FORCE-DEFORMATION HYSTERESIS CURVES OF REINFORCED CONCRETE SHEAR WALLS

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SUMMARY

Experimental studies on twenty framed shear wall models of reinforced concrete are presented. Mainly load-horizontal displacement and load-vertical displacement curves are discussed. Vertical displacements at the shear wall model yielding in bending are not so small as they can be neglected. Influences of load history and hoop reinforcement of these curves are clearly shown by the tests. The hoop reinforcement ratio of column is useful for keeping load after the maximum load points. The force-displacement curves can be analyzed by simple assumptions.

INTRODUCTION

Shear walls are effective structural members in buildings against earthquake forces. Many experimental and analytical studies have been done in U.S.A., Japan etc.. However, inelastic behavior of reinforced concrete shear walls subjected to bending, shear force and vertical load can not be predicted accurately. The object of this paper is to discuss "force-deformation" hysteresis curves of reinforced concrete shear walls referring to the test results.

Thirty seven reinforced concrete shear wall models[*3] were tested as a part of Japanese National Project "Establishment of New Aseismatic Design Method". Among these tests, twenty models with the same scale were tested by authors. Hereafter, experimental and analytical studies on the twenty models are described.

OUTLINE OF EXPERIMENTAL STUDIES

Test Specimens Dimensions and reinforcements of the twenty test specimens are shown in the Table 1 and Fig. 1. Mechanical properties of concrete and steel are shown in Table 2. Fig. 1 shows the test specimen W7102. Main parameters of the twenty test specimens are described as follws; 1) Height of wall; 2m and 3m, 2) Thickness of wall; 5cm to 10cm, 3) Main longitudinal reinforceing bars in columns; four D13 bars, eight D13 bars and twelve D13 bars, 4) Web reinforcement ratio in column; 0.22% to 1.02%, 5) Reinforcement ratio of wall, 0.23% to 0.70%.

Loading and Measuring The loading arrangement is shown in Fig.2. The all specimens were tested at Building Research Institute in Tokyo.

Three specimens were applied monotonic loadings but the other were applied cyclic loadings. The horizontal displacements and vertical ones at the intersectional points of center lines of beams and columns were measured. The elongations over the certain intervals along column center lines were also measured in order to obtain flexural deflections. Strains of steel and concrete were measured by means of electric strain gauges.

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Test Results The several critical steps are shown in Table 3, in which Q means horizontal load, and R is given by ratio of horizontal displacement at the top to height of the specimen. The critical steps are at stages of first flexural crack, first shearing crack, yielding of main bars in column, maximum load and limit deflection. Yielding of main bars were determined by the strain gauge data. The limit deflection is defined as the top horizontal deflection at the load step where the load drops suddenly or decreases lower than 75 percent of its maximum. The final failure patterns are shown in Fig. 3.

Strength of Shear Wall The critical loads corresponding to flexural cracks, shearing cracks, yielding of steel and the maximum load are compared with the calculated values given by the following formulae in Table 4.

The bending moment when the first flexural crack occures, can be calculated by the following formura be on "beam theory".

$$M = 1.8\sqrt{F_c}\cdot Z_e + N\cdot Z_e / A_e \qquad \text{---(1)}$$

in which A_e , Z_e: Area and coeffient of equivalent section (cm^2), (cm^3)
N : Axial force (kg)
F_c : Compressive strength of concrete (kg/cm^2)

The calculated values fairly agree to the experimental values.

Sugano proposed the following formula[*1] predicting the shear stress at diagonal cracking, τ_c.

$$\tau_c = [0.043\cdot P_g\ (\%) + 0.051]\ F_c \qquad \text{---(2)}$$

in which P_g : $A_g / A_w \times 100$
A_g : Reinforcement area of a surrounding column (cm^2)
A_w : Area of wall $(= t\cdot l)$ (cm^2)

Eq. (2), predicts the shear stress at diagonal cracking of the test excellently.

Sugano[*1] and Hirosawa[*2] proposed the equations(Eq. 3, Eq. 4) for shear strength of wall individually from the existing experimental data. Ultimate shear strengthes, Q_{u_1} and Q_{u_2}, are given by the following equations.

$$Q_{u1} = Q_c + Q_s \qquad \text{---(3)}$$

in which Q_c : Shear force carried by concrete (kg)
$Q_c = Q_d\cdot l / L$
L : Length of imaginary brace (cm)
Q_d : Axial force at brace (kg)
Q_s : Shear force carried by wall reinforcement (kg)

$$Q_{u2} = [0.0679\cdot P_t^{0.23}(F_c + 180)/\sqrt{M/QD + 0.12} + 2.7\sqrt{\sigma_{wh}\cdot P_{wh}} + 0.1\cdot\sigma_0]b_e\cdot j \qquad \text{---(4)}$$

in which b_e : Width of equivalent rectangular section which has the same area as I shape section (cm)
$P_t = 100 \times a_{tl} / b_e\cdot d$
a_{tl} : Reinforcement area of column (cm^2)
d : $D - D_{cr}/ 2$ (cm) D_{cr} : Depth of one column (cm)
σ_{wh}: Yield strength of horizontal wall steel (kg/cm^2)
P_{wh}: Horizontal reinforcement ratio of wall panel in which b_e is considered as thickness of wall
σ_0 : Average stress due to axial force (kg/cm^2)

The both calculated values are listed in the Table 4. Sugano's Eq. gives a little smaller load than the experimental one and Hirosawa's Eq., but Hirosawa's gives a little higher load. Moreover, if the longitudinal bars in beams are considered as horizontal wall steel, the analytical values by Hirosawa's Eq. increase, as be shown in Table 4(Eq. 4'). The above two equations well predict the maximum load of the walls mostly failing in shear after flexural yielding. However, it is not clear that these equations can predict shear strength of walls with smaller shear span ratios.

FORCE-DEFORMATION CURVES

Shear force-horizontal displacement curves and shear force-vertical displacement curves measured at the top beam of the principal models are shown in Fig. 4.

Cyclic Loading and Monotonic Loading Three pairs of specimens were designed at the same ones in which one was applied monotonic load and the other one was applied cyclic loads in each pair, that is, W7401:W7102, W7403:W7402, and W7502:W7503. Force-displacement curves of each pair of specimens can be easily compared in Fig. 4. The ratios of the maximum load of the monotonic specimens to that of cyclic one are from 1.02 to 1.11, which can be thought as 1. However, the ductility of the pair is quite different from each other. Namely the force-horizontal displacement curve of the monotonic test has a long flat part and suddenly drops thereafter, but the other has the shorter flat part and the force decreases gradually.

Reinforcement Ratio and Shear Span Ratio One example of the effect by main reinforcement of column can be shown through the comparison of the model W7102(4-D13) with W7503(8-D13). In this example W7102 has not larger strength but nearly equal ductility. W7606 was designed the same as W7605 except the height of specimen. The ratio of the maximum loads is 1.44 but that of the member rotation angles at the limit displacement is 0.69.

Hoop Reinforcement Ratio One of main objects of the test series is to discuss the effect by the hoop reinforcement of column. The hoop reinforcement ratio of the W7102's columns is 0.22%, that of W7402 is 0.45% but that of W7404 is 1.02%. The hysteresis curves of the three models are shown in Fig.4. The horizontal load of W7102 suddenly drops after 20mm horizontal displacement, due to shear failure at the column on the compressive side. The load of W7402 also drops after 20mm but keeps 70% of the maximum. The failure of column on compressive side progresses gradually together with the crash at the corner of the wall panel. The highest reinforced model, W7404, has nearly the same curves within 20mm horizontal displacement, but the horizontal load is kept 90% of the maximum.

ANALYSIS OF FORCE-DEFORMATION CURVES

The experimental force-flexural deflection curves are spindle shape hysteresis loops, but the force-remainder curves, which are given by the subtraction of the flexural deflection from the total deflection, are close to slip models. The example of the force-flexural deflection curves is shown in Fig. 5.

Stress-Strain Curves of Concrete and Steel Stress-strain relation of steel is assumbed as shown in Fig. 6 based on Jirsa's equations[*4]. E function curve for concrete proposed by Umemura is adopted to the analysis.

Moment-Curvature Curves The curvature of the critical section of the shear wall specimen is calculated with above-mentioned relations of the materials by using the discrete element method, where the horizontal section is discretized into thirty elements.

Horizontal Displacement Distribution of curvatures along the center line of the model is assumed as shown in Fig. 7. Flexural deformation can be easily calculated using the moment area method. Analytical hysteresis curves are added to Fig. 5. Both curves are similar to each other.

The above-mentioned remainder deflection is assumed as shear deformation. The assumed hysteresis rule of the shear deformation is shown in Fig. 8, referring to the experimental deflection.

The lateral displacement is the sum of the flexural and the shear deformations. Fig. 9 shows the example of the horizontal displacement hysteresis curves calculated on the assumption that the analytical flexural displacement follows the same amplitude as the experimental one.

Vertical Displacement The analytical vertical displacements are calculated on the following assumptions. 1) Elastic deformation is neglected, 2) Only cracked part of the wall has vertical strain, 3) The elongation at the center of the critical section is calculated by the above-mentioned discrete element method, 4) Distribution of the strain is assumed as shown in Fig. 7, 5) Summation of the above-mentioned strain is the vertical displacement.

The example of the analyzed vertical displacement corresponding to the above-mentioned horizontal displacement is shown in Fig. 9. It can be thought that the anlyzed curve fits the experimental one.

CONCLUSION

1) The ductility of the shear wall subjected to monotonic load is different from that subjected to cyclic loads. The load-horizontal displacement curves of the former one have long flat parts and they suddenly drop, but the latter ones proceed to the lower gradually.

2) The hoop reinforcement of the surrounding columns is effective to keep the horizontal and vertical loads after the load reaches the maximum point.

3) The horizontal and vertical displacement of shear wall yielding in bending can be predicted by means of the method in this paper.

ACKNOWLEDGEMENTS

The researches described in this paper were carried supported by Ministry of Construction, Government of Japan. The authors are indebted to Prof. A. Ono, Prof. Y. Kanoh, Dr. M. Hirosawa, Prof. T. Okada and Mr. Gotoh for their valuable advices.

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Table 1. Test Specimens

Specimens	M/QD	Column N-φ	Column Pg(%)	Hoop φ-@	Hoop Pw(%)	Wall T(cm)	Wall φ-@	Wall Ps(%)	Comment
W7101	0.833	4-D13	0.81	9-@50	1.02	8	6-@50	0.70	
W7102	"	"	"	6-@100	0.22	"	6-@150	0.23	
W7103	"	"	"	"	"	"	"	"	1 Story
W7104	"	"	"	"	"	5	"	0.37	
W7401	"	"	"	"	"	8	"	0.23	Monotonic Load
W7402	"	"	"	6-@50	0.45	"	"	"	
W7403	"	"	"	"	"	"	"	"	Monotonic Load
W7404	"	"	"	9-@50	1.02	"	"	"	
W7501	"	8-D13	1.63	"	"	"	"	"	
W7502	"	"	"	6-@100	0.22	"	"	"	Monotonic Load
W7503	"	"	"	"	"	"	"	"	
W7504	"	4-D13	0.81	9-@50	1.02	5	"	0.37	
W7505	"	8-D13	1.63	"	"	"	"	"	
W7506	"	"	"	6-@100	0.22	"	"	"	
W7601	"	"	"	"	"	8	6-@100 Double	0.70	
W7602	"	12-D13	2.44	9-@50	1.02	10	6-@125	0.22	
W7603	"	"	"	6-@100	0.22	"	6-@80 Double	0.70	
W7604	1.28	8-D13	1.63	"	"	8	6-@150	0.23	3 Stories
W7605	"	12-D13	2.44	"	"	10	6-@125	0.22	3 Stories
W7606	0.833	"	"	"	"	"	"	"	

M/QD: Shear span ratio T(cm): Thickness cm

Table 2. Mechanical Property of Materials

Concrete

No.	σ	No.	σ
W7101	265	W7503	222
W7102	251	W7504	224
W7103	265	W7505	207
W7104	251	W7506	182
W7401	206	W7601	204
W7402	235	W7602	204
W7403	290	W7603	243
W7404	244	W7604	353
W7501	279	W7605	276
W7502	233	W7606	266

σ(kg/cm²): Compressive strength

Steel

No.	6φ (SR24)	9φ (SR24)	D13 (SD30)
W7101 –W7104	4560	3640	3660
W7401 –W7404	4220	3240	3830
W7501 –W7506	3740	2940	3240
W7601 –W7603	4514	3341	4173
W7604 –W7606	4310	3569	3346

σ(kg/cm²): Yield strength

Table 3. Test Results

* : No experimental values

Specimen No.		Flexural Crack Q ton	Flexural Crack R×10⁻³ rad.	Shearing Crack Q ton	Shearing Crack R×10⁻³ rad.	Yield of Main Bar Q ton	Yield of Main Bar R×10⁻³ rad.	Maximum Load Q ton	Maximum Load R×10⁻³ rad.	Limit Deflection Q ton	Limit Deflection R×10⁻³ rad.
W7101	+	30.0	0.67	30.0	0.67	52.6	4.85	66.7	12.05	51.7	19.68
	-	35.0	0.64	20.0	0.35	49.8	2.34	60.8	5.22	-	-
W7102	+	25.0	0.67	25.0	0.67	49.4	4.95	49.4	4.95	-	-
	-	30.0	0.61	30.0	0.61	51.7	4.96	57.0	9.49	56.6	12.21
W7103	+	*	*	26.4	0.62	50.6	5.15	53.5	7.31	51.5	9.71
	-	*	*	25.0	0.59	-	-	50.2	4.98	37.2	11.41
W7104	+	30.0	0.85	22.5	0.41	46.3	5.12	47.6	7.25	43.5	10.99
	-	35.0	0.96	22.5	0.33	51.4	5.14	52.3	8.32	51.3	10.03
W7401		24.0	0.35	22.0	0.27	54.8	8.46	58.1	20.23	58.0	28.00
W7402	+	26.0	0.34	26.0	0.34	52.9	4.92	52.9	4.92	51.5	10.56
	-	20.0	0.24	20.0	0.24	50.2	4.43	54.9	9.75	-	-
W7403		26.0	0.26	30.0	0.39	59.4	10.04	61.2	18.15	59.6	33.23
W7404	+	34.0	0.46	36.0	0.62	52.4	5.02	53.4	10.10	40.0	29.01
	-	26.0	0.14	28.0	0.22	42.4	2.13	57.0	10.01	-	-
W7501	+	26.0	0.29	36.0	0.64	65.8	4.95	65.8	4.95	65.7	10.11
	-	26.0	0.25	28.0	0.38	52.8	2.59	63.8	9.99	63.8	9.99
W7502		20.0	0.27	36.0	0.96	62.0	3.75	68.1	11.14	65.5	12.98
W7503	+	28.0	0.34	43.0	1.46	58.5	5.00	66.8	10.05	66.8	10.05
	-	28.0	0.18	32.0	0.44	59.0	3.26	63.5	5.01	55.7	6.25
W7504	+	30.0	1.36	16.0	0.59	46.3	4.99	48.7	10.17	48.7	10.17
	-	28.0	0.87	16.0	0.25	43.1	3.21	50.4	9.51	50.4	9.51
W7505	+	27.0	1.37	16.0	0.59	-	-	52.3	5.05	52.3	5.05
	-	23.4	0.87	12.7	0.28	-	-	42.7	3.83	42.7	3.83
W7506	+	20.0	0.73	20.0	0.61	-	-	56.5	5.07	56.5	5.07
	-	24.0	0.96	14.0	0.41	-	-	57.5	5.16	57.5	5.16
W7601	+	40.0	0.85	30.0	0.47	75.5	4.69	85.3	10.02	85.3	10.02
	-	32.0	0.43	38.0	0.71	65.7	2.67	77.2	5.00	68.0	10.06
W7602	+	24.0	0.26	44.0	1.08	74.1	5.00	82.5	5.02	77.3	7.65
	-	38.0	0.53	38.0	0.53	65.9	3.17	85.9	8.34	85.6	9.91
W7603	+	40.0	0.42	44.0	0.60	87.5	3.32	104.7	9.39	104.7	9.39
	-	42.0	0.39	46.0	0.51	95.7	3.87	100.8	4.97	97.1	6.29
W7604	+	12.0	0.30	26.0	0.80	47.8	5.09	50.3	10.05	43.3	13.18
	-	12.0	0.08	26.0	0.59	49.2	5.03	50.2	10.14	-	-
W7605	+	12.0	0.32	12.0	0.32	59.0	3.67	65.3	14.96	65.3	14.96
	-	12.0	0.13	26.0	0.58	60.2	5.16	61.5	10.25	-	-
W7606	+	32.0	0.28	48.0	0.68	94.0	4.71	94.3	5.01	90.7	10.35
	-	32.0	0.11	44.0	0.61	-	-	86.0	4.96	79.7	9.83

Table 4. Comparison of Experimental Values with Calculated Values

Specimen No.		Flexural Crack		Shearing Crack		Maximum Load			
		Test	Eq.(1)	Test	Eq.(2)	Test	Eq.(3)	Eq.(4)	Eq.(4)'
W7101	+	30.0	34.9(0.86)	30.0	27.6(1.09)	66.7	76.9(0.87)	72.9(0.91)	78.1(0.85)
	-	25.0	(0.72)	20.0	(0.72)	60.8	(0.79)	(0.83)	(0.78)
W7102	+	25.0	34.2(0.73)	25.0	26.1(0.96)	49.4	52.2(0.95)	61.5(0.80)	70.3(0.70)
	-	30.0	(0.88)	30.0	(1.15)	57.0	(1.09)	(0.93)	(0.81)
W7103	+	-	34.8 -	26.4	27.6(0.96)	53.5	52.2(1.02)	63.0(0.85)	63.6(0.84)
	-	-	-	25.0	(0.91)	50.2	(0.96)	(0.80)	(0.79)
W7104	+	30.0	33.9(0.88)	22.5	18.4(1.22)	47.6	37.2(1.28)	53.1(0.90)	60.9(0.78)
	-	35.0	(1.03)	22.5	(1.22)	52.3	(1.41)	(0.98)	(0.86)
W7401		24.0	31.9(1.33)	22.0	21.4(1.03)	58.1	57.1(1.02)	58.9(0.99)	67.0(0.87)
W7402	+	26.0	33.4(0.78)	26.0	24.5(1.06)	52.9	57.1(0.93)	61.9(0.85)	70.1(0.75)
	-	20.0	(0.60)	20.0	(0.82)	54.9	(0.96)	(0.87)	(0.78)
W7403		26.0	35.6(0.73)	30.0	30.2(0.99)	61.2	57.1(1.07)	67.6(0.91)	75.9(0.81)
W7404	+	34.0	33.7(1.01)	36.0	25.4(1.42)	53.4	57.1(0.94)	62.8(0.85)	71.0(0.75)
	-	26.0	(0.77)	28.0	(1.10)	57.0	(1.00)	(0.91)	(0.80)
W7501	+	26.0	36.7(0.71)	36.0	35.1(1.03)	65.8	69.6(0.95)	73.8(0.89)	81.4(0.81)
	-	26.0	(0.71)	28.0	(0.80)	63.8	(0.92)	(0.86)	(0.78)
W7502		20.0	35.0(0.57)	36.0	29.3(1.23)	68.1	69.6(0.98)	68.2(1.00)	75.7(0.90)
W7503	+	28.0	34.2(0.82)	43.0	28.0(1.54)	66.8	69.6(0.96)	66.9(1.00)	74.4(0.90)
	-	28.0	(0.82)	32.0	(1.14)	63.5	(0.91)	(0.95)	(0.85)
W7504	+	30.0	32.8(0.91)	16.0	16.4(0.98)	48.7	36.4(1.34)	51.7(0.94)	58.3(0.84)
	-	28.0	(0.85)	16.0	(0.98)	50.4	(1.38)	(0.97)	(0.86)
W7505	+	27.0	33.8(0.80)	16.0	19.7(0.81)	52.3	34.7(1.51)	56.1(0.93)	62.8(0.83)
	-	23.4	(0.69)	12.0	(0.61)	42.7	(1.23)	(0.76)	(0.68)
W7506	+	20.0	32.1(0.62)	20.0	17.3(1.16)	56.5	32.1(1.76)	53.5(1.06)	60.2(0.94)
	-	24.0	(0.75)	14.0	(0.81)	57.5	(1.79)	(1.07)	(0.96)
W7601	+	40.0	33.1(1.21)	30.0	25.7(1.17)	85.3	101.4(0.84)	69.5(1.23)	82.4(1.04)
	-	32.0	(0.97)	38.0	(1.48)	77.2	(0.76)	(1.11)	(0.94)
W7602	+	24.0	34.7(0.69)	44.0	34.3(1.28)	82.5	102.6(0.80)	78.3(1.05)	87.0(0.95)
	-	38.0	(1.10)	38.0	(1.11)	85.9	(0.84)	(1.10)	(0.99)
W7603	+	40.0	37.1(1.08)	44.0	40.9(1.08)	104.7	153.0(0.68)	88.7(1.18)	102.8(1.02)
	-	42.0	(1.13)	46.0	(1.12)	100.8	(0.66)	(1.14)	(0.98)
W7604	+	12.0	25.7(0.47)	26.0	44.5(0.58)	50.3	53.9(0.93)	72.4(0.69)	81.3(0.62)
	-	12.0	(0.47)	26.0	(0.58)	50.2	(0.93)	(0.69)	(0.62)
W7605	+	12.0	24.7(0.49)	12.0	46.4(0.26)	65.3	69.2(0.94)	76.7(0.85)	85.5(0.76)
	-	12.0	(0.49)	26.0	(0.56)	61.5	(0.89)	(0.80)	(0.72)
W7606	+	32.0	37.6(0.85)	48.0	44.7(1.07)	94.3	93.8(1.01)	86.9(1.09)	94.3(1.00)
	-	32.0	(0.85)	44.0	(0.98)	86.0	(0.92)	(0.99)	(0.91)
$\bar{X}$			0.81		1.00		1.03	0.94	0.84
σ			0.21		0.27		0.26	0.13	0.11

Test: Experimental value (ton)　Eq: Calculated value (ton)　( ): Ratio

$\bar{X}$: Mean value of the ratios　σ : Standard deviation of the ratios

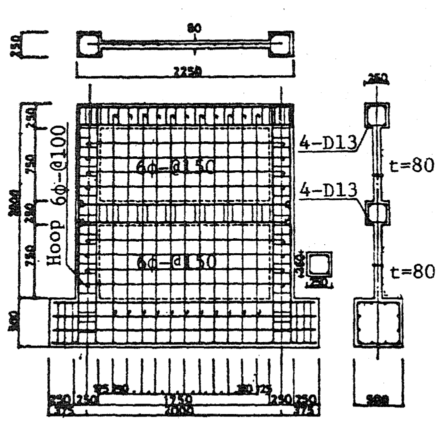

Fig. 1. Details of Specimens (W7102)

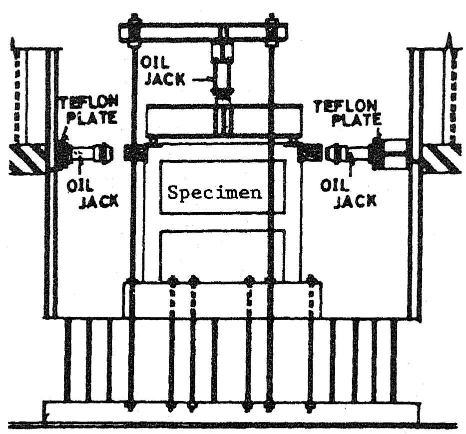

Fig. 2. Loading Arrangement

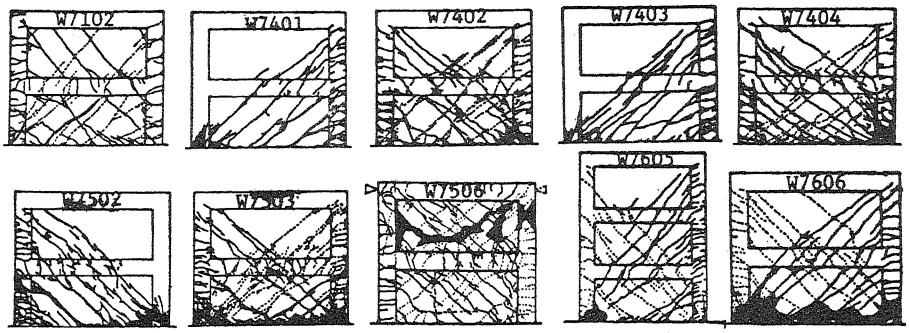

Fig. 3. Final Failure Patterns

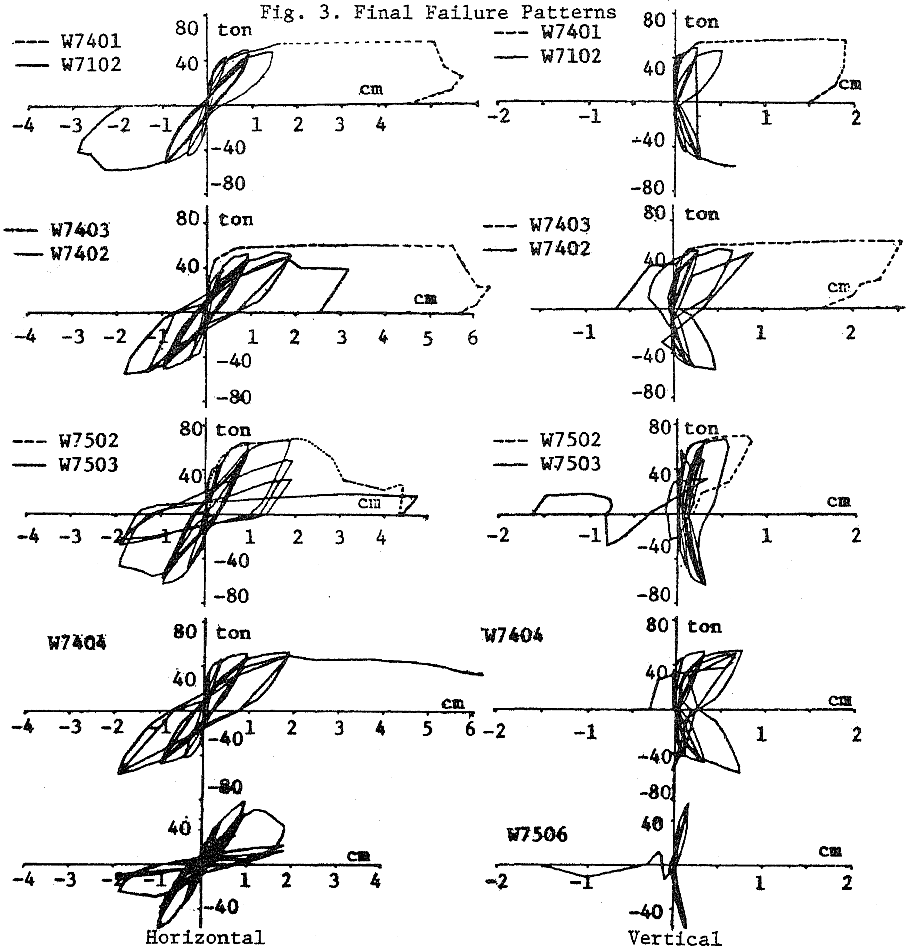

Fig. 4-1. Experimental Force-Deformation Curves

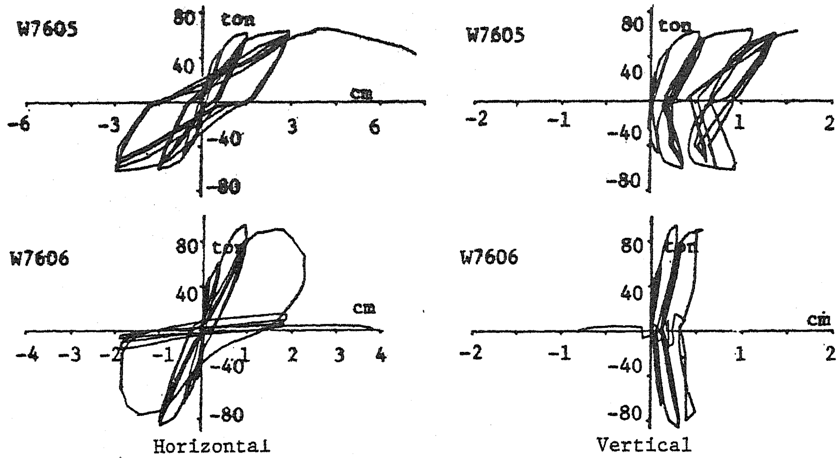

Fig. 4-2. Experimental Force-Deformation Curves

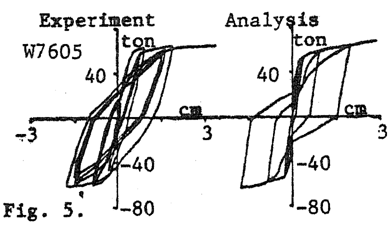

Fig. 5. Force-Flexural Deflection Curves

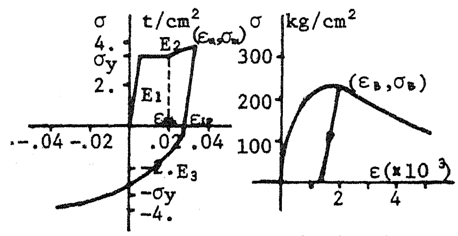

Fig. 6. Stress-Strain Relation Curves

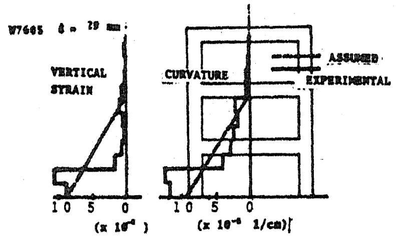

Fig. 7. Distribution of Curvature and Vertical Strain

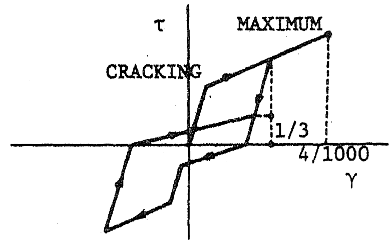

Fig. 8. Shear Deformation

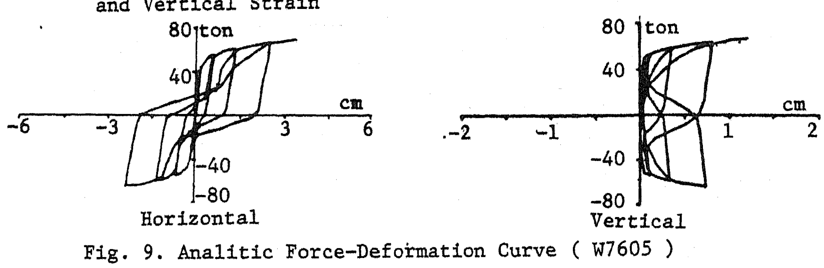

Fig. 9. Analitic Force-Deformation Curve (W7605)