METHODS FOR THE INVESTIGATION OF EARTHDAMS UNDER SEISMIC LOADS

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SUMMARY

The growth of world population requires a steady increase in the number of dams for irrigation and fresh-water supply, also in seismic risk areas. For the time being there is only little practical experience with seismic effects on earthdams. Various methods have been developped to evaluate the stress-strain behaviour of these dams, but those methods still have to be improved. An overview on the current state of the art and the research work in seismic dam design at Braunschweig Technical University is given here.

Principally the behaviour of earthdams during earthquake loading can be studied by pseudostatic or dynamic methods. The pseudostatic method can easily be applied, but it does not take into account a number of factors importantfor the stability analysis of earthdams in a seismic region. The dynamic analysis can be realized with the numerical algorithms of modal response analysis or the step-by-step method using the finite elements.

To find an economic solution of the problem, dynamic analysis with finite elements allows the parametric investigation of different influences with desirable accuracy. The present study shows differences in stresses and strains which depend on various soil parameters of the embankment materials. Diverse element types and element meshes have been compared to select a finite element simulating model.

1. METHODS OF ANALYSIS

1.1 Pseudostatic Method

For this method the earthquake effect is taken into account by using a static equivalent load P_E in a conventional stability analysis (Fig. 1a). P_E is divided into a horizontal component force P_{Eh} and a vertical one P_{Ev}. For these additional forces static stability has to be shown. P_{Eh} or P_{Ev} is given by the product of the dimensionless seismic coefficient k_h or k_v and of the gravity force P_G of a given sliding body. The seismic coefficient k_h, k_v gives the ratio of seismic to gravitational acceleration g.

This method is hence designed to simulate the maximum stresses occurring in a dam section by introducing a purely statical equivalent load. The main problem of the pseudostatic method is the choice of the static coefficient. Usually seismic coefficients are used which are based on accelerometer measurements for structures in the relevant seismic region. The magnitude of these coefficients has also been supported by vibration tests on structures and by theoretical approaches. Empirical values for k_h from 5 % to 15 % are commonly used. k_v is usually neglected.

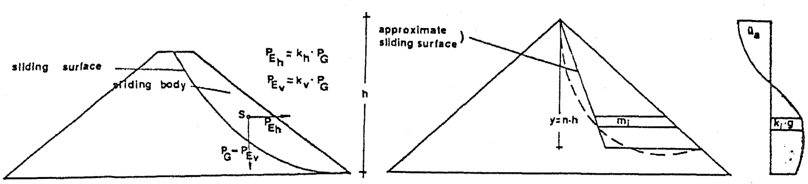

Fig. 1 a): Assumption of earthquake forces for the pseudostatic method
b): Sliding wedge as simplified form of the sliding body for the calculation of an average seismic coefficient

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The theoretical methods for the computation of k-values are either based on the assumption of a rigid body and an elastic continuum for the dam model respectively. The rigid body method assumed an acceleration which is stationary and constant over dam height and whose magnitude equals the seismic acceleration. This method is conservative and can only be regarded as a very rough tool for seismic dam analysis.

In contrary the method of elastic continuum analysis can represent the time history of seismic loading as well as the distribution of acceleration over the dam weight for every sliding body (Fig. 1b). The distribution of the acceleration $\ddot{u}_a$ is calculated by Eq. 1 after Ambraseys (1960):

$$\ddot{u}_a(y,t) = \sum_{n=1}^{\infty} \phi_n(y) \cdot s_a \qquad \left[\frac{m}{s^2}\right] \qquad \text{(Eq. 1)}$$

where Φ_n are the mode shapes and s_a is the spectral acceleration. The sliding body under consideration is subdivided into shear layers for which the relevant seismic coefficient k are calculated according to Eq. 2

$$P_E = \frac{\Delta P_G}{g} \cdot \ddot{u}_a(y,t) = \Delta P_G \cdot k \qquad \left[\frac{kN}{m}\right] \qquad \text{(Eq. 2)}$$

For every possible sliding body an average seismic coefficient k_m can be calculated. A dependence from the material properties is influenced by the dynamic shear modulus G and the damping factor λ. Fig. 2 shows the average seismic coefficient for sliding wedges of different heights as a function of the undamped fundamental period T_o for a dam founded on rock and agitated by the N-S component of the El Centro earthquake (Ambraseys, Sarma, 1967).

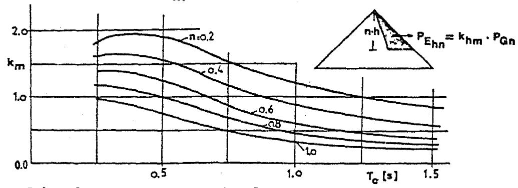


Fig.2 : Average simultaneous seismic coefficient k_m

1.2 Modal response analysis

This method of analysis for seismic loading cases of earth dams has been derived from analytical methods for structures under earthquake loads. Basis for the calculation of the vibration behaviour is the dynamic differential equation (Eq. 3) which describes equilibrium under dynamic loading for a certain calculation model

$$M \cdot \ddot{u}(t) + C \cdot \dot{u}(t) + K \cdot u(t) = R(t) \qquad \text{(Eq. 3)}$$

where M, C and K are matrices describing mass, damping and stiffness respectively and R(t) is the load vector at time t.

The calculation can be based on a multiple-mass-system or a finite element model. For the multiple-mass-model assumed parallel and horizontal layers are interconnected by damped springs. The disadvantage of this model is that the kinematic parameters acceleration, velocity and displacement have to be assumed constant for a horizontal layer at the time t. For a finite element analysis the dam and the foundation are discretised by introduction of finite elememts .

Modal response analysis calculates the natural frequencies ω_i of the earth structure. For every natural frequencies ω_i a definite value of acceleration can be derived from the response spectrum of the design earthquake. By superposition of the Eigenvectors, which are not phaseconstant, the deformation diagram of the earth dam is found. There are several numerical methods available to solve the vibration equation for the natural frequencies analytically.

Such a method was developed by Ojalvo, Newman (1970). It is a reduction method which determines the low fundamental periods of large structures with a great number of degrees of freedom by solving the Eigenvalue problem with small matrices. The method of Ojalvo, Newman (1970) is based on the method of Householder for symmetrical matrices. They additionally found a procedure to reduce the effort in building up the triangular matrix. Thus the computational efficiency is increased without reducing the stability of the calculation method.

The modal response analysis renders only the maximal values of stress and deformation inside the earth structure but not in phase because the seismic loading is derived from a response spectrum. This method in contrary to the pseudo-static method does account for the vibration behaviour by considering fundamental frequencies but cannot include the deformations occurring during an earthquake in dependence of the variation of stress exertion with time as given by a seismogram (Papakyriakopoulos, Simons, 1978).

1.3 Time history analysis

The direct integration of the dynamic motion equation of the dam agitated from the foundation below applied to a finite element model avoids-although at the expense of increased computational effort-the disadvantages of modal analysis. Hereby the time history of a vibrating system can be analysed for finite time steps (step-by-step) more suitably and non-linear soil behaviour can be modelled.

Below same numerical methods of direct integration shall be discussed. Explicit and implicit integration methods are distinguished. In the first case for the solution of the differential equation the system is considered at time t for the second type of analysis at time t + Δ t (Bathe, Wilson 1976).

1.3.1 Method of Central Differencies

In this explicit method the unkown displacements u (t+ Δt) are calculated from the state of equilibrium at time t. Velocity and acceleration are found from:

$$\dot{u}(t) = 0.5 \cdot \frac{u(t+\Delta t) - u(t-\Delta t)}{\Delta t} \qquad \text{(Eq. 4)}$$

$$\ddot{u}(t) = \frac{u(t-\Delta t) - 2 \cdot u(t) + u(t+\Delta t)}{(\Delta t)^2} \qquad \text{(Eq. 5)}$$

1.3.2 Houbolt's Method

Equilibrium at time t+st is considered and the displacements u (t+st) are found in dependence of u(t), u(t- Δt) and u(t-2 Δt). Velocity and acceleration are given by:

$$\dot{u}(t+\Delta t) = [\, 11u(t+\Delta t) - 18u(t) + 9u(t-\Delta t) - 2u(t-2\Delta t)\,] / 6\Delta t \qquad \text{(Eq. 6)}$$

$$\ddot{u}(t+\Delta t) = [\, 2u(t+\Delta t) - 5u(t) + 4u(t-\Delta t) - u(t-2\Delta t)\,] / (\Delta t)^2 \qquad \text{(Eq. 7)}$$

1.3.3 θ -Method after Wilson

This method of integration developed by Wilson is based on the assumption that acceleration changes inearly during the time interval between t and t + θ· Δt

$$\ddot{u}(t+\Delta t) = \ddot{u}(t) + \frac{\Delta \tau}{\theta \Delta t} (\ddot{u}(t+\theta \Delta t) - \ddot{u}(t)) \; ; \; 0 \leq \Delta\tau \leq \theta \cdot \Delta t \qquad \text{(Eq. 8)}$$

For reasons of computational stability θ has to be greater than 1.37.

1.3.4 Newmark's Method

After Newmark one obtains

$$u(t+\Delta t)=u(t)+\dot{u}(t)\Delta t+((0{,}5-\beta)\ \ddot{u}(t)+\beta\ddot{u}(t+\Delta t))/(\Delta t)^2 \qquad \text{(Eq. 9)}$$

$$\dot{u}(t+\Delta t)=\dot{u}(t)+((1-\gamma)\ddot{u}(t)+\gamma\ddot{u}(t+\Delta t))\Delta t \qquad \text{(Eq.10)}$$

Stability can be guaranteed if $\gamma \geq 0{,}5$ and $\beta \geq 0{,}25 \cdot (0{,}5+\gamma)^2$.

1.4 Critical Comparison of the Methods

1.4.1 Quasi-static Method

It is usual for the calculation of resistance of a building against dynamic loading to introduce static equivalent loads (e.g. wind loads). For structures in seismic regions therefore the pseudostatic method is often applied. For the analysis of earth dams by a pseudostatic method the following factors of importance for dam stability are not considered with sufficient accuracy :

1. Influence of pulsating dynamic forces on soil properties.
2. Interaction between dam and foundation deformations.
3. Influence of time variations of earthquake forces inside a dam on its deformation behaviour and thus also on its stability.
4. The usual constructional features of a dam, especially the zones of different materials.

Therefore the pseudostatic method cannot be recommended in general. Frequently dams have been classified as "safe" which later, however, were heavily damaged during earthquakes of comparable magnitude (e.g. San Fernando Dam). Past experience has shown that the calculation results can be conservative as well as non-conservative.

1.4.2 Dynamic Methods

The dynamic behaviour of a dam can be analysed by the methods of modal response analysis and time history analysis. In model response analysis only one stress-and deformation state throughout the whole earthquake is calculated. Additionally soil properties such as strength and damping can only be considered as linear function of deformations. Despite of these shortcomings the method is often applied. In order to determine a tolerance range several variations of soil properties are required. The second method of dynamic analysis - time history analysis - allows the geometric and material properties of every dam and any desired degree of accuracy to be modelled. Also can be stress and deformation behaviour with time for a given seismogram be represented with greater exactness than in modal analysis.

For that a stable numerical method of direct integration with minimum computational effort is necessary. The Central Difference Method has the advantage that it can be applied on element level. The necessary storage capacity for the analysis of large meshes is thus reduced. However, the method shows only low stability and requires a short time interval which depends on the stiffness and mass distribution of the structure. For the three other implicit methods of Houbolt, Wilson and Newmark the effective stiffness matric has to be triangularised. Computational effort can be reduced if optimised and computer-suitable routines are used.

Most important is that all three methods fulfil both stability and accuracy criteria. Wilson, Farhoomand and Bathe (1973) have dealt with this problem. The accuracy of results mainly depends on the ratio of time interval Δt to the natural period of the considered system. Because seismograms usually do not show earthquake periods below 0,05 secs for T, always the lowest period of the system but a higher fundamental period is chosen. For Newmark s suggestion to keep $\Delta t/T \leq 1/10$ errors were reduced to 5 to 7 % already. Investigations have shown that for $\Delta t/t = 0{,}003$ numerical deviations became negliaible. For $\Delta t/T = 0.2$ deviations amount to 20 to 30 %, values of the order of the damping ratio. The time interval Δt should be chosen so that all acceleration peaks of the seismogram are accounted for.

Summarising it can be noted that at present dynamic dam analysis for earthquake loading can be based satisfactorily and realistically on Wilson' s and Newmark' s methods.

2. THE CONCEPT OF DYNAMIC ANALYSIS

The concept of dynamic analysis comprises the following points:

1. Selection of a typical cross-section and development of a suitable FE-model.
2. Calculation of the initial stress state for the structure (static loads and dead weight). For this task a most appropriate calculation method should be applied.
3. Experimental determination of dynamic soil properties and the stress-strain behaviour for different initial stress states.
4. Analysis of the vibration behaviour of the structure.
5. Determination of the time dependent load.
6. Computational investigation by dynamic method for economical design of the structure for dynamic loading.

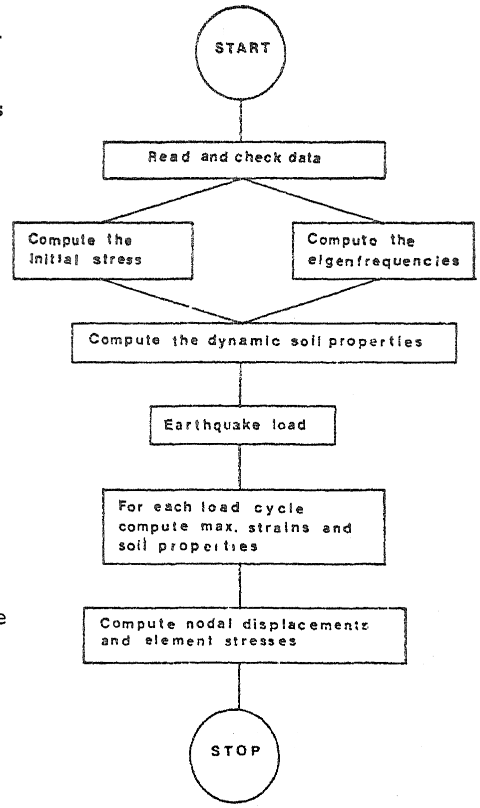


Fig. 3 : Flow chart of dynamic FE-Program

This concept allows stresses and deformations of an earth dam to be analysed for earthquake loading. The following explanations have to be added: A typical cross-section of the dam is selected and subdivided into finite elements. On the basis of this FE-model the initial stress state from dead weight and static loading is computed. The progress of construction is represented by the addition of suitable incremental layers until the top of the dam is reached. Application of a nonlinear stress-strain law in this part of the analysis is desirable. Then the dynamic behaviour of the dam and its fundamental periods or frequencies respectively are analysed. It follows the calculation of the parameters of viscous, velocity-proportional damping for the dynamic analysis. Great care has to be taken in the sslection of an appropriate design seismogram. Finally stresses and deformations of the dam for a dynamic loading case given by a seismogram are computed. Another requirement is the application of a FE-computer program (Fig. 3) with a built-in nonlinear stress-strain soil law. Reliable dynamic soil properties must be used in these computations.

3. PARAMETER STUDIES

3.1 Example 1 Wooden beam

Geometry and especially the choice of a suitable FE-mesh considerably influence the calculation results of a structure under earthquake loading. Therefore the FE-mesh has to be chosen so that the criteria of stability for the various numerical algorithms are fulfilled. Regarding this problem extensive investigations have been carried out. For testing purposes a wooden beam was chosen which was agitated harmonically.

The numerical algorithms of Wilson (θ -Method) and Newmark have been investigated. The results have been compared with the results of an analytical solution. The degree of agreement between both solutions depends on the number of elements chosen for the discretisation of the beam (Fig. 4).

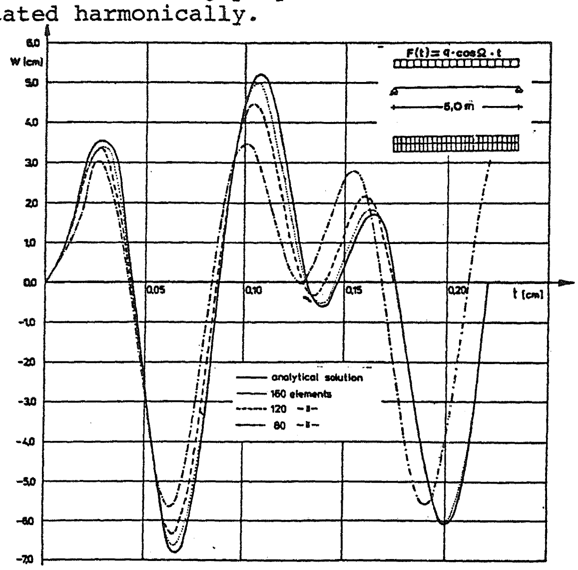


Fig.4 : Comparison of results

The number of elements for the FE-mesh has to be chosen so that thw actual frequencies of the calculation model are represented. Only if this condition is satisfied, meaningful values for acceleration, velocity, and displacement can be calculated (Simons, Papakyriakopoulos 1980).

3.2 Example 2 Homogeneous Dam-Section

Because the problem of FE-discretisation for reliable results and the determination of the actual natural frequencies are complete, different numerical methods of modal response analysis for the calculation of the natural frequencies ω_i have been applied. A homogeneous dam section with different numbers of finite elements has been used for this parameter study. The calculated natural frequencies showed stability for different overall element numbers. Therefore natural frequencies can also be determined for coarse meshes without a considerable loss in accuracy (Fig. 5).

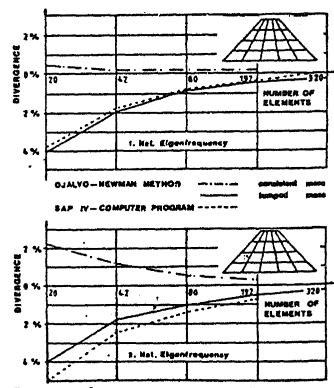
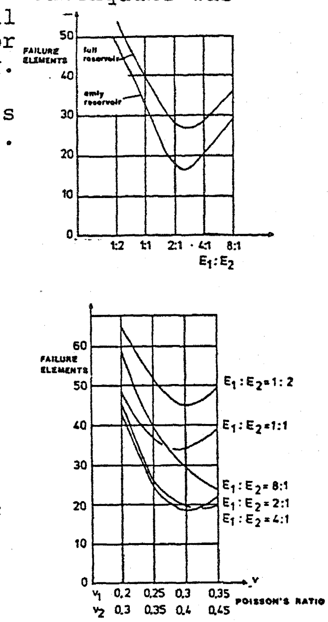


Fig.5 : Divergence of 1. and 2. natural Eigenfrequency

3.3 Example 3 Dam with Zones of Different Soil Properties

Soil properties play an important rôle for dam behavior. Above all the ratio E_1/E_2 of shell to core stiffness is significant. Therefore a typical cross-section of a dam of 91,5o m height has been investigated (Fig. 6). As load the horizontal component of the 1940 "El Centro" earthquake was applied to every mode. The influence of full or empty reservoir and Poisson's ratio ν for different stiffness ratios are shown in Fig. 7a and 7b in dependence of the number of failure elements. The most favourable stress distribution is found for $E_1/E_2=1/4$ (Fig.8).



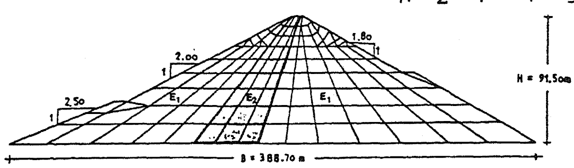


Fig.6 : Dam-Section for Example 3

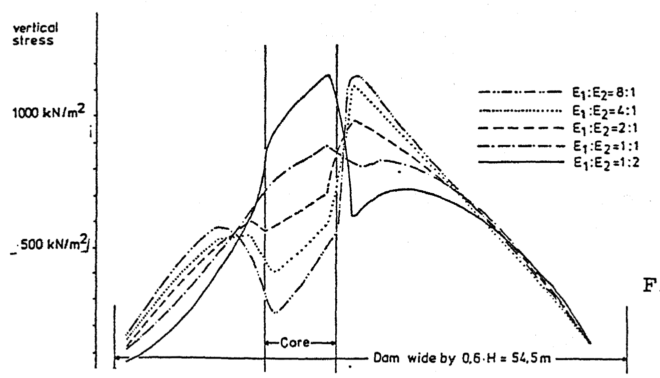


Fig.8 : Vertical stress distribution for different stiffnes ratios

Fig.7 a,b: Influence of water level and Poisson s ratio

4. APPROACHES FOR THE IMPROVEMENT OF PRESENT METHODS

The discussed methods of calculation and above all dynamic analysis require a most exact description of the seismic load and reliable dynamic soil parameters. The development of mathematical approaches or numerical algorithms respectively has reached a sufficient state of development and reliability. The testing methods for the determination of dynamic soil properties and stress-strain laws, however, have to be further improved. Also the stochastic methods for the evaluation of design earthquakes can only rely on insufficient data collections and seismograph recordings. Not before these problems have been solved satisfactorily a three-dimensional consideration of the problem and the necessary computational effort can be justified.

5. RANGE OF APPLICATION - FURTHER DEVELOPMENTS

The time history of motion parameters and stresses in an earth dam can be modelled by a FE-program using the method of direct integration and a non-linear stress-strain law for the soil. With this method the stress-strain behaviour of dams of every geometry and foundation characteristic can be analysed for construction states or various states of operation under earthquake or any other dynamic loading. The methods of dynamic analysis also account for different dynamic soil parameters within the zones of a dam. In seismic risk areas usually zoned dams are built because they show greater resistance against earthquake loading than dams with surface sealing blankets. In the case of zoned dams the behaviour of the pore-water and the pore-water pressure especially within the cohesive fill material play a decisive role under earthquake loading. It is therefore necessary to investigate the pore-water pressure and mainly the excess pore-water pressure development within a dam during dynamic loading. The findings of such investigations should be introduced into the stability analysis.

Under certain circumstandes the computational requirements for the dynamic analysis of earth dams under earthquake loads cannot be fulfilled. In order to model non-linear behaviour an equivalent linear stress-strain-law can be introduced. Thus a simple but also exact design method for practical applications can be developed.

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