A STUDY OF HYSTERETIC CURVE OF REINFORCED CONCRETE MEMBERS UNDER CYCLIC LOADING

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SUMMARY

Research work in connection with this papier includes experimental investigation of local contact effects of cracked section during reloading and development of a computer program for the load-deflection hysteresis loops of reinforced concrete flexural member with constant axial load.

INTRODUCTION

In the study of inelastic behaviour of reinforced concrete members under seismic action, a theoretical analysis of the hysteretic curve of flexural members subjected to axial load and transverse cyclic loading is of great interest to many investigators. In current method for the calculation of the moment-curvature hysteretic curve, there are two basic assumptions concerning concrete in compression:

(a) Cracked concrete, when subjected to compression under reloading, works as if uncracked.

(b) For concrete in compression zone a stress-strain relation for repeated axial compressive cyclic loading (Fig.1) is adopted instead of that for reversed loading.

On the basis of the above assumptions, the calculate moment-curvature (M-φ) curve under large cracked deformation will present a discontinuous pinching point A (See Fig.2); however such a point does not exist at the corresponding position in the experimental curve. It is necessary to point out that owing to the influence of secondary moment, the calculated results for load-deflection (P $-\Delta$) curve errs more than that for moment-curvature (M $-\varphi$) curve (Fig.3). If we consider the local contact effects of cracked sections during reloading, the discontinuous pinching point A will disappear from the calculated hysteresis loops.

EXPERIMENTAL INVESTIGATION OF LOCAL CONTACT EFFECTS OF CRACKED SECTIONS

A cracked section of a reinforced concrete member during reloading undergoes a process from local contact to entire closure. Local contact of cracked concrete section will transmit local pressure whose magnitude increases with the level of closure.

(1) Outline of Tests In order to study the local contact effects of cracked sections, four reinforced concrete beams with opening at mid-span were tested under reversed loading. The dimensions of test beams are shown in Fig.4. The actual length and depth of the part above opening and the cube strength of concrete are shown in Table 1.

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Table 1

Beam No.	Cube Strength of Concrete R (kg/cm²)	The Part above Opening			
		Length l (mm)	Depth h (mm)	Ultimate Force N (t)	Strength of Concrete (kg/cm²)
LCE-1-1	310	100	28	7	210
LCE-1-2	310	100	28	7.3	217
LCE-2-1	300	140	34	10.1	246
LCE-2-2	300	150	36.5	11.2	256

(2) Test Results Fig. 5 shows the local contact effects of cracked sections with large cracks, Fig. 6 shows the contact effects of cracked sections with small cracks and Fig. 7 shows the contact effects of cracked sections with small cracks to start with, but then suddenly became large cracks. From available test results, the following physical phenomena can be summarized:

(a) As crack widths become larger, the contact effects become more evident:

(b) If crack widths were varied within a limited range (1.75 mm～2.3 mm) the transmitted stress decreased (Fig. 6). It means that the fatigue phenomena existed.

(c) The ratio of crack width which produced contact effects to the historical maximum crack width decreased with the increase of historical maximum crack width.

(3) Equivalent Stress-Strain Relations Fig. 8 presents the equivalent stress-strain relation obtained from a combination of various results ($\bar{\varepsilon}$ - average concrete strain, ε_o - maximum strain corresponding to the ultimate strength of concrete). The difference between Fig. 1 and Fig. 8 is: the former is only suited to uncracked sections subjected to repeated loading while the latter is suited to cracked section subjected to reversed loading. The relation between $\bar{\varepsilon}_{max}$ (corresponding to maximum crack width) and $\bar{\varepsilon}_B$ (corresponding to crack width produced contact effects) is shown in Fig. 9 :

$$\varepsilon_B = \varepsilon_{max}\left(0.1 + \frac{0.9\varepsilon_o}{\varepsilon_o + \varepsilon_{max}}\right) \quad (1)$$

(a) The reloading formulas for concrete stress σ_c are given as follow:

1. $\bar{\varepsilon}_B \leq \bar{\varepsilon} \leq 0$:

$$\sigma_c = \sigma_{co}\left(1 - \frac{2\bar{\varepsilon}}{\bar{\varepsilon}_B + \bar{\varepsilon}}\right) \quad (2)$$

where σ_{co} - maximum contact stress corresponding to $\bar{\varepsilon} = 0$.

The relation between σ_{co} / σ_p and $\bar{\varepsilon}_B / \varepsilon_o$ is shown in Fig. 10:

$$\sigma_{co} = 0.3\,\sigma_p \left[2 + \frac{(\bar{\varepsilon}_B/\varepsilon_o - 4)}{(\bar{\varepsilon}_B/\varepsilon_o + 2)} \right] \quad (3)$$

2. $\bar{\varepsilon} > 0$:

if $\varepsilon_1 < \varepsilon_o$

$$\sigma_o = \sigma_{co}\left(1 - \frac{\bar{\varepsilon}}{\varepsilon_o}\right) + \frac{2\bar{\varepsilon}}{\varepsilon_o + \bar{\varepsilon}}\sigma_p \quad (4)$$

if $\varepsilon_1 > \varepsilon_o$

$$\sigma_c = \sigma_{co}\left(1 - \frac{\bar{\varepsilon}}{\varepsilon_1}\right) + \frac{2\bar{\varepsilon}}{\varepsilon_1 + \bar{\varepsilon}}\left(\frac{2\varepsilon_o}{\varepsilon_1 + \varepsilon_o}\right)\sigma_{c1} \quad (5)$$

Where ε_1, σ_{c1} - concrete strain and stress before unloading.

Before crack formation:

$$\sigma_c = 2\sigma_p \frac{\bar{\varepsilon} - 0.2\varepsilon_1}{\varepsilon_o + \bar{\varepsilon} - 0.2\varepsilon_1} \quad (6)$$

(b) The unloading formulas for concrete stress are given as follow:

if $\varepsilon_1 \leqslant \varepsilon_o$

$$\sigma_c = \frac{(\bar{\varepsilon} - 0.2\varepsilon_1)\,\sigma_{c1}}{1.8\varepsilon_1 - \bar{\varepsilon}} \quad (7)$$

if $\varepsilon_1 > \varepsilon_o$

$$\sigma_c = \left(\frac{2\bar{\varepsilon} - \varepsilon_1}{3\varepsilon_1 - 2\bar{\varepsilon}}\right)\sigma_{c1} \quad (8)$$

(c) The proposed formulas of skeleton curve (See Fig.8) are given as follow

if $\bar{\varepsilon} \leqslant \varepsilon_o$

$$\sigma_c = \frac{2\sigma_p\bar{\varepsilon}}{\varepsilon_o + \bar{\varepsilon}} \quad (9)$$

if $\varepsilon_o \leqslant \bar{\varepsilon} \leqslant \varepsilon_u$

$$\sigma_c = \sigma_p\left\{1 - [200(\bar{\varepsilon} - \varepsilon_o)]^2\right\} \quad (10)$$

if $\bar{\varepsilon} > \varepsilon_u$

$$\sigma_c = 0.3\,\sigma_p \quad (11)$$

where ε_u - maximum concrete strain.

THEORETICAL ANALYSIS OF MOMENT-CURVATURE HYSTERETIC CURVE

(1) Basic Assumption A computer program was developed to investigate the moment-curvature hysteretic curve of the critical section. The following assumptions were adopted:

(a) The strain distribution throughout the depth of section is linear.

(b) The local contact effects should be taken into account in calculation and the formulas (1) ~ (11) should be used for concrete in compression zone.

(c) The stress-strain relation of concrete in tension before crack formation is given in our previous paper [2] .

(d) In the stress-strain relation of steel, Baushinger effect is considered and on the basis of experimental deta, the following formulas are adopted:

$$\sigma = \pm \sigma_S \frac{\varepsilon - \varepsilon_Y}{\varepsilon - \varepsilon_Y \pm C} \tag{12}$$

where σ_S - the ultimate strength of steel;

ε_Y - historical maximum residual deformation of steel;

C - parameter:

if $\varepsilon_Y < 0.02$

$$C = 0.0014 + 0.0393 \frac{|\varepsilon - \varepsilon_Y|(|\varepsilon_Y| + 0.001)}{(|\varepsilon - \varepsilon_Y| + 0.06)(|\varepsilon_Y| + 0.0035)} \leqslant 0.007$$

if $\varepsilon_Y \geqslant 0.02$

$$C = 0.007 .$$

(2) Comparison of Experimental and Theoretical Moment-Curvature Curve. Fig.11 shows the calculated moment-curvature hysteresis loops. Upon comparing the experimental and theoretical results (Fig.11), it is found that the discontinuous pinching point disappeared, and good agreement between the two exists.

THEORETICAL ANALYSIS OF LOAD-DEFLECTION HYSTERETIC CURVE

(1) Basic Assumptions A computer program was developed to investigate the load-deflection characteristic of reinforced concrete members under cyclic loading. On the basis of our previous papers [2][3] . the following additional assumptions are adopted:

(a) The maximum calculated value of curvature extends through the whole plastic region and the length of latter may be determined by reference to our previous paper [2] .

(b) The stiffness of all sections of a member, except the plastic region, takes a constant value.

(2) Computer Program To develop a computer program for the calculation of load-deflection curve the following two problems must be solved:

(a) How to distinguish the different loading conditions and give to them corresponding formulas?

(b) How to realize the automatic calculation procedure?

In order to solve the above problems, we adopt the following means:

(a) Consider a variable sx: = sx + 1 (See Fig. 12), if sx is an odd number, the calculated curve goes from the first to the third quadrant if sx is an even number, the direction of calculated curve will be just the reverse. According to the sx value, one can select the corresponding formulas (1)~(12).

(b) Consider the same curvature increament $\Delta\varphi$ for the calculation of moment-curvature ($M-\varphi$) and load-deflection ($P-\Delta$). According to predicted deflection value $[\Delta]$ (See Fig. 13) the reloading curve can automatically change to the unloading.

(3) Comparison of Experimental and Theoretical Results for Load-Deflection Curves Fig. 14 ~ 16 show the hysteresis loops, good agreement between experimental and theoretical results was obtained.

CONCLUSION

1. The local contact effects of cracked section is an important physical phenomenon of reinforced concrete members with large cracks and subjected to cyclic loading. The magnitude of such effects depends upon the historical maximum crack width.

2. In order to obtain good agreement between experimental and theoretical hysteresis loops, consideration must be given to the local contact effects of cracked section.

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[2] Zhu Bolong, Wu Mingshun, 1979. 5 "A Study of Plastic Hinges and Hysteretic Curves of Reinforced Concrete Members". AICAP-CEB Symposium "Structural Concrete under Seismic Action".

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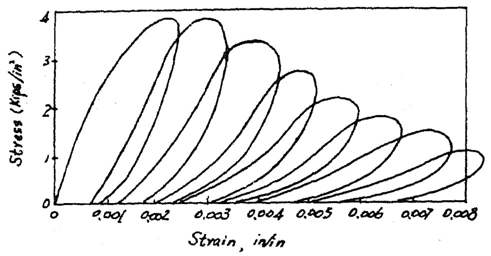

Fig. 1 Stress-strain curve for concrete cylinder with repeated loading

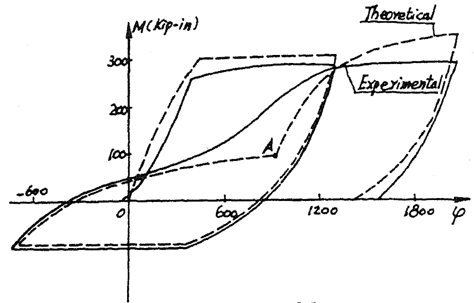

Fig. 2 Moment-curvature curves [1]

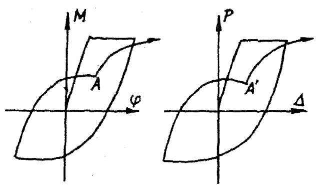

Fig. 3

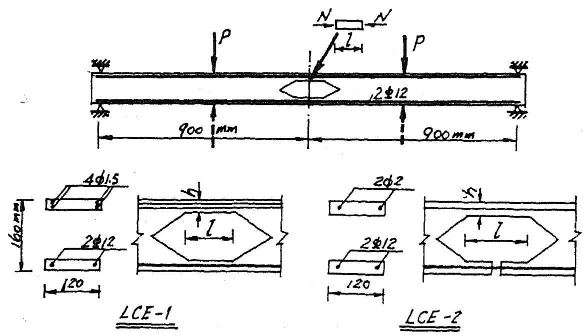

Fig. 4 Specimens LCE-1、LCE-2

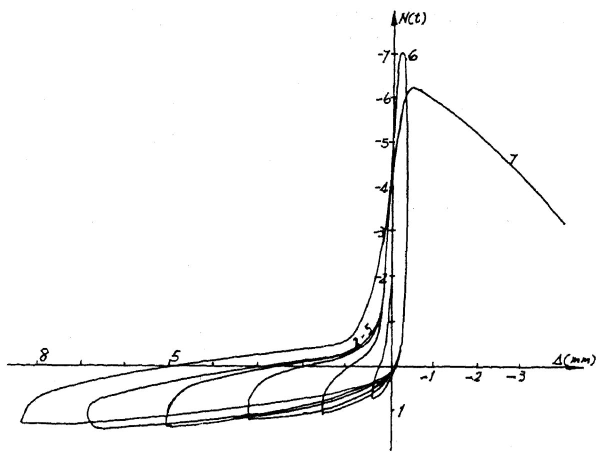

Fig. 5 Contact effects of cracked section (LCE-1-1)

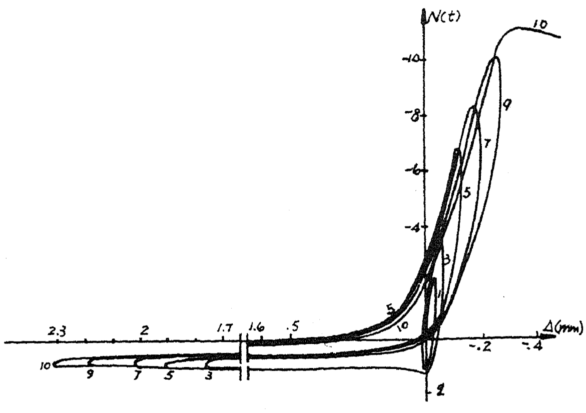

Fig.6 Contact effects of cracked section (LCE-2-2)

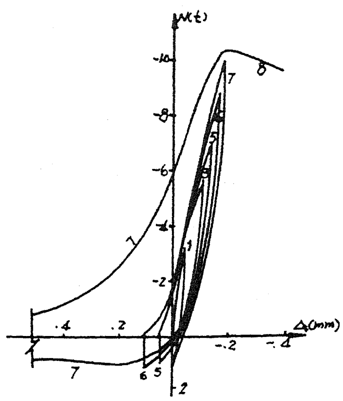

Fig.7 Contact effects of cracked section (LCE-2-1)

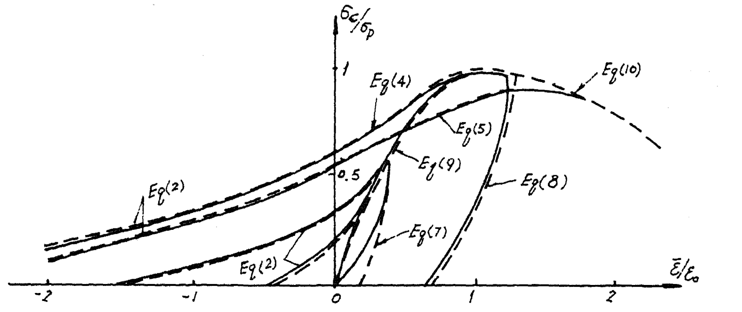

Fig.8 Equivalent stress-strain curve considering local condact effects of cracked section

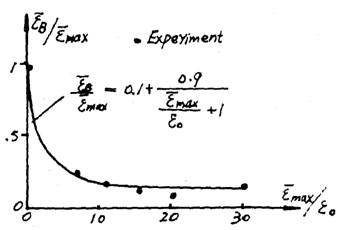

Fig.9 Relation between $\bar{\varepsilon}_{max}$ and $\bar{\varepsilon}_B$

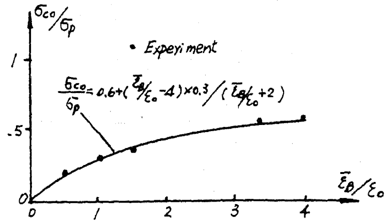

Fig.10 Relation between σ_{co} and $\bar{\varepsilon}_B$

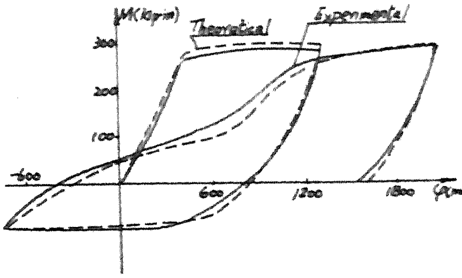

Fig. 11 Moment-curvature curve

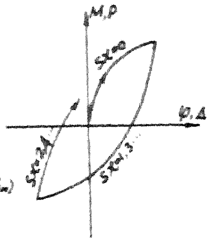

Fig. 12

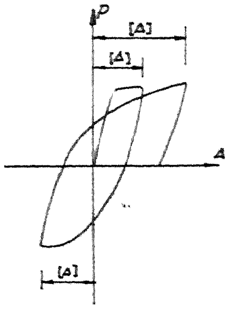

Fig. 13

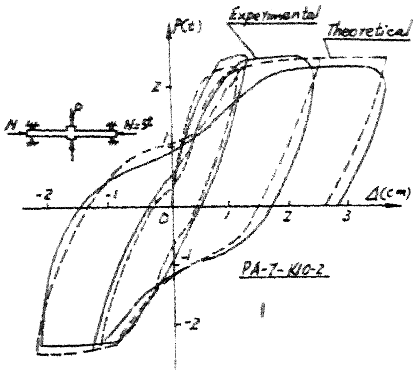

Fig. 15.

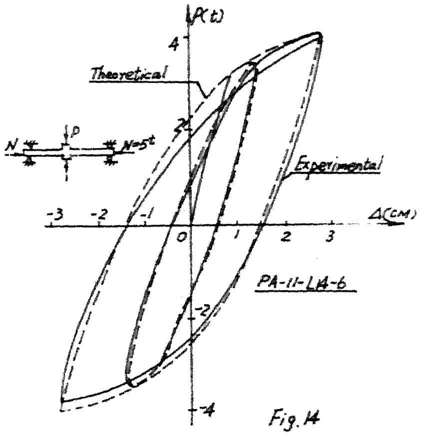

Fig. 14

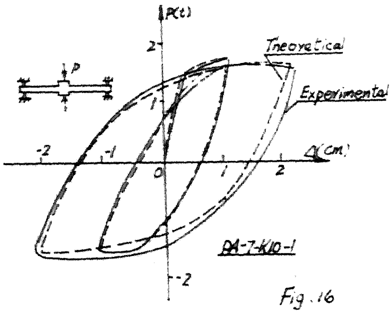

Fig. 16