RESPONSE OF HYSTERETIC SYSTEMS TO RANDOM EXCITATIONS WITH TIME-DEPENDENT POWER SPECTRA

by

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SUMMARY

Here is presented a new analytical technique available for random response analysis of hysteretic systems with strong non-linearity. After the investigation of the usefulness of this technique through the simulation analysis, response characteristics of bi-linear hysteretic systems are studied to make clear the effect of time-dependent power spectra of earthquake motions on inelastic response of building structures. Stochastic earthquake-type motions considered herein are random processes whose non-stationary and non-white spectral characteristics are prescribed by those of the non-stationary absolute acceleration response of the visco-elastic system under pseudo-stationary white random excitations.

INTRODUCTION

In connection with seismic damages of building structures, little attention has been paid to time-dependence of power spectra of strong earthquake motions. However, this time dependence ought to have a significant effect on inelastic structural response, because among inelastic structures under strong earthquake motions there probably exists a kind of resonant phenomenon induced by the fact that the lengthened predominant period, sometimes included in the latter section of earthquke motions with high intensity, becomes successively and nearly equal to the lengthened natural period accompanied by the increase of response level of inelastic structures.

It is well known that earthquake motions have random nature, and then it is quite natural that this point should be taken into consideration in discussing these problems. But there doesn't exist a useful analytical technique for the response analysis of inelastic systems which have hysteretic characteristics such as bi-linear or tri-linear one and multi-degree-of freedom and which are subjected to non-white random excitations with only a few exceptions available for the limited cases 1], 2].

In this paper, therefore, an analytical technique available for these problems is first developed with the aid of two original ideas associated with the non-linear hysteretic system and the analytical technique based on the theory of continuous markov processes 3], 4]. Secondly, after the investigation of the usefulness of the presented technique through the simulation analysis, the results and discussions are presented concerning r.m.s. values of non-stationary relative displacement response of single-degree-of freedom bi-linear hysteretic systems under non-white random excitations by considering those time-dependence of power spectra.

ANALYTICAL TECHNIQUE OF HYSTERETIC RANDOM RESPONSE

To avoid an excess of detail, here is presented an analytical technique of random response of a single-degree-of freedom system with bi-linear hysteretic characteristics in Fig. 1, which can be indicated by a collection

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of the elastic element and the slip one illustrated in Fig. 2. In these figures r , δ and h mean respectively the second slope, the elastic limit deformation of bi-linear hysteretic characteristics and the critical viscous damping ratio of the system. On referring to Fig. 2, the dimensionless equation of motion of this system can be derived as follows:

$$\ddot{x}+2h\dot{x}+rx+r_1 g_1=f \quad : \quad r_1=1-r \qquad \cdots\cdots (1)$$

$$g_1=y\{u(y+\delta)-u(y-\delta)\}+\delta\{u(y-\delta)u(\dot{x})-u(-y-\delta)u(-\dot{x})\} \qquad \cdots\cdots (2)$$

$$\dot{y}=g_2=\dot{x}\{u(y+\delta)-u(y-\delta)+u(-\dot{x})u(y-\delta)+u(\dot{x})u(-y-\delta)\} \qquad \cdots\cdots (3)$$

$$f=-\sum_{j=1}^{n}\alpha_j\,(2h_{g_j}\omega_{g_j}\dot{z}_j+\omega_{g_j}^2 z_j) \qquad \cdots\cdots (4)$$

$$\ddot{z}_j+2h_{g_j}\omega_{g_j}\dot{z}_j+\omega_{g_j}^2 z_j=-\ddot{w} \qquad \cdots\cdots (5)$$

where x , $x-y$ and f are respectively the relative displacement of the mass, the slip element and the non-white random excitation which is composed of linear combinations of the absolute acceleration response $\ddot{z}_j+\ddot{w}$ of a single-degree-of freedom visco-elastic system subjected to pseudo-stationary white random excitation $\ddot{w}$ having zero mean, stationary power spectral density S_0 and an energy envelope function $e(t)$. The coefficients α_j , h_{g_j} and ω_{g_j} are respectively the amplitude, the viscous damping and the frequency parameters defining the type of non-white spectrum of random earthquake-type excitations. Two functions g_1 and g_2 , newly developed herein, mean the following two physical conditions associated with motions of the two elements: first, the elongations of the second spring element are; 1) y for $|y|<\delta$; 2) δ for $y\geq\delta$ and $\dot{x}\gtrless 0$ and $-\delta$ for $y\leq-\delta$ and $\dot{x}\lesseqgtr 0$, and secondly the velocities of the slip element are; 1) zero for $|y|<\delta$; 2) $\dot{x}$ for $y\geq\delta$ and $\dot{x}\geq 0$ or for $y\leq-\delta$ and $\dot{x}\leq 0$.

Denoting $\{x\}=\{x_1(=x), x_2(=\dot{x}), x_3(=y), x_{4_j}(=z_j), x_{5_j}(=\dot{z}_j)\}^T$ $(j=1\sim n)$ and choosing these as state variables, the Fokker-Planck equation, governing the non-stationary probability distribution function $p\equiv p(\{x\};t)$, can be derived as follows:

$$\frac{S_0}{2}e^2(t)\sum_{j=1}^{n}\sum_{l=1}^{n}\frac{\partial^2 p}{\partial x_{5_j}\partial x_{5_l}}-\frac{\partial}{\partial x_1}(x_2 p)+\frac{\partial}{\partial x_2}\Big[\Big\{2hx_2+rx_1+r_1 g_1+\sum_{j=1}^{n}\alpha_j\,(a_j x_{5_j}+b_j x_{4_j})\Big\}p\Big]$$
$$-\frac{\partial}{\partial x_3}(g_2 p)-\sum_{j=1}^{n}\frac{\partial}{\partial x_{4_j}}(x_{5_j}p)+\sum_{j=1}^{n}\frac{\partial}{\partial x_{5_j}}\{(a_j x_{5_j}+b_j x_{4_j})p\}=\frac{\partial p}{\partial t} \qquad \cdots\cdots (6)$$

in which a_j and b_j denote $2h_{g_j}\omega_{g_j}$ and $\omega_{g_j}^2$ respectively. Introduce the characteristic function corresponding to the probability distribution function defined by

$$\phi\equiv\phi(\{s\};t)=\int_{D_{2n+3}}p\exp(i\{s\}\{x\}^T)\prod_{\kappa=1}^{3}dx_\kappa\prod_{\nu=1}^{n}dx_{4_\nu}dx_{5_\nu} \quad : \quad i=\sqrt{-1} \qquad \cdots\cdots (7)$$

, and the Fokker-Planck equation can be transformed into the equation in a domain of the characteristic function:

$$-\frac{S_0}{2}e^2(t)\phi\sum_{j=1}^{n}\sum_{l=1}^{n}s_{5_j}s_{5_l}+s_1\frac{\partial\phi}{\partial s_2}-rs_2\frac{\partial\phi}{\partial s_1}-2hs_2\frac{\partial\phi}{\partial s_2}-s_2\sum_{j=1}^{n}\alpha_j\Big(a_j\frac{\partial\phi}{\partial s_{5_j}}+b_j\frac{\partial\phi}{\partial s_{4_j}}\Big)+\sum_{j=1}^{n}s_{4_j}\frac{\partial\phi}{\partial s_{5_j}}$$
$$-\sum_{j=1}^{n}s_{5_j}\Big(a_j\frac{\partial\phi}{\partial s_{5_j}}+b_j\frac{\partial\phi}{\partial s_{4_j}}\Big)-\frac{r_1}{\pi}s_2\Big[\int_{D_1}\sin\delta\lambda\frac{\partial}{\partial s_3}\phi(s_1,s_2,s_3+\lambda,\{s_{4_j}\},\{s_{5_j}\})\frac{d\lambda}{\lambda}$$
$$+\frac{\delta}{2}\int_{D_1}\phi(s_1,s_2+\lambda,s_3,\{s_{4_j}\},\{s_{5_j}\})\frac{d\lambda}{\lambda}+\frac{\delta}{2}\int_{D_1}\cos\delta\lambda\,\phi(s_1,s_2,s_3+\lambda,\{s_{4_j}\},\{s_{5_j}\})\frac{d\lambda}{\lambda}$$
$$-\frac{\delta}{2\pi}\int_{D_2}\sin\delta\lambda\,\phi(s_1,s_2+\mu,s_3+\lambda,\{s_{4_j}\},\{s_{5_j}\})\frac{d\mu d\lambda}{\mu\lambda}\Big]+$$

$$+\frac{s_3}{2}\left[\frac{\partial\phi}{\partial s_2}+\frac{1}{\pi}\int_{D_1}\sin\delta\lambda\frac{\partial}{\partial s_2}\phi(s_1,s_2,s_3+\lambda,\{s_{4_j}\},\{s_{5_j}\})\frac{d\lambda}{\lambda}\right.$$

$$\left.+\frac{1}{\pi^2}\int_{D_2}\cos\delta\lambda\frac{\partial}{\partial s_2}\phi(s_1,s_2+\mu,s_3+\lambda,\{s_{4_j}\},\{s_{5_j}\})\frac{d\mu d\lambda}{\mu\lambda}\right]=\frac{\partial\phi}{\partial t} \quad \cdots\cdots (8)$$

Note that the following identities for two functions g_1 and g_2 are used in deriving (8) from (5):

$$g_1=\frac{y}{\pi}\int_{D_1}\sin\delta\lambda\,\exp(i\lambda y)\frac{d\lambda}{\lambda}+\frac{\delta}{2\pi i}\left\{\int_{D_1}\exp(i\lambda\dot{x})\frac{d\lambda}{\lambda}+\int_{D_1}\cos\delta\lambda\,\exp(i\lambda y)\frac{d\lambda}{\lambda}\right.$$

$$\left.-\frac{1}{\pi}\int_{D_2}\sin\delta\lambda\,\exp\{i(\mu\dot{x}+\lambda y)\}\frac{d\mu d\lambda}{\mu\lambda}\right\} \quad \cdots\cdots (9)$$

$$g_2=\frac{\dot{x}}{2}\left\{1+\frac{1}{\pi}\int_{D_1}\sin\delta\lambda\,\exp(i\lambda y)\frac{d\lambda}{\lambda}+\frac{1}{\pi^2}\int_{D_2}\cos\delta\lambda\,\exp\{i(\mu\dot{x}+\lambda y)\}\frac{d\mu d\lambda}{\mu\lambda}\right\} \quad \cdots\cdots (10)$$

in which D_n indicates an n-dimensional full space of state variables $\{x\}$.

While the characteristic function can be developed in series including the moments of the distribution, the first order moments are equal to zero because of a symmetric relation of bi-linear hysteretic characteristics about its origin and zero mean assumption of white random excitations, and if the attention of investigation is focused on the second order moments, rather sufficient for the analysis of the earthquake engineering problems, the following approximate development for ϕ would be valid:

$$\phi(\{s\};t)=\exp\left(-\tfrac{1}{2}\{s\}[m]\{s\}^T\right) \quad \cdots\cdots (11)$$

in which

$$[m]\equiv[m_{\nu\kappa}]=[\mathrm{E}(x_\nu x_\kappa)] \quad \cdots\cdots (12)$$

and $\mathrm{E}(\cdot)$ denotes an ensemble average and $\nu,\kappa=1,2,3,\{4_j\},\{5_j\}$ $(j=1\sim n)$. Substituting (11) into (8) and comparing the coefficients having the second order dummy variables $\{s\}$, the simultaneous non-linear differential equation of moments can be obtained as follows:

$$\dot{m}_{11}=2m_{12} \quad \cdots\cdots (13)$$

$$\dot{m}_{22}=2\{(r_1c_1-2h)m_{22}+r_1c_2m_{23}-rm_{12}-\sum_{\nu=1}^{n}\alpha_\nu(a_\nu m_{25\nu}+b_\nu m_{24\nu})\quad \cdots\cdots (14)$$

$$\dot{m}_{33}=2(c_3m_{23}-c_4m_{33}) \quad \cdots\cdots (15)$$

$$\dot{m}_{4_j4_l}=m_{4_j5_l}+m_{4_l5_j} \quad \cdots\cdots (16)$$

$$\dot{m}_{5_j5_l}=S_0e^2(t)-(a_j+a_l)m_{5_j5_l}-b_jm_{4_j5_l}-b_lm_{4_l5_j} \quad \cdots\cdots (17)$$

$$\dot{m}_{12}=m_{22}-rm_{11}+(r_1c_1-2h)m_{12}+r_1c_2m_{13}-\sum_{\nu=1}^{n}\alpha_\nu(a_\nu m_{15\nu}+b_\nu m_{14\nu}) \quad \cdots\cdots (18)$$

$$\dot{m}_{13}=m_{23}+c_3m_{12}-c_4m_{13} \quad \cdots\cdots (19)$$

$$\dot{m}_{14_j}=m_{24_j}+m_{15_j} \quad \cdots\cdots (20)$$

$$\dot{m}_{15_j}=m_{25_j}-a_jm_{15_j}-b_jm_{14_j} \quad \cdots\cdots (21)$$

$$\dot{m}_{23}=-rm_{13}+(r_1c_1-2h-c_4)m_{23}+r_1c_2m_{33}+c_3m_{22}-\sum_{\nu=1}^{n}\alpha_\nu(a_\nu m_{35\nu}+b_\nu m_{34\nu}) \quad \cdots\cdots (22)$$

$$\dot{m}_{24_j}=-rm_{14}+(r_1c_1-2h)m_{24_j}-\alpha_j(a_jm_{4_j5_j}+b_jm_{4_j4_j})+m_{25_j}+r_1c_2m_{34_j} \quad \cdots\cdots (23)$$

$$\dot{m}_{25_j}=-rm_{15_j}+(r_1c_1-2h-a_j)m_{25_j}-\alpha_j(a_jm_{5_j5_j}+b_jm_{4_j5_j})-b_jm_{24_j}+r_1c_2m_{35_j} \quad \cdots\cdots (24)$$

$$\dot{m}_{34_j}=m_{35_j}+c_3m_{24_j}-c_4m_{34_j} \quad \cdots\cdots (25)$$

$$\dot{m}_{35_j}=c_3m_{25_j}-b_jm_{34_j}-(a_j+c_4)m_{35_j} \quad \cdots\cdots (26)$$

$$\dot{m}_{4_j5_l}=m_{5_j5_l}-a_lm_{4_j5_l}-b_lm_{4_j4_l} \quad \cdots\cdots (27)$$

in which

$$c_1 = -\frac{\delta}{\sqrt{2\pi m_{22}}}\mathrm{erfc}\left(\frac{\delta}{\sqrt{2m_{33}\beta}}\right) \qquad (28)$$

$$c_2 = \frac{\delta}{\sqrt{2\pi m_{33}}}\exp\left(-\frac{\delta^2}{2m_{33}}\right)\mathrm{erfc}\left(\frac{\gamma_3\delta}{\sqrt{2m_{22}\beta}}\right)-\mathrm{erf}\left(\frac{\delta}{\sqrt{2m_{33}}}\right) \qquad (29)$$

$$c_3 = \frac{1}{2}\left\{1+\mathrm{erf}\left(\frac{\delta}{\sqrt{2m_{33}}}\right)\right\}-\frac{\sqrt{\gamma_2\gamma_3}}{\pi}\exp\left(-\frac{\delta^2}{2m_{33}}\right) \qquad (30)$$

$$c_4 = \frac{\gamma_3\delta}{\sqrt{2\pi m_{33}}}\exp\left(-\frac{\delta^2}{2m_{33}}\right)\left\{1+\mathrm{erf}\left(\frac{\gamma_3\delta}{\sqrt{2m_{22}\beta}}\right)\right\}+\frac{1}{\pi}\sqrt{\frac{m_{22}}{m_{33}\beta}}\left[\exp\left(-\frac{\delta^2}{2m_{33}\beta}\right)\right.$$
$$\left.-\gamma_2\gamma_3\exp\left\{-\frac{\delta^2}{2}\left(\frac{1}{m_{33}}+\frac{\gamma_3^2}{m_{22}\beta}\right)\right\}\right] \qquad (31)$$

and $\gamma_j=m_{23}/m_{jj}$ $(j=2,3)$, $\beta=1-\gamma_2\gamma_3$, and $\mathrm{erf}(\cdot)$ and $\mathrm{erfc}(\cdot)$ are an error function and a complementary error one respectively. The subsequent analysis is made by using the non-white random excitation in the form of r.m.s. value of f in (4), and then here is needed the expression for this form which can be obtained by making use of (4) as follows:

$$\sigma_f^2(t)=\sum_{j=1}^{n}\sum_{l=1}^{n}\alpha_j\alpha_l(a_ja_lm_{5_j5_l}+b_jb_l\,m_{4_j4_l}+a_jb_l\,m_{4_l5_j}+a_lb_j\,m_{4_j5_l}) \qquad (32)$$

If the attention of the investigation is focused on non-white power spectra of random excitation with more than two peaks, some iterative procedures are necessary for transforming the term $S_0e^2(t)$ in (17) into the term $\sigma_f^2(t)$ as expressed in (32), but on the one with one peak, no such procedures are necessary 5].

INVESTIGATION OF THE USEFULNESS OF THE TECHNIQUE

Prior to the discussions on time-dependence of power spectra of earthquake motions, the usefulness of the presented technique is investigated. Computational response analysis was carried out for one-degree-of freedom bi-linear hysteretic systems which have the elastic limit deformation equal to one and which are subjected to stationary white random excitations with power spectral intensity S_0 by using the presented technique and the simulation one. In Fig.3 (a) and (b) are shown the typical r.m.s. values of transient displacement response against the duration normalized by the undamped natural elastic period t_0 of the system. From these figures it appears that the presented technique is capable of giving considerably accurate results even for the case of r=0.1where pretty much amounts of plastic flow is generated in the system.

Similar computational analysis was made for two-degree-of freedom systems which have no viscosity, the elastic limit deformations δ_j $(j=1,2)$ equal to one and the optimum elastic dynamical parameters by using the same analytical procedure as the one in the preceeding section 6]. The typical r.m.s. values of transient displacement response of the first and the second floors are shown in Fig.4 (a)~(d) in which r_j $(j$=1,2$)$ and t_0 denote the corresponding second slope of bi-linear hysteretic characteristics and the first undamped natural elastic period of the system respectively. These results indicate almost the same tendency as the one shown in Fig.3. Similar results revealing the same accuracy as these, not presented herein on account of space limitations of the paper, were obtained for the case of tri-linear hysteretic systems 7].

Therefore it is concluded that the presented technique, on the whole, is sufficiently applicable for random response analysis of various kinds of hysteretic systems with strong non-linearity.

COMPARATIVE DISCUSSIONS ON HYSTERETIC RESPONSE

In order to investigate the effect of time-dependent power spectra of eathquake motions on hysteretic response, here is considered rather ideal and infavourable situation to the system. In Fig.5 are shown two typical power spectra appropriate for this situation, namely one is power spectrum with two peaks corresponding to the predominant angular frequencies of random excitations designated by ω_{g_1} and ω_{g_2} , between which the dimensionless undamped elstic natural frequency of the system equal to one is located, and the other time-dependent power spectrum with one peak whose predominant angular frequency varies from ω_{g_1} to ω_{g_2} with time as shown in the upper half part of Fig.6. In the lower part of this figure are shown the variations of r.m.s. value $\sigma_f(t)$, the maximum value of which is denoted by σ_0 , selected upon a typical energy envelope function formulated on the basis of a number of actual strong earthquake motions 8]. Corresponding to the variation of the predominant angular frequency, there are illustrated five cases of an energy envelope function whose attenuation begins at the quadrisected sections between ten and twenty of dimensionless time in order to pay special attention to problems of the time when the elongation of the predominant period of earthquake motions, having a significant effect on inelastic responses, begins in connection with the variation of an energy envelope function.

In Fig.7 (a) $\sim$ (d) are shown the typical r.m.s. values of non-stationary relative displacement response against the dimensionless time. The solid lines represent the results from neglecting time-dependence of power spectra of random excitations corresponding to the case in Fig.5 (a) , while the dotted ones results from considering time-dependence of those corresponding to the case in Fig.5 (b). From these results it can be seen that structural systems experience the higher response level having a longer duration, considering time-dependence of power spectra of random excitations, and also that the effect of time-dependence of power spectra on the maximum of r.m.s. value $\sigma_x(t)$ becomes more pronounced when the elongation of the predominant period of earthquake motions begins at the early section of those with high intensity. This is more remarkable with the smaller second slope of bi-linear hysteretic characteristics, the larger magnitude of earthquake motions and the larger frequency band width between ω_{g_1} and ω_{g_2} . This is due to the fact that these factors are more apt to bring about the pseudo resonant phenomena in inelastic structures under strong earthquake motions.

Moreover, it appears that the elongation of the predominant period of strong earthquake motions, with its beginning after the high intensity section of these, has little effect on inelastic response of structures. This is because almost no resonant phenomena occur in inelastic systems with the shorter pseudo natural period accompanied with the decrease of their response levels.

CONCLUDING REMARKS

Here was developed a new analytical technique for the assesment of random response of the hysteretic systems. Through the simulation analysis , this technique has been proved to be effectively applicable even for the hysteretic systems with pretty much amounts of plastic flow generated under strong random excitations. It should be noted that this technique can be applicable for both structural systems with multi-degree-of freedom and poly-linear hysteretic characteristics and random excitations with non-stationary and non-white spectral characteristics.

Non-stationary response analysis of bi-linear hysteretic systems with single-degree-of freedom was carried out by considering time-dependence of power spectra of random earthquake-type excitations. The results indicated that the effects of these time-dependence could not be neglected in connection with inelstic response of hysteretic systems and that these effects were more pronounced with the smaller second slope of bi-linear hysteretic characteristics, the larger magnitude of earthquake motions and the higher speed of the attenuation of the predominant angular frequency in the neighborhood of the section of high intensity earthquake motions.

However, much research remains to be investigated concerning the effect of time-dependent spectra of earthquake motions on inelastic structures by choosing more general and actual types of systems and random excitations than those in this study in connection with seismic damages of building structures.

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REFERENCES

1] Wen, Y.-K., "Method of Random Vibration of Hysteretic Systems", Proc. of ASCE, Vol.102, No.EM2, 1976, 249-263.

2] Kobori, T., Minai, R. and Suzuki, Y., "Stochastic Seismic Response and Reliability of Hysteretic Structures", Proc. of 6th WCEE, Vol.2, 1083-1088.

3] Iwan, W. D., "A Distributed-Element Model for Hysteresis and Its Steady-State Dynamic Response", Jour. of Applied Mechanics, Vol.33, No.4, 1966, 893-900.

4] Sveshnikov, A. A., "Application of the Theory of Continuous Markov Processes to the Solution of Non-linear Problems of Applied Gyroscopy", Proc. of 5th International Conference on Non-linear Osscilations, Vol.3, 1970, 659-665, (in Russian).

5] Asano, K., "Effect of Time-Dependent Spectra of Random Earthquake Excitation on Hysteretic Structural Response", Proc. of 5th Japan Earthquake Engineering Symposium, 1978, 1129-1136.

6] Asano, K., "Stochastic Earthquake Response of the Lumped Mass Building Structure with Elasto-plastic Hysteretic Characteristics (Part 2)", Trans. of AIJ, No.256, 1974, 25-32, (in Japanese).

7] Asano, K., "Stochastic Earthquake Response of the Lumped Mass Building Structure with Elasto-plastic Hysteretic Characteristics (Part 4)", Trans. of AIJ, No.258, 1974, 39-45, (in Japanese).

8] Amin, M. and Ang, H.-S., "Non-stationary Stochastic Model of Earthquake Motion", Proc. of ASCE, Vol.94, No.EM2, 1968, 559-583

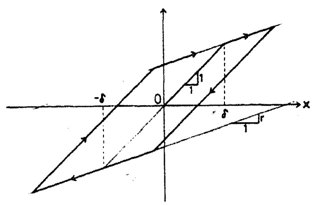


Fig.1 Bi-linear hysteretic characteristics

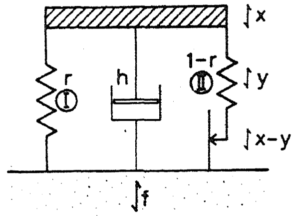
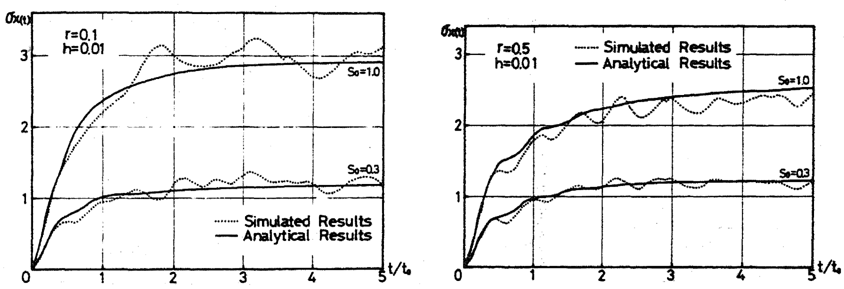
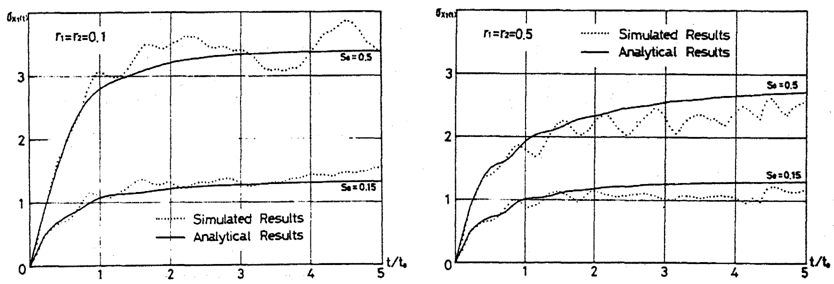


Fig.2 System with bi-linear hysteretic characteristics

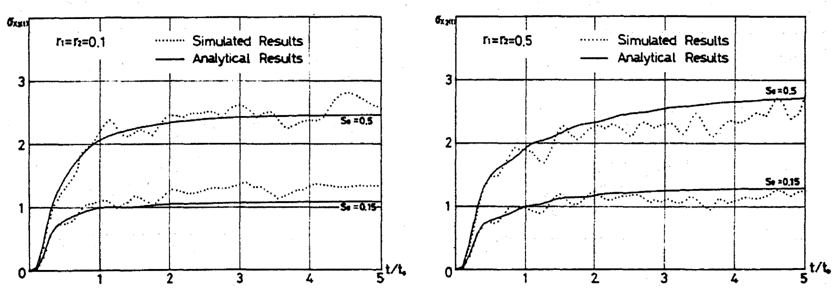


(a) (b)

Fig.3 R.m.s. value of displacement response of 1-dof bi-linear hysteretic system for stationary random excitation

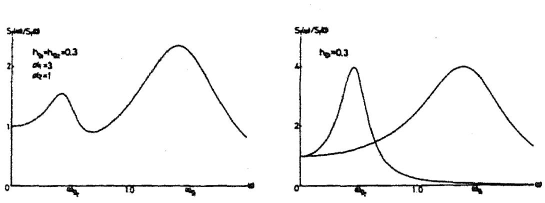


(a) 1st floor (b) 1st floor



(c) 2nd floor (d) 2nd floor

Fig.4 R.m.s. value of displacement response of 2-dof bi-linear hysteretic system for stationary random excitation



(a) (b)

Fig.5 Power spectra of random excitation
(a) stationary one with two peaks
(b) time-dependent one with one peak

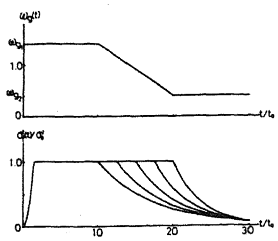


Fig.6 Variation of predominant angular frequency and energy envelope function

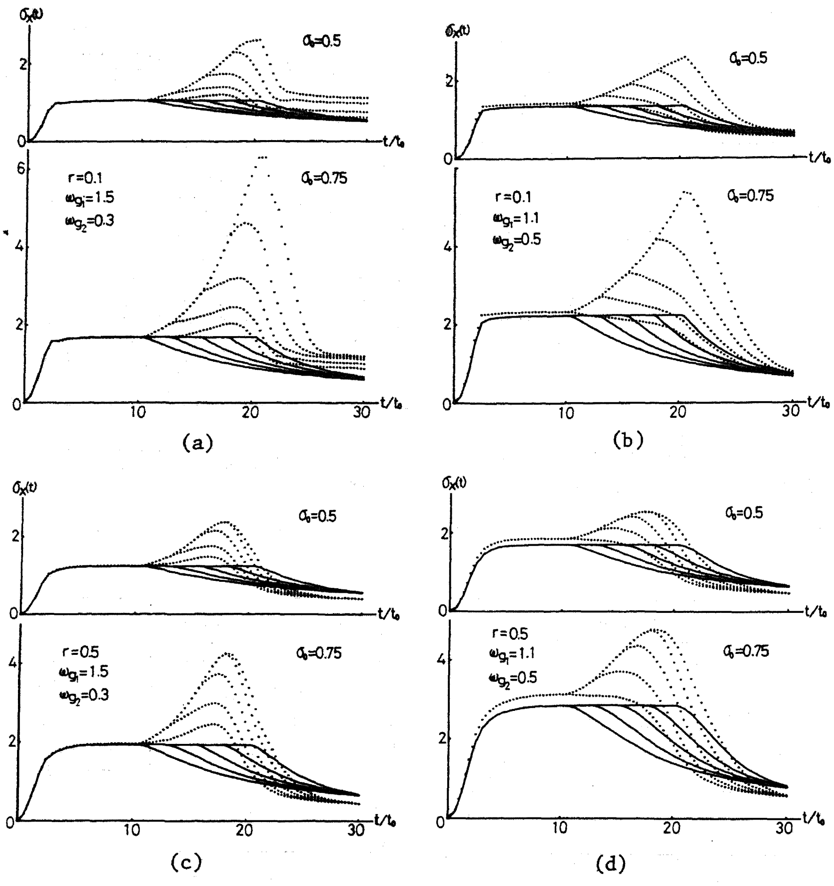


Fig.7 R.m.s. value of displacement response: time-dependence of spectra considered (·······) and neglected (——)