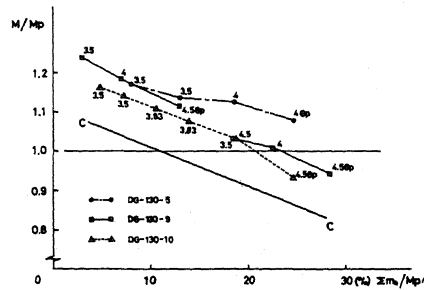


Fig.13 Total Strength Deterioration of Beams on Positive Side

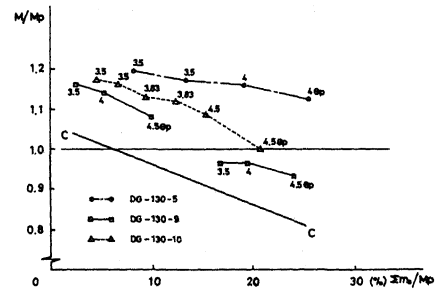


Fig.14 Total Strength Deterioration of Beams on Negative Side