

EARTHQUAKE RESISTANCE OF DYWIDAG PRESTRESSED CONCRETE BRIDGE  
BEING CONSTRUCTED BY CANTILEVER METHOD

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SYNOPSIS

In this paper the stability to overturning of the bridge which is being constructed by Dywidag method is described. The results of vibration tests using the vibration machine are shown. Considering that the ground is a non-linear body, simulation analyses of the bridge are done by using the results of tests and linear acceleration method. The results of the non-linear analyses are compared with those of the linear analyses, and it is found that the non-linear analysis is very effective to the earthquake resistance.

INTRODUCTION

The purpose of this paper is to investigate the earthquake resistance of a bridge which is being constructed by Dywidag method. The bridge is a six-span continuous box type bridge with full length 389 meters, constructed by the following methods. "T-shaped structures" shown in Fig. 1 are firstly constructed at five points and each "T-shaped structure" (it is called "T-structure" hereafter) is rigidly connected one another to build up a complete continuous bridge. While "T-structure" is not yet connected, the cantilever beam of "T-structure" is supported only by a pier in state of temporary fixed support. Besides, the pier is constructed on a small base mat. The "T-structure" is, then, very unstable in balance and it is necessary to investigate the dynamical behavior of "T-structure" during earthquake.

By means of vibration machine (the maximum output power is about 38 ton at 10 Hz), vibration tests were conducted in vertical direction, axial direction and direction perpendicular to the bridge axis with regard to the following three steps under construction: (i) "T-structure", (ii) two "T-structures" are connected when the bridge is in state of two-span continuous bridge (see Table 2), and (iii) all "T-structures" are connected when the bridge is completed. With respect to rocking vibration of "T-structure" that is the first mode of the free vibration, the simulation analyses to several inputs of ground acceleration are conducted, considering that "T-structure" is most easy to overturn in the rocking vibration. In the analysis, non-linear restoring ground reaction-deformation relation, which is often used in the analysis of rocking vibration in such structures as building<sup>1)</sup> and caisson foundation<sup>2)</sup>, is used and it is found that the non-linear analysis is effective to the analysis of stability toward overturning, comparing with the results of linear analysis.

RESULTS OF VIBRATION TEST

(i) Vibration Characteristics Table 1 shows that vibrations of "T-structure"

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have several vibration modes as rigid body and beam structure. The former is represented by rocking vibrations as Mode No.I and VI in Table 1 and torsional vibration of Mode No.V in Table 1; the latter, by Mode No.II, III, IV and VII in Table 1. Rocking vibration of Mode No.I, in which "T-structure" is most unstable in earthquake, has the first natural frequency  $f=1.21$  Hz. As it is a very small value, the investigation of stability to overturning is needed for earthquake with long period. The stability to overturning will be discussed later, considering the case of rocking vibration in the direction perpendicular to the bridge axis, too (Mode No.VI). Vibration of Mode No.II, of which natural frequency  $f=2.73$  Hz is the first natural frequency of cantilever beam of "T-structure", has the smallest value of all the damping constants obtained from vibration tests. The strain on slab surface at fixed end of the cantilever beam in this resonant mode was  $1 \times 10^{-6}$ .

In a case in which two "T-structures" were connected, when the bridge was in state of two-span continuous beam, vibration tests were conducted. The steel bars, then, which bolt up bridge piers and cantilever beams to be in state of temporary fixed support, were taken away, but there remained the concrete columns between the piers and the beams. Although the support conditions of beam were uncertain, the support conditions are considered similar to pin support, judging from the first mode shape (see Table 2). The vertical vibrations of the completed bridge have ten natural frequencies between 1 Hz and 6 Hz. With regard to the horizontal vibration, the third, fourth and fifth modes were obtained, though the first and second modes were not obtained (see Table 3).

(2) Non-linearity of Ground Reaction Rocking vibration of Mode No.I in Table 1 is corresponding to the first mode of rocking vibration. It may be considered from the vibration test results that the vibration is within the linear range and that the rocking center is almost equal to the center of base mat. Then, the first natural frequency of the rocking vibration is given by

$$\omega^2 = k_v - \frac{I_g}{J_g}, \quad \omega = 2\pi f \quad 1$$

where  $k_v$ : coefficient of vertical ground reaction,  $J_g$ : moment of inertia of "T-structure" with respect to the base mat center,  $I_g$ : moment of inertia of area of base mat with respect to the base mat center. Substituting the experimental value  $f=1.21$  Hz and the calculated values of  $J_g$  and  $I_g$  into Eq. 1,  $k_v$  is obtained as  $k_v=20.39$  kg/cm<sup>3</sup>. Looking similarly at the rocking vibration of Mode No.VI, the value of  $k_v$  is obtained as  $k_v=28.50$  kg/cm<sup>3</sup>. Now, displacements of ground are about  $1.25 \times 10^{-2}$  mm in Mode No. I and  $0.5 \times 10^{-2}$  mm in Mode No.VI. In such a range of very small displacements non-linearity is recognized. It is, then, necessary to consider non-linear ground reaction-displacement relation in the overturning analysis.

#### OVERTURNING OF "T-STRUCTURE"

(1) Model of "T-Structure" and Equations of Motion The model, which expresses approximately the rocking vibration of "T-structure", is shown in Fig. 2 and it's contents are: (i) the mass of "T-structure" is concentrated in the center of gravity, (ii) the slab (A) in Fig. 2 has the same area of actual base mat with it's rigidity infinite, and (iii) the spring which ties the concentrated mass  $m$  and the slab (A), too has infinite stiffness.

The distribution of ground reaction applied to the base mat and a restoring reaction-deflection relation in the analysis are shown in Fig. 3. The coefficient of ground reaction  $k$  in elastic range in Fig. 3 is determined as follows. From torsional vibration in a horizontal plane of Mode No.V in Table 1 the coefficient of lateral ground reaction  $k_H$  is calculated in the same manner as the above-mentioned  $k_V$ . The obtained value is  $8.14 \text{ kg/cm}^3$ .  $k_V=20.39 \text{ kg/cm}^3$  and  $k_H=8.14 \text{ kg/cm}^3$  are used here as the values of coefficient of ground reaction. The free rocking vibration in axial direction of model was solved, and the first natural frequency  $f=1.19 \text{ Hz}$  was obtained by using the above values of  $k_V$  and  $k_H$ . As it agrees approximately with experimental value  $f=1.21 \text{ Hz}$ , the estimated values of  $k_V$  and  $k_H$  are reliable.

After the completion of the bridge, a bridge with caisson foundations were constructed on the same bedrock of the lower basin. The results of loading tests are shown in Fig. 4. The stress-displacement relation, when displacement  $\delta$  is over  $5 \text{ mm}$ , is unknown for lack of loading capacity. The value of  $\delta$  is, however, the order of  $\text{cm}$  when stability to overturning will be discussed later on. Accordingly, such a stress-displacement relation as Fig. 4, in which  $\delta$  is under  $5 \text{ mm}$ , may be regarded as linear not to affect the analysis. The coefficient of ground reaction in plastic range which is to have more effects upon the analysis of overturning, will be explained later on.

Under these assumptions, equations of motion for the rocking vibration of "T-structure" are given for axial direction as follows:

$$m(\ddot{x} + H\ddot{\theta}) + c_1\dot{x} + abk_H(x) = -m\ddot{z}$$

$$J_0\ddot{\theta} + c_2\dot{\theta} + b\int_{\frac{1}{4}}^{\frac{3}{4}} k_V(\eta\theta)\eta d\eta - abk_H(x)H = 0$$

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where  $x=y-H\theta$ ,  $y$ : horizontal displacement of mass,  $\theta$ : rotation angle of base mat,  $H=13.16\text{m}$ : distance of base mat from mass,  $m=250.33\text{t}\cdot\text{sec}^2/\text{m}$ : mass of "T-structure",  $c_1, c_2$ : coefficients of damping,  $J_0=69430.6\text{t}\cdot\text{sec}^2\cdot\text{m}$ : moment of inertia of "T-structure" with respect to center of gravity,  $a=7.0 \text{ m}$ : width of base mat,  $b=11.0\text{m}$ : depth of base mat,  $\ddot{z}$ : acceleration of ground.

(2) Linear Analysis The following accelerations of ground are considered in the analysis: (i) the three cycles of a sinusoidal wave with the resonant frequency and (ii) response spectrum analysis. The former is a method in which "T-structure" is applied to such acceleration of ground  $z$  as the three cycles of a sinusoidal wave with the same frequency as the resonant frequency of "T-structure". Although this may be too severe in practice, this is used as a method to discuss the stability to overturning, and the horizontal acceleration of mass  $m$  is calculated as shown Table 4. With respect to the latter, acceleration response in horizontal direction is obtained from the mean response spectrum for  $200 \text{ gal}$  acceleration used in Japan, as shown in Table 4. The moment, when line of action of resultant force composed of horizontal acceleration of mass  $m$  and gravity acceleration passes the edge of base mat, is considered as the overturning condition. Comparing this horizontal acceleration in the moment,  $\alpha_c$ , with the preceding horizontal acceleration in (i) and (ii), critical accelerations of ground  $\ddot{z}_c$  satisfying overturning condition are obtained as shown in Table 4. From

Table 4, it will be found that  $\ddot{z}_c$  in (i) is about one third as large as  $\ddot{z}_c$  in (ii) and the method of the three cycles of a sinusoidal wave is far strict in comparison with the response spectrum analysis. The "T-structure" is inclined to overturn more easily in direction perpendicular to the bridge axis than in axial direction. Though it seems unexpected, the reason is that the value of moment of inertia in the latter is ten times as large as in the former.

### (3) Non-linear Analysis

(i) Restoring force-displacement relation and overturning condition The rocking vibration in axial direction alone is considered and ground reactions acting on base mat are simulated by twenty springs in this analysis. Fig. 5 shows the relations between maximum vertical displacement of base mat edge,  $\delta_{max}$ , and acceleration of ground,  $\ddot{z}$ , which is ten seconds of step acceleration pulse applied to base mat with the amplitude constant. Parameter  $\lambda$ , which is corresponding to spring constant of plastic range in bilinear hysteresis, has much effects upon overturning of "T-structure" as shown in Fig. 5. The value may be assumed equal to 1/1000 with most safety, as is obvious from Fig. 5, considering the value is unknown for lack of loading capacity. This is identical with elasto-plastic restoring characteristics. At the same time the following overturning condition is assumed: The moment when forces applied to twenty vertical springs are about to pass the yield point. The condition is expressed by the dotted line in Fig. 5.

(ii) Pseudoelastic spring to equivalent bi-linear spring and acceleration of ground  $\ddot{z}$  What kind of equivalent spring should be considered in comparison with the results in non-linear analysis. The way of determining the equivalent spring constant  $k_0$  is as follows. First, the potential energy  $E_1$  stored in the all springs in the instance of overturning is calculated. Secondly, all bilinear type springs are replaced by the equivalent springs of which displacements are equal to those of the bi-linear type springs. And the potential energy  $E_2$  stored in the springs is, then, calculated. The equivalent spring constant  $k_0$  is judged from the relation  $E_1=E_2$ . Then, the value of  $k_0$  is given by  $k_0=2.86 \text{ kg/cm}^3$ .

The following three inputs are considered as acceleration of ground  $\ddot{z}$ ;  
 Input (A) : Step acceleration pulse,  $\ddot{z}=\ddot{z}_0=\text{constant}$  ( $0 < t < t_0$ ),  $\ddot{z}=0$  ( $t > t_0$ ).  
 Input (B) : Three cycles of sinusoidal waves with resonant frequency of equivalent spring system,  $\ddot{z}=\ddot{z}_0 \sin 2\pi f_0 t$ ,  $f_0=0.442 \text{ Hz}$  ( $0 < t < t_0$ ),  $\ddot{z}=0$  ( $t > t_0$ ).  
 Input (C) : El Centro California earthquake of May 18, 1940 N-S component, in which maximum acceleration is expressed by  $\ddot{z}_0$  and several values of  $\ddot{z}_0$  are considered intentionally.  
 The values of  $\ddot{z}_0$  and  $t_0$  in the above three Inputs (A), (B) and (C) are shown in Fig. 6.

(iii) Stability to Overturning Under the above conditions (i) and (ii), Eq. 2 is solved by using linear acceleration method with the coefficients of damping neglected, and the solutions to the three above-mentioned inputs are shown in Fig. 6. The relations between maximum displacement at base mat edge,  $\delta_{max}$ , and amplitude of acceleration of ground,  $\ddot{z}_0$ , are represented by several curves in Fig. 6. Those relations of the Inputs (A), (B) and (C) are shown by curve-A, B and C in Fig. 6, respectively. Curve-B<sub>1</sub> expresses  $\delta_{max}$  which appears during the first cycle in the vertical displacement response

and curve-B<sub>a</sub> expresses  $\delta_{max}$  through three cycles in the response. Overlapping portions of the curves represent the same relations as curve-B<sub>a</sub>. The number of yield springs are shown as line-10, line-9 and line-8 in Fig. 6. The line-10, for example, expresses that all springs applied to the half part of base mat yield. In this case,  $\delta_{max}$  is 9.5 cm and "T-structure" is about to overturn. The overturnings occur when  $\ddot{z}_0$  is equal to 390 gal in case of Input(A) and 615 gal in case of Input(B) as shown curve-A, B and line-10 in Fig. 6. "T-structure" does not overturn in case of Input(C) but the amount of yield springs reaches to 70 % to 500 gal amplitude.

The results of equivalent linear analysis in case of Inputs(A),(B) and (C) are shown in the curve-D,E and F, respectively in Fig. 6. "T-structure" overturns when  $\ddot{z}_0$  reaches to the same acceleration, 390 gal as non-linear analysis in case of Input(A), but mere 85 gal, which is about one seventh as large as in the preceding non-linear analysis, is enough in case of Input(B) as shown curve-D,E and line-10 in Fig. 6. Although "T-structure" does not overturn in case of Input(C), only 270 gal is sufficient for the same amount of springs as in the preceding non-linear analysis to yield.

#### CONCLUSION

1. "T-structure" vibrations except rocking vibration are represented by the vibrations of cantilever beam, the torsional vibration of rigid body round vertical axis and the torsional vibration of slab.
2. The horizontal vibrations of complete bridge are uncertain in the region of low frequencies, while the vertical vibrations have ten natural frequencies between 1 Hz and 6 Hz.
3. "T-structure" is inclined to overturn more easily in direction perpendicular to the bridge axis than in axial direction.
4. The non-linear analysis is effective to the analysis of stability toward overturning of "T-structure", and if fears are entertained for the stability judging from the results of linear analysis, the "T-structure" will not readily overturn from the view-point of non-linear analysis, as follows;
  - (i) In case of three cycles of a sinusoidal wave with the resonant frequency, "T-structure" overturns when the magnitudes of ground acceleration are 615 gal in the non-linear analysis and mere 85 gal in the equivalent linear analysis. The former is about seven times as large as the latter.
  - (ii) In case of El Centro California earthquake, "T-structure" does not overturn but the maximum values of the acceleration, when the amount of yield springs reaches 70 %, are 500 gal in the non-linear analysis and only 270 gal in the linear analysis.

#### REFERENCES

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- 2) T. Tanaka and T. Kunii, "Non-linear Restoring Force Characteristics of the Pier Having a Well Foundation", Proc. of the 5th Japan Earthquake Engineering Symposium, November 1978, pp. 553-560

Table 2 Vibration modes of two-span continuous bridge

	f (Hz)	h (%)	Mode Shape
A	1st	1.54	
	2nd	2.06	
	3rd	2.82	

Table 4 Critical accelerations of ground by linear analysis

Analysis Direction	I		II	
	A	B	A	B
$\ddot{z}_0$	100gal $\sin 7.6t$	100gal $\sin 9.75t$	200gal	200gal
$\ddot{z}_0 + \ddot{y}_{max}$ (gal)	430	850	269	551
$\alpha_c$ (gal)	261	409	261	409
$\ddot{z}_c$ (gal)	60.7	48.1	190	149

A= Vertical Direction  
 B= Direction Perpendicular to Bridge Axis  
 f= Natural Frequency  
 h= Coefficient of Damping  
 I= The Three Cycles of a Sinusoidal Wave with the Resonant Frequency  
 II= Response Spectrum Analysis

Table 1 Vibration modes of "T-structure".

	f (Hz)	h (%)	Mode No.	Mode Shape
A	1st	1.21	I	
	2nd	2.73	II	
	3rd	3.78	III	
	4th	9.15	IV	
B	1st	1.79	V	torsional vibration of rigid body round vertical axis.
	2nd	3.39	VI	rocking vibration in direction perpendicular to bridge axis.
	3rd	8.54	VII	torsional vibration of slab.

Table 3 Vibration modes of completed bridge

	f (Hz)	h (%)	Mode Shape
B	3rd	3.18	
	4th	3.50	
	5th	4.67	

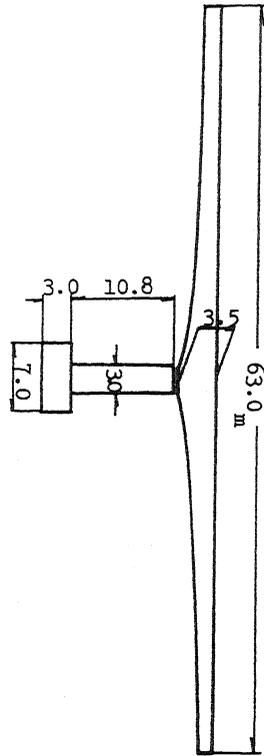


Fig. 1 "I"-shaped structure"

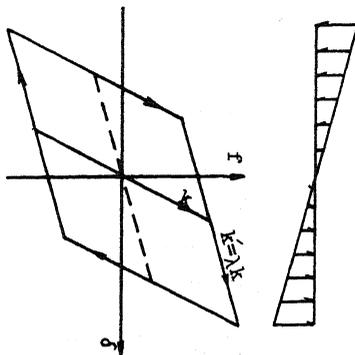


Fig. 3 Distribution of ground reaction and restoring reaction deflection relation

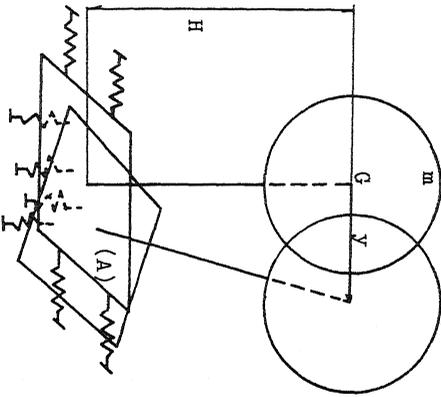


Fig. 2 Model of "I"-structure"

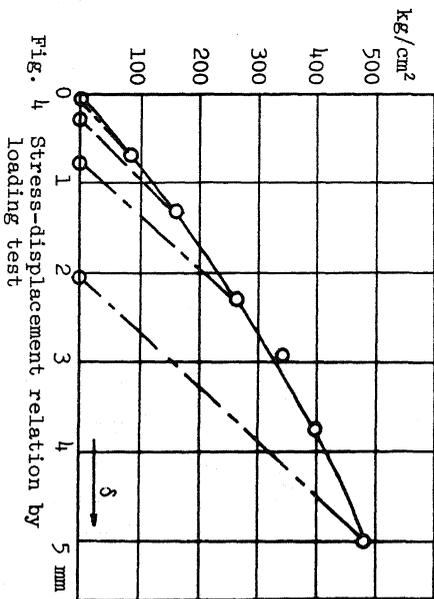


Fig. 4 Stress-displacement relation by loading test

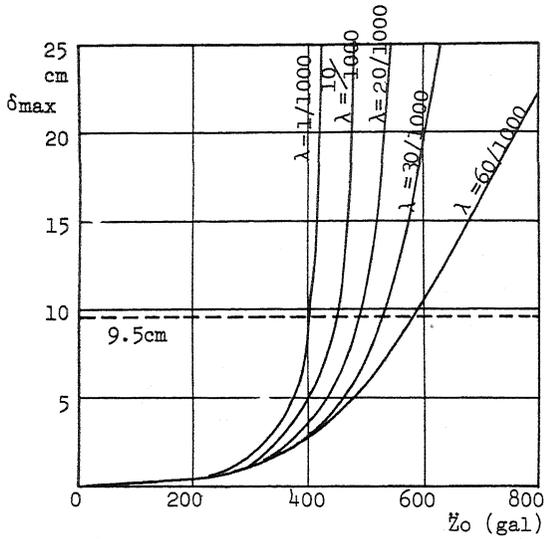
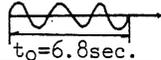
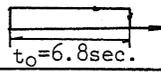


Fig. 5 Relations between maximum vertical displacement of base mat edge,  $\delta_{max}$  and acceleration of ground,  $\ddot{z}_0$  which is ten seconds of step pulse

Input	
(B)	 $t_0 = 6.8 \text{ sec.}$
(A)	 $t_0 = 6.8 \text{ sec.}$
(C)	$t_0 = 10.0 \text{ sec.}$

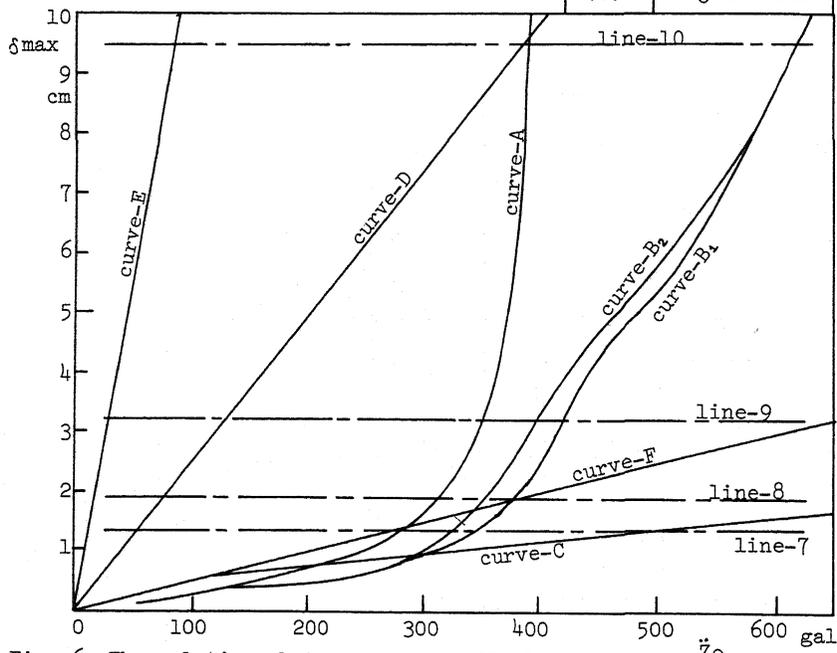


Fig. 6 The relations between maximum displacement at base mat edge,  $\delta_{max}$ , and amplitude of acceleration of ground,  $\ddot{z}_0$ , to Inputs (A), (B) and (C)