STATISTICAL ANALYSIS OF DISTRIBUTIONS AND REGULATION OF PARAMETERS OF NONLINEAR SYSTEMS UNDER SEISMIC EFFECTS

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SUMMARY

A nonlinear different equation describing the motion of a single-mass design scheme of structures under intensive seismic effects has been solved analytically in a statistical presentation. Nonlinearity is described by a hysteresis curve depending on four parameters, each having a certain physical meaning. The influence of parameters of the hysteresis curve upon the value of the seismic force is analysed numerically and the mechanism of the transition of the structure to the limit state due to damages is studied.

The paper deals with the research results on regulating parameters of limit states of multi-storeyed frame buildings under intensive seismic effects. A three-level model of optimization and regulation of parameters of limit states of frame buildings is analysed. The results are presented of selection of optimal parameters of multi-mass nonlinear systems. Specific features of assessment of reliability in the design of earthquake-resistant structures are discussed.

The first part of the paper deals with the statistical method of the analysis of structures for seismic effects as an essentially nonlinear systems. As is known, under intensive seismic effects the fabric of building structures works outside the elasticity range. In this case the relationship restoring force - displacement can be expressed by nonlinear functions only. The motion of such systems is described by nonlinear differential equations which can, as a rule, be solved only by means of a digital computer.

The paper claims to put forward an analytical nonlinear relationship restoring force - displacement, which parameters are readily found by tests and can be used to obtain explicit solutions for nonlinear differential equations in the form of spectral densities and distributions of output processes for given characteristics of input processes and parameters of the system.

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The general solution for a nonlinear system permits to investigate the laws of seismic forces formulation due to various nonlinearities, the mechanism of the structure's transition to the limit state and its safety, optimal parameters of structures for certain seismic areas.

A single-of-freedom model of a structure is considered the motion of which is described by the following differential equation

$$\ddot{u} + \mathcal{V}(u) = q(t) \tag{1}$$

$\ddot{u}, \dot{u}, u$ - are acceleration, velocity and displacement of the system mass;
$\mathcal{V}(u)$ - is the nonlinear relationship restoring force - displacement;
$q(t)$ - is the input stationary random process simulating the seismic effect;
$\mathcal{V}(u)$ is expressed as follows

$$\mathcal{V}(u) = \frac{u}{\beta + \alpha u^2 + \gamma \dot{u} u} + \delta \dot{u} \tag{2}$$

Eq.2 describes a hysteresis curve, the form of which can change within a wide range due to the selection of parameters β, α, γ and δ. For the sake of illustration Fig.1 presents a curve for input in the form of a sinusoid $q(t) = At\sin \omega t$ with a uniformly increasing amplitude when $\gamma = 0$. In case of a more complex effect $\mathcal{V}(u)$ will have a more complicated form in conformance with the ratios of displacements and velocities of the system.

The geometrical meaning of parameters of $\mathcal{V}(u)$ is as follows: β characterizes the initial slope of the hysteresis loop at a zero displacement, α permits the slope of the hysteresis loop to be varied due to the magnitude of the system displacement; γ is a parameter helping to change the width of the hysteresis loop; δ characterizes the width of the hysteresis loop in the middle portion. These parameters can also be some functions of parameters of the system's motion (number of cylces, magnitude of displacement amplitudes, fatigue damages, etc.). The above parameters are found in testing structures to failure.

Eq.1 and 2 describe the motion of a single-of-freedom nonlinear system with the general hysteresis curve. With $\gamma = \delta = 0$ a differential equation is obtained which describes the motion a nonlinear - elastic system:

$$\ddot{u} + 2\varepsilon \dot{u} + \frac{u}{\beta + \alpha u^2} = q(t) \tag{3}$$

In this equation the damping factor ε is introduced which is absent in the case when the hysteresis curve is considered. With $\alpha = 0$ a linear system is obtained.

All the further obtained results have ultimate transfers from the solution of the system with a hysteresis curve to the solution of a nonlinear - elastic system and then to the solution of a linear system.

Let's consider the way of solving Eq.3, describing the motion of a nonlinear - elastic system.

Spectral presentations of functions in Eq.3 will be used:

$$U(t)=\int_{-\infty}^{\infty} U(\omega)e^{i\omega t}d\omega\,;\; q(t)=\int_{-\infty}^{\infty} Q(\omega)e^{i\omega t}d\omega$$

where $U(\omega)$ and $Q(\omega)$ are random spectra.

The assumed presentations will be substituted in Eq.3, the denominator being first eliminated. In the terms with three integrals convolution will be done, then we differentiate by ω and reduce by $e^{i\omega t}$. The obtained equation is multiplied in turns by $U(\bar{\omega}$, $Q(\bar{\omega})$, $\bar{\omega}^2 U(\bar{\omega})$ and in each of the three equations the operation of averaging will be done. In averaging we think it feasible to use the hypothesis of the quasi-Gaussian law, i.e. the moments of a higher order are expressed through the moments of a second order and $\langle U(\bar{\omega})\,U(\omega)\rangle = S_u(\bar{\omega})\,\delta(\bar{\omega}+\omega')$ as $\bar{\omega}=-\omega'$, $\delta(\bar{\omega}+\omega')$ is the delta-function, $S_u(\bar{\omega})$ is the spectral density.

Further the obtained equations are integrated on account that:

$$x(t)=\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau;\; \int_{-\infty}^{\infty}\omega S(\omega)\,d\omega=0$$

$\int_{-\infty}^{\infty}\omega^2 S_u(\omega)\,d\omega=\sigma_{\dot{u}}^2$; $\sigma_{\dot{u}}$ is the mean square velocity of the system. As a result three equations with three unknowns are obtained:

$$S_{qu}(\omega)=\frac{S_q(\omega)}{L_2}\;;\; L_1=(-\omega^2-2\varepsilon\omega i+n)$$

$$S_{\dot{u}}(\omega)=\frac{\omega^2 S_q(\omega)}{L_1 L_2}\;;\; L_2=(-\omega^2+2\varepsilon\omega i+n)$$

$$S_u(\omega)=\frac{S_q(\omega)}{L_1 L_2}\;;\; n=\frac{\omega_*^2-2\mathcal{L}(\sigma_{\dot{u}}^2+\sigma_{qu}^2)}{1+\mathcal{L}\sigma_u^2} \tag{4}$$

Eq.4 are resolved to an integral equation with one unknown

$$\sigma_u^2=\int_{-\infty}^{\infty}\frac{S_q(\omega)\,d\omega}{[(n-\omega^2)^2+4\varepsilon^2\omega^2]} \tag{5}$$

where σ_u is the unknown mean square deviation of the system; $S_q(\omega)$ is the spectral density of the input process; $n = \omega_*^2/(1+3\alpha\sigma_u^2)$; ω_*^2 is the initial natural frequency of the system.

Eq.5 gives the solution of the nonlinear differential Eq.3. Eq.5 can be written in an explicit form, solving the integral by means of the theory of subtractions or by using a computer.

With the help of Eq.5 calculations were performed for earthquakes of intensity 8-9 presented in the form of a narrow-band stationary random process with a spectral density:

$$S_q(\omega) = \frac{\sigma_q^2 a}{\pi} \cdot \frac{\omega^2 + a^2 + \varphi^2}{(\omega^2 + a^2 + \varphi^2) - 4\varphi^2\omega^2}$$

and the following values of parameters σ_q = 88 cm/s^2 for earthquake of intensity 9 and σ_q = 44 cm/s^2 for the earthquake of intensity 8, a = 2.5. The predominant input frequency is assumed φ = 12, similar to the natural circular frequency of the initial linear system ω_* = 12.2 (α = 0).

Fig.2 givea graphs found from Eq.5 - values of mean square deviations σ_u of the system and mean square values of the seismic (restoring) force σ_v depending on the nonlinearity factor α , characterizing the extent of damage.

As can be seen from the plots, with the growth of the value α , the values σ_u decrease slightly at first, and then rise abruptly as the structure loses rigidity. Comparing the plots for earthquakes of intensity 8 and 9, one can see that when the intensity of the effect becomes twice as little, the factor α , that is damage of the structure, can be increased almost by as much as 5 times. Values of the restoring force reduce significantly with the increase of α as compared to the elastic system (α = 0).

As a result, the normal distribution for the force $v(u)$ can be written down within the assumed hypotheses for the chosen allowable value $/\alpha/$.

At the second stage of the investigation there was considered a hysteresis curve assigned by Eq.2 at = 0, i.e. there were considered hysteresis loops with a constant slope angle. Using for Eq.1 with characteristic (2) at = 0 the same method as for Eq.3 we arrive at a solution in the form of a system of two integral equations as regards the unknowns $\sigma_{\ddot{u}}$ and σ_u .

$$\sigma_{\ddot{u}}^2 = \int_{-\infty}^{\infty} \frac{\omega^2 S_q(\omega)\, d\omega}{(n-\omega^2)^2 + m^2\omega^2}\,; \quad \sigma_u^2 = \int_{-\infty}^{\infty} \frac{S_q(\omega)\, d\omega}{(n-\omega^2)^2 + m^2\omega^2} \qquad (6)$$

$$n = \frac{1-\delta\gamma\sigma_u^2}{\beta} \; ; \quad m = \frac{\gamma(1-\delta\gamma\sigma_u^2)\sigma_u^2}{\beta^2}$$

By means of the obtained system of integral equations for an earthquake of intensity 9 with the same parameters σ_q, a and φ calculations are done to search σ_u for various γ and δ values with $\omega_* = 1/\beta = 12,2$, ω_* is the natural circular frequency of the linear system occurring with degradation of the hysteresis curve into a straight line ($\gamma = \delta = 0$).

The plots of σ_u values depending on δ and γ are given in Fig.3. As can be seen from the plots with the growth of the hysteresis loop width (δ), the displacements of the system reduce. With the growth of the parameter δ, the influence of γ parameter on the σ_u value reduces substantially and starting with $\delta > 1$ the γ parameter can be excluded from the calculations which facilitates greatly mathematical operations.

From the given analysis of the nonlinearity parameters in Eq.2, it can be inferred that further investigation should be done on account of parameters β, α and δ only.

The presented problem will be further developed towards obtaining a non-stationary general solution for a nonlinear differential equation. In this case it would be possible to analyse the mechanism of failure of the system under transient conditions depending on the intensity of the earthquake effect and to determine such allowable parameters α and δ, characterizing the damage of the system that provide the appropriate safet of the system and at the same time permit the design seismic load to be reduced substantially.

The second part of the paper regards problems of regulating parameters of limit states of multi-storey frame buildings under intensive earthquake effects.

Such an approach to designing structures for seismic regions is attributed to specific features of their dynamic behaviour, i.e. to dependence of response parameters upon the dynamic scheme as well as upon strength, rigidity and energy dissipation characteristics of the structure. Thus, buildings are simulated as essentially nonlinear systems, whose parameters depend upon the characteristics of individual structural members and their connections.

The considered methods of regulating limit states of structures present one of feasible ways of putting into practice principles of active seismic protection [1].

Limit states are regulated at the stage of designing buildings with the assigned parameters of limit states. This stage is preceded by stages of analysis of limit states (the analysis itself) and synthesis of constructive decisions (basing on optimal, in terms of certain criteria, distributions of the system's parameters). All the three stages of the design are united within the purposeful approach to designing earthquake-resistant buildings [2,3] .

Such a design, in fact, strives to predict and assess, at the stage of preliminary analysis and alternative design, possible schemes and quantitative characteristics of exhausting load-bearing and operation capacities of structures under intensive seismic effects and to ensure the required structural safety by taking particular anti-seismic precautions.

Using this approach buildings are regarded as dynamic systems whose behaviour under intensive seismic effects is provided so that they could easily take up seismic loads with minimum efforts and so that the system's motion itself could help to fulfil prescribed purposes of aseismic protection. Thus, here the nature-relevant principle of economy of efforts in resisting environmental effects is used as a criterion of optimization of structural schemes of buildings. The essense of the assumed approach is expressed quite clearly by M.Mesarovich, that with the purposeful approach in the theory of systems it is supposed that some invariant aspects of the system's behaviour are known reflecting its purpose, and we realize fully the actions of the system providing for the achievement of the purpose [4].

K.A.Lurie's minimax approach helps to solve the problem of extreme control of the process of elastic-plastic deformation of multi-mass system of a hysteresis type under a seismic effect in the form of an earthquake accelerogramm $y_0(t)$. There were obtained recurrent integral relationships of optimal parameters of limit restoring forces $R_{i,max}$ in the level of different storeys.

The problem resolves itself to a joint solution of the system of equations of motion:

$$\ddot{u}_1 + 2\xi\dot{u}_1 + R_{1,max}\,\rho_1(u_1) - R_{2,max}\,\rho_2(u_2) = -\ddot{y}_0(t)$$

$$\ddot{u}_i + 2\xi\dot{u}_i + R_{i,max}\,\rho_i(u_i)\cdot\left(\frac{1}{m_{i-1}} + \frac{1}{m_i}\right) - R_{i-1,max}\frac{\rho_{i-1}(u_{i-1})}{m_{i-1}} - R_{i+1,max}\frac{\rho_{i+1}(u_{i+1})}{m_i} = 0$$

and equations of relationships

$$R^{opt}_{i,max} = \frac{m_i}{m_{i-1}} R^{opt}_{i-1,max} \frac{\int_0^{t_b} \rho_{i-1}(u_{i-1})\,dt}{\int_0^{t_b} \rho_i(u_i)\,dt} - \frac{m_i}{\int_0^{t_b} \rho_i(u_i)\,dt} \sum_{k=1}^{i-1} \frac{\int_0^{t_b} \rho_k(u_k)\,\dot{u}_k\,dt}{\int_0^{t_b} \rho_k(u_k)\,dt}.$$

Here u_i, $\dot{u}_i$ and $\ddot{u}_i$ are relative displacement velocity and acceleration, respectively, of the i-th floor; $\rho_i(u_i)$ are functions characterizing the nonlinear dependence of restoring forces upon the system's displacements, with hysteresis loops inclusive; ξ is the parameter of viscous damping; $m = \frac{m_i}{m_1}$ is the parameter of distribution of masses over the height of the system; t_e is the time of the effect.

The above problem was analized by eng. K.A.Tonoyan for two-mass and four-mass nonlinear-elastic and elastic-plastic systems for harmonic effects of frequencies 0.32 - 2.24 Hz as well as for records of the ground motions during the Gazli, 1976 and the Carpatian, 1977, earthquakes (components N-S) by digital and analogue computers. The mass ratio varied within the range of 0.25 to 2, the damping of the system was 5% of the critical one.

Fig.4 presents the optimal ratios of $R_{2,max}/R_{1,max}$ as a function of m_2/m_1 parameter for a two-mass system. The line 1 corresponds to harmonic excitation with frequency 0,32 Hz, lines 2-6, respectively, to frequencies 0,64;0,96; 1,28; 1,92 and 2,24 Hz; lines 7 and 8 - to accelerograms of the Gazli and Carpatian earthquakes. The analysis of relationships shows that for structures with a smaller mass m_2/m_1 of the upper storey under effects with definitely expressed predominant periods, more optimal are systems with a lower strength of structures in the upper storey. The extent of reduction of $R_{2,max}/R_{1,max}$ depends greatly on ratios of the input excitation frequency and partial frequencies of the upper storey. At effects in the form of the registered accelerograms these ratios have a smaller range of variation (0.8 -1.0) which is accounted for by the presence of rather great accelerations in a wide range of periods in the spectra of each earthquake.

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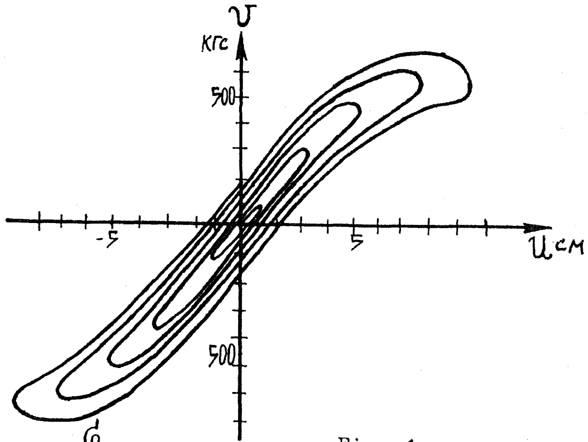

Fig. 1

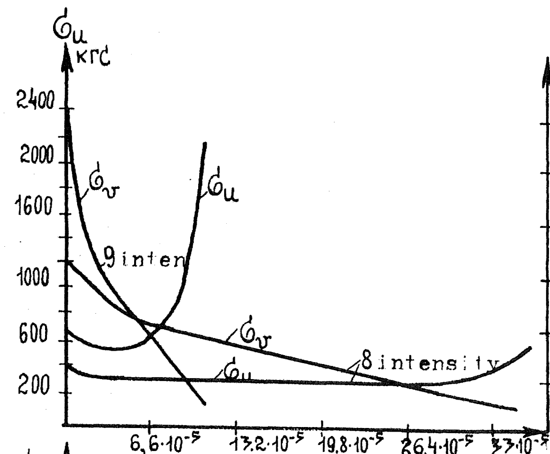

Fig 2

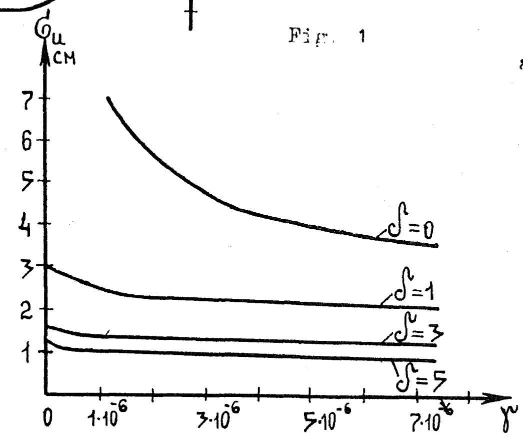

Fig 3

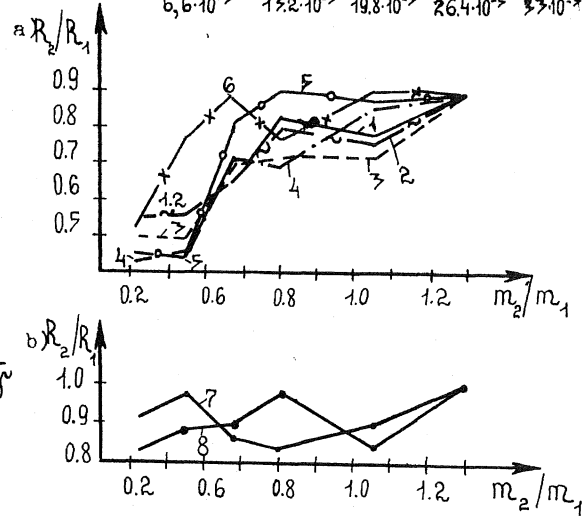

Fig 4