

EARTHQUAKE RESPONSE CHARACTERISTICS OF
JOINTED AND CONTINUOUS BURIED LIFELINES

Joseph P. Wright^I
and
Shiro Takada^{II}

SUMMARY

Dynamic response curves of displacement versus frequency are calculated for the problem of forced harmonic response of jointed cylindrical structures in a homogeneous linearly elastic medium of infinite extent. Consistent with experimental observations, the results of a parametric study show that, for frequencies in the earthquake range, dynamic effects can be neglected for jointed or continuous buried pipelines made of concrete, cast iron, or steel, and having commonly used sizes.

INTRODUCTION

The dynamic response of a physical system depends on its frequency and damping characteristics. If the system is highly damped or its resonant frequencies are outside the predominant frequency band of the excitation, dynamic effects may be neglected and the response can be analyzed by means of static computations. Continuous tunnel and pipeline structures were studied in Ref. 1, based on elasto-dynamics, and it was found that dynamic effects can be neglected for buried pipelines subjected to earthquake excitations.

More recently, jointed pipelines were considered in Refs. 2 and 3, based on the theory of Ref. 1. This was done by choosing a low value for the modulus of elasticity of the continuous pipe material to represent the effect of joints, which are generally quite flexible. It was found that dynamic effects can be considerable, particularly for jointed buried pipes whose diameters are not small compared to the wave lengths predominant in earthquakes. The purpose of this paper is to examine the findings of Refs. 2 and 3 by considering a more refined model for jointed pipes.

FORMULATION OF THE PROBLEM

Consider the forced harmonic response problem shown in Fig. 1. The

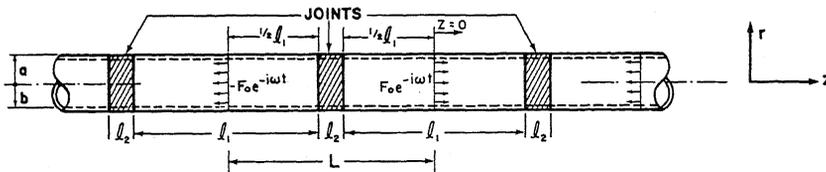


FIG. 1 BURIED JOINTED PIPELINE MODEL; FORCED HARMONIC RESPONSE

I Associate Partner, Weidlinger Associates, Consulting Engineers, New York.
II Associate Professor, Department of Civil Engrg., Kobe Univ., Kobe.

pipe-soil system is represented by an infinite, homogeneous, linearly elastic soil material which surrounds an infinitely long, periodic chain of cylindrical, elastic pipe segments that are connected by cylindrical elastic joint segments.

The pipe and joint segments have outer radius a , and inner radius b . It is assumed that the inner surfaces are stress free and that the outer surfaces are in complete (no slip) contact with the soil. For simplicity the pipe and joint segments are assumed to be radially rigid, as in Refs. 1, 2, with only longitudinal displacements $U(z, t)$. The pipe (joint) segments have density $\rho_1(\rho_2)$, Young's modulus $E_1(E_2)$, and length $\ell_1(\ell_2)$.

The length $L = \ell_1 + \ell_2$ is the distance between concentrated forces that act longitudinally at the middle of the pipe segments and are 180 degrees out of phase. Hence the system is axi-symmetric, as well as periodic in the z direction with period $2L$. The forces are assumed periodic in time with frequency f , so that the applied forces, $F(z, t)$, can be represented by

$$F(z, t) = F_o \delta_p(z) e^{-i\omega t} \quad (1)$$

where $\omega = 2\pi f$, and $\delta_p(z)$ denotes the periodic set of Dirac delta functions that alternate in sign and are separated by the distance L .

The soil material has density ρ , shear wave speed c_s , dilatational wave speed c_p , longitudinal displacements $U_z(r, z, t)$ and radial displacements $U_r(r, z, t)$. As a result of periodicity in z and axi-symmetry in r , the soil displacements may be expanded in Fourier-Hankel series involving Hankel functions, $H_n^{(1)}$, of the first kind of orders $n=0$ and 1, as shown in Ref. 1, with

$$U_r(r, z, t) = \sum_{m=1}^{\infty} \left[A_m \frac{h_m^*}{h^2} H_1^{(1)}(h_m^* r) + B_m \frac{\alpha_m}{k^2} H_1^{(1)}(k_m^* r) \right] \sin \alpha_m z e^{-i\omega t} \quad (2)$$

$$U_z(r, z, t) = \sum_{m=1}^{\infty} \left[A_m \frac{\alpha_m}{h^2} H_0^{(1)}(h_m^* r) - B_m \frac{k_m^*}{k^2} H_0^{(1)}(k_m^* r) \right] \cos \alpha_m z e^{-i\omega t} \quad (3)$$

where $\alpha_m = (2m-1)\pi/L$, $h = \omega/c_p$, $k = \omega/c_s$, and $h_m^{*2} = h^2 - \alpha_m^2$, $k_m^{*2} = k^2 - \alpha_m^2$.

The assumptions of radial rigidity and complete contact result in the condition $U_r(a, z, t) = 0$ which, combined with Eq. 2, implies that

$$B_m = - \left[\frac{h_m^*}{h^2} \frac{k^2 H_1^{(1)}(h_m^* a)}{\alpha_m H_1^{(1)}(k_m^* a)} \right] A_m \quad (4)$$

Hence the longitudinal soil displacements may be expressed as

$$U_z(r, z, t) = \sum_{m=1}^{\infty} A_m U_{zm}(r) \cos \alpha_m z e^{-i\omega t} \quad (5)$$

where

$$U_{zm}(r) = \frac{\alpha_m}{h^2} H_0^{(1)}(h_m^* r) + \frac{h_m^* k_m^*}{h^2 \alpha_m} \frac{H_1^{(1)}(h_m^* a)}{H_1^{(1)}(k_m^* a)} H_0^{(1)}(k_m^* r) \quad (6)$$

The assumption of complete contact also implies that $U(z, t) = U_z(a, z, t)$; thus the pipe and joint displacements are given by

$$U(z, t) = \sum_{m=1}^{\infty} A_m U_{zm}(a) \cos \alpha_m z e^{-i\omega t} \quad (7)$$

As a result of $U_r(a, z, t) = 0$, the shear stress at $r = a$ is given by

$$\tau(a, z, t) = \mu \left. \frac{U_z(r, z, t)}{\partial r} \right|_{r=a} \quad (8)$$

where $\mu = \rho c_s^2$ is the shear modulus of the soil. By using the identity

$$\frac{dH_0^{(1)}(x)}{dx} = -H_1^{(1)}(x) \quad (9)$$

the shear stress may be expressed as

$$\tau(a, z, t) = \mu \sum_{m=1}^{\infty} A_m U'_{zm}(a) \cos \alpha_m z e^{-i\omega t} \quad (10)$$

where

$$U'_{zm}(a) = -\frac{h_m^* k_m^*}{h^2 \alpha_m} H_1^{(1)}(h_m^* a) \quad (11)$$

The governing equation of the pipe and joint segments can be written as

$$\bar{\rho} \frac{\partial^2 U(z, t)}{\partial t^2} - \bar{E} \frac{\partial^2 U(z, t)}{\partial z^2} = \frac{2\tau(a, z, t)}{\bar{a}} + \frac{F_0}{A} \delta_p(z) e^{-i\omega t} \quad (12)$$

where $\bar{a} = (a^2 - b^2)/a$, $A = \pi(a^2 - b^2)$, $E = E_1(E_2)$ and $\rho = \rho_1(\rho_2)$ for z in pipe (joint) segments. Substitution of Eqs. 7 and 10 into Eq. 12 gives

$$\sum_{m=1}^{\infty} A_m \{ [\bar{E}\alpha_m^2 - \bar{\rho}\omega^2] U_{zm}(a) - \frac{2\mu}{\bar{a}} U'_{zm}(a) \} \cos \alpha_m z = \frac{F_0}{A} \delta_p(z) \quad (13)$$

which is the same as Eq. 15 in Ref. 2, except for the modified definitions of \bar{E} , $\bar{\rho}$ and \bar{a} .

Multiplication of Eq. 13 by $(2/L) \cos \alpha_n z$, integration from $z = -L/2$ to $z = +L/2$, and the use of trigonometric identities and other manipulations produces

$$\sum_{m=1}^{\infty} R_{nm} A_m = \frac{2F_0}{AL} \quad \text{for } n = 1, 2, \dots \quad (14)$$

where, for $m = n$,

$$R_{nn} = \{ [E_1 \alpha_n^2 - \rho_1 \omega^2] - \Delta_n \left[\epsilon - \frac{\sin(2n-1)\pi\epsilon}{(2n-1)\pi} \right] \} U_{zn}(a) - \frac{2\mu}{a} U'_{zn}(a) \quad (15)$$

and, for $m \neq n$,

$$R_{nm} = (-1)^{n-m+1} \Delta_m \left[\frac{\sin(n-m)\pi\epsilon}{(n-m)\pi} - \frac{\sin(n+m-1)\pi\epsilon}{(n+m-1)\pi} \right] U_{zm}(a) \quad (16)$$

with ϵ defined by $\epsilon = l_2/L$, and $\Delta_n = [(E_2 - E_1)\alpha_n^2 - (\rho_2 - \rho_1)\omega^2]$.

EFFECT OF JOINT SIZE

Equations 14 represent an infinite set of simultaneous equations that are linear in the unknowns, A_m . Solution of these equations would be difficult, if not impossible, for arbitrary values of physical parameters. However, the relative joint size parameter, $\epsilon = l_2/L$, is generally quite small with $0 < \epsilon < 0.03$.

For $\epsilon = 0$, Eqs. 14 reduce to the case of a buried continuous pipe. In this case, the $m \neq n$ terms are zero and the A_m can be found directly for all m . This was done in Ref. 2 and, because the A_m are proportional to $1/m^2$, 15 terms were found to insure sufficient accuracy. In evaluating the pipe displacement, Eq. 7, at $z = 0$, the point where the force is applied. Study of Eqs. 15 and 16 shows that the coefficients R_{nm} are analytic functions of ϵ and that the difference between a continuous pipe and one with small joints is terms that are proportional to ϵ^3 . Hence for sufficiently small ϵ , it is reasonable to assume that a finite subset of Eqs. 14 will be adequate for numerical studies.

Continuous dependence of the solution of ϵ also implies that, for any case of physical interest, there will be a range of ϵ values for which the displacement response will be nearly the same as for $\epsilon = 0$. Since the case $E_2 = E_1$, $\rho_2 = \rho_1$ reduces Eqs. 14 to the case of a buried continuous pipe, independent of ϵ , the ratios $e = E_2/E_1$ and $d = \rho_2/\rho_1$ are also important in studying the effects of joint size. These ratios are quite variable and rather uncertain for actual joint materials with $0 < e < 1$ and $0.1 < d < 1$ being typical. The case of joints without stiffness, $e = 0$, $d = 1$, is of particular interest since it should provide a lower bound estimate of the range of ϵ values for which the displacement response is nearly the same as $\epsilon = 0$.

NUMERICAL STUDIES

Based on the above considerations, the computational strategy used was to solve specific problems with $e = 0$, $d = 1$, first with $\epsilon = 0$, then with $\epsilon = 0.004, 0.01, 0.02, 0.03$. This was done using 15, 20 and 40 terms in Eqs. 14 with the finite subset of complex linear equations being solved by Gauss elimination with complete pivoting. Other cases, with $e = 0.001, 0.01, 0.1$ and $d = 0.1, 0.4$, were also studied.

Two quantities were calculated for frequencies $f < 20$ Hertz: (1) the pipe maximum displacement at $z = 0$, denoted here by $W(f)$; (2) the dynamic amplification ratio $R(f) = W(f)/W(0)$. Since the series for $W(f)$ converges like $\Sigma(1/m^2)$, the numerical error committed by using M terms goes to zero as $(1/M)$. In all cases reported here, $W(f)$ increased by less than two per cent as M increased from 15 to 40 terms. The amplification ratio was even less sensitive to M , with the changes being less than two-tenths of a per cent in all cases studied.

The first case studied was based on parameters suggested by Refs. 2 and 3. Soil properties: $\rho = 1.5 \text{ g/cm}^3$, $c_s = 30 \text{ m/sec}$, $\nu = 0.3$, where ν is Poisson's ratio defined by $c_p^2/c_s^2 = 2(1-\nu)/(1-2\nu)$. Pipe dimensions: $L = 6\text{m}$, $a = 1.4\text{m}$, $b = 1.2\text{m}$. Pipe properties (concrete-like): $\rho_1 = 2.4 \text{ g/cm}^3$, $E_1 = 2,000 \text{ kg/mm}^2$. For this case, the joint size was varied between zero and 18 cm with no significant change in W for any frequency, for all cases with $0 \leq \epsilon \leq 0.03$, $0 \leq e \leq 1$, $0 \leq d \leq 1$. The amplification ratio increased by less than one per cent, from 1 to 1.01. Essentially the same behavior was obtained for $a = 0.5\text{m}$, $b = 0.4\text{m}$.

Next, a parameter study was undertaken to try to find larger amplifications for cases of physical interest. Starting from the parameter values of the first case, the following trends were observed. R increases as E_1 decreases. R increases as L increases. R increases as c_s increases. R increases as (b/a) increases toward one. Amplifications were unchanged for $0 \leq e \leq 1$, $0 \leq d \leq 1$, as long as $\epsilon \leq 0.03$.

Based on the results of the parameter study, the following cases, corresponding to relatively long and thin-walled concrete pipes buried in a relatively stiff soil, were chosen. Soil properties: $\rho = 1.6 \text{ g/cm}^3$, $c_s = 300 \text{ m/sec}$, $\nu = 0.4$. Pipe properties: $\rho_1 = 2.4 \text{ g/cm}^3$, $E_1 = 2,000 \text{ kg/mm}^2$. The amplification increased from one at $f = 0$ to maximum values at about 16 Hz, with maximum R values of 1.06, 1.09, 1.13, 1.14, 1.13, 1.10 and 1.08, for each radius, respectively. As before, the joint properties did not affect the response for $\epsilon \leq 0.03$.

Other case studies for cast iron with $\rho_1 = 7.2 \text{ g/cm}^3$, $E_1 = 10,000 \text{ kg/mm}^2$, and steel with $\rho_1 = 7.8 \text{ g/cm}^3$, $E_1 = 20,000 \text{ kg/mm}^2$, showed similar behavior with maximum R values being even less than those for concrete pipes.

CLOSING REMARKS

The results of the present parameter study indicate that, for frequencies in the earthquake range ($f < 20$ Hertz), dynamic effects can be neglected for jointed or continuous buried pipelines made from materials such as concrete, cast iron, or steel, and having commonly used sizes. This is consistent with experimental observations, such as Refs. 4 and 5.

An exhaustive parametric study was not attempted because of the number of parameters in the model. However, the cases studied are believed to include those of most interest. In fact, an attempt was made to use parameters that would accentuate dynamic effects by increasing L and c_s , decreasing E_1 , and making (b/a) as close to one as seemed reasonable, considering current materials and design practices. Maximum amplifications, taken from displacement versus frequency curves, never exceeded 14 per cent in the cases studied.

It is interesting to note that the displacement response at the center ($z=0$) of the pipe is essentially the same for any joint size satisfying $\epsilon < 0.03$. From this point of view, the response of jointed and continuous buried pipelines is essentially the same. However, the displacement field in the immediate vicinity of the joints of jointed pipelines is quite different from that of continuous pipes in the same location. This highly localized behavior can not be properly accounted for by using a low value for the modulus of elasticity of a continuous buried pipe, as was done in Refs. 2 and 3.

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