

## DYNAMIC ANALYSES OF LIQUID STORAGE TANKS

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### ABSTRACT

Theoretical and experimental investigations of the dynamic behavior of ground-supported, deformable, cylindrical liquid storage tanks were conducted. The study was carried out in three phases: A) a detailed theoretical treatment of the coupled liquid-shell system, B) an experimental investigation of the dynamic characteristics of full-scale tanks, and C) a development of an improved design procedure based on an approximate analysis.

### INTRODUCTION

Seismic damage of liquid storage tanks during recent earthquakes demonstrates the need for a reliable technique to assess their seismic safety. Past analyses have often assumed unrealistic conditions and, hence, are not strictly applicable to real tanks. In addition, the lack of experimental confirmation of the theoretical concepts has raised doubt among engineers about their applicability in the design stage. With few exceptions, current design procedures are based on the mechanical model derived by Housner for rigid tanks [1].

The following study develops a method for analyzing the seismic response of cylindrical tanks by means of a digital computer. The reliability of the theoretical analysis was confirmed by conducting vibration tests on full-scale tanks. In addition, approximate solutions were developed to provide practicing engineers with simple, fast, and sufficiently accurate tools for estimating the seismic behavior of storage tanks. The details of the study can be found in [2].

Throughout the investigation the liquid is assumed to be homogeneous, inviscid, and incompressible. In addition, the amplitudes of vibration are assumed to be small. The strain energy expression of the shell includes the effects of both stretching and bending. It should be noted that this analysis is applicable only to tanks that are anchored to a rigid base; the strong-motion response of unanchored tanks is different.

### HYDRODYNAMIC PRESSURE

A first step in analyzing the dynamic response of liquid-filled tanks is to evaluate the hydrodynamic pressures exerted on their walls. These pressures are:

- $p_1$  = the long period component contributed by the "convective" fluid motion (sloshing);
- $p_2$  = the "impulsive" pressure component which varies in synchronism with the horizontal ground acceleration;
- $p_3$  = the short period component contributed by the  $\cos\theta$ -type vibrations of the tank walls; and

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$p_4$  = the contributions of the  $\cos n\theta$ -type vibrations ( $n \geq 2$ ) of the tank walls.

#### FREE LATERAL VIBRATIONAL MODES

Knowledge of the natural frequencies of vibration and the associated mode shapes is an essential step in computing the earthquake response of deformable tanks. These were determined [3] by means of a discretization scheme in which the elastic shell is modeled by finite elements (Fig. 2) and the liquid region is treated as a continuum by boundary solution techniques. In this approach, the number of unknowns is substantially less than in those analyses where both tank wall and liquid are subdivided into finite elements.

The analysis is first applied to compute the  $\cos\theta$ -type modes for which there is a single cosine wave of deflection in the circumferential direction; these modes are strongly excited by rigid base motion. Figure 3 shows the fundamental mode shape of two different classes of tanks, namely, "tall" and "broad" tanks. A significant difference can also be seen in Fig. 4 where the impulsive hydrodynamic pressure distribution along the tank length is displayed.

The analysis is extended to compute the  $\cos n\theta$ -type modes; however, these modes are influenced by the initial hoop stress due to the hydrostatic pressure. To incorporate this, it was necessary to use the second order nonlinear strain-displacement equations, to modify accordingly the strain energy expression of the shell, and to develop the "added" stiffness matrix. Figure 5 shows a comparison between the computed natural frequencies and those measured experimentally in [4].

The analysis is also extended to investigate the coupling between liquid sloshing modes and shell vibrational modes. The quiescent liquid free surface is subdivided into concentric annular "elements"; the degrees of freedom being the free surface displacements. It is found that the coupling is weak (Fig. 6); and consequently, the convective pressure can be evaluated with reasonable accuracy by considering the tank wall to be rigid.

Tanks are usually covered by either a fixed or a floating roof. Due to the high in-plane rigidity of the fixed roof system, only insignificant in-plane deformations usually occur as the tank vibrates. This constraint affects the dynamic characteristics of the tank as can be seen in Fig. 7.

#### VIBRATION TESTS OF FULL-SCALE TANKS

Adequate understanding of the behavior of complex systems is enhanced by the combined use of theoretical and experimental techniques in support of each other. Therefore, a series of ambient and forced vibration tests of three full-scale tanks was conducted [5] to determine the natural frequencies and, if possible, the mode shapes of vibrations and to select two tanks on which permanent instruments would be installed to record future earthquakes. Figure 8 shows schematic sections of these tanks and their foundations.

Measurements of the ambient and forced vibrations were made at selected points along the shell height, at the roof circumference, and around the tank bottom. The first series of tests were conducted to measure the axial pattern of shell vibrational modes. The objective of the second series of tests was to monitor the motion around the circumference. A vibration generator was used in the sinusoidal steady-state resonant tests;

it was anchored to a concrete slab resting on the ground adjacent to the tank. The horizontal sinusoidal force exerted by the vibration generator was transmitted through the ground and produced small amplitude vibrations of the tank. Figure 9 is a schematic diagram showing the experimental set-up and the instrumentation used in the tests.

One phenomenon that was clearly observed in the recorded motion was that  $\cos n\theta$ -type vibrations of the tank wall were developed. This can be seen in Fig. 10 in which a Fourier spectrum of the radial velocity of the tank wall is displayed. These modes were anticipated in the ambient tests because of the nature of the excitation which tends to excite many modes. However, in a forced vibration test, a perfect circular cylindrical shell should exhibit only  $\cos\theta$ -type modes; other modes are attributed to initial irregularities of the shell. Figure 11 shows the axial and circumferential patterns of one of the mode shapes based on ambient and forced vibration measurements; it is clear that the roof does restrain the top of the tank against radial deformations. The computed mode shape is also presented for comparison. Figure 12 shows a comparison between the computed natural frequencies and those measured for tank no. (3).

It should also be noted that the foundation conditions had an influence on the response of the  $\cos\theta$ -type modes. Rocking motion was observed in tank no. (1); however, such motion was not observed for tank no. (3) which had a very rigid foundation. Tank no. (2), which is not anchored to the foundation, exhibited behavior slightly different from the other two tanks. However, it is believed that it would behave much differently with a high level of excitation.

#### EARTHQUAKE RESPONSE ANALYSIS

The only special feature of the earthquake response problem of liquid-filled tanks compared with any other type of structures is the method of defining the effective external load vector for the tank wall. This vector can be evaluated [6] by employing the expression of the work done by external loads (inertia forces and hydrodynamic pressure) through arbitrary virtual displacements.

For a perfect circular tank, the external loads are functions of  $\cos\theta$  only; and consequently, the earthquake response can be obtained by superposition of the vertical modes corresponding to  $n=1$  only. It is found that the flexibility of the tank walls has a significant effect on the seismic response of both tall and broad tanks. The dynamic stresses are much greater than those computed assuming rigid walls. This is due to the fact that the impulsive loads arise through acceleration of the wall which, in a flexible tank, has two components: 1) the acceleration of the undeformed shell, i.e., the ground acceleration and 2) the relative acceleration of the deformed shell. These results were further substantiated by comparing the computed response of aluminum tank models with that measured in [7,8]. This comparison also indicated that the computed fundamental frequency is higher than the measured frequency. Figure 13 shows the time history of the axial membrane force resultant in a tall tank subjected to the N-S component of the 1940 El Centro earthquake. The maximum value of the stress is 8375 lb/in which is much greater than that of 3443 lb/in induced in a similar rigid tank.

As was mentioned earlier,  $\cos n\theta$ -type modes cannot be excited in a perfect circular tank; however, fabrication tolerances in civil engineering

tanks permit a departure from a nominal circular cross section and this tends to excite these modes. A complete analysis of the effect of irregularity of flexible tanks can be found in [2]; the fact remains that the magnitude and distribution of fabrication error cannot be predicted, and consequently, only a hypothetical analysis can be made. It is important to note that a recent study [4] showed that buckling of tank models depends largely on the stresses associated with the  $\cos\theta$ -type modes.

#### SEISMIC DESIGN OF LIQUID STORAGE TANKS

The principal aim of the final phase research was to devise a practical approach which would allow, from the engineering point of view, a simple and satisfactorily accurate estimate of the dynamic response of storage tanks to earthquakes. To achieve this, some simplified analyses were developed [9]. As a natural extension of Housner's model, the effect of the soil deformability on the seismic response of rigid tanks was investigated. The analysis revealed that rocking motion of rigid tall tanks accounts for a significant part of the overall seismic behavior of such tanks.

To account for the flexibility of the container, the tank was assumed to behave as a cantilever beam with bending and shear stiffness. The analysis follows the same method used for the liquid-shell system but in a simplified manner. To further simplify the design procedure, a mechanical model shown in Fig. 14 was developed and its parameters were displayed in charts. These curves facilitate the calculations of effective masses, their centers of gravity, and the periods of vibration. The effective masses  $m_r$ ,  $m_f$ , and  $m_s$  correspond to the forces associated with ground motion, wall deformation, and liquid sloshing, respectively. Once the parameters of the mechanical model of the particular tank under consideration are found, the maximum seismic loading can be predicted by means of a response spectrum characterizing the design earthquake.

Approximate analyses are being developed to investigate the response of tanks to vertical ground excitations and to estimate the amount of uplift induced in unanchored tanks subjected to strong horizontal ground motions.

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DYNAMIC ANALYSES OF LIQUID STORAGE TANKS

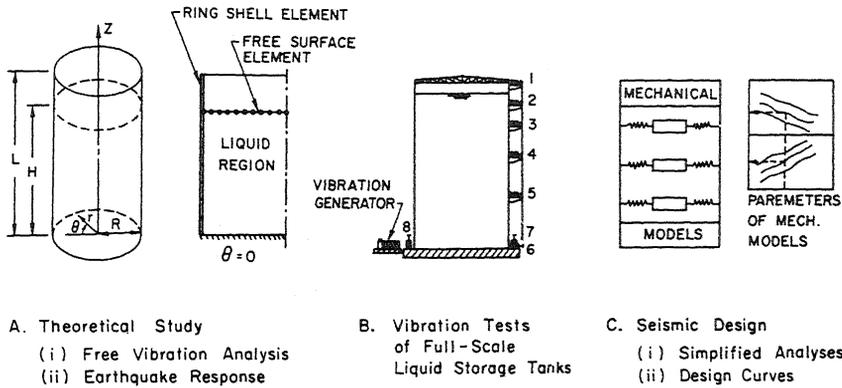


Figure 1

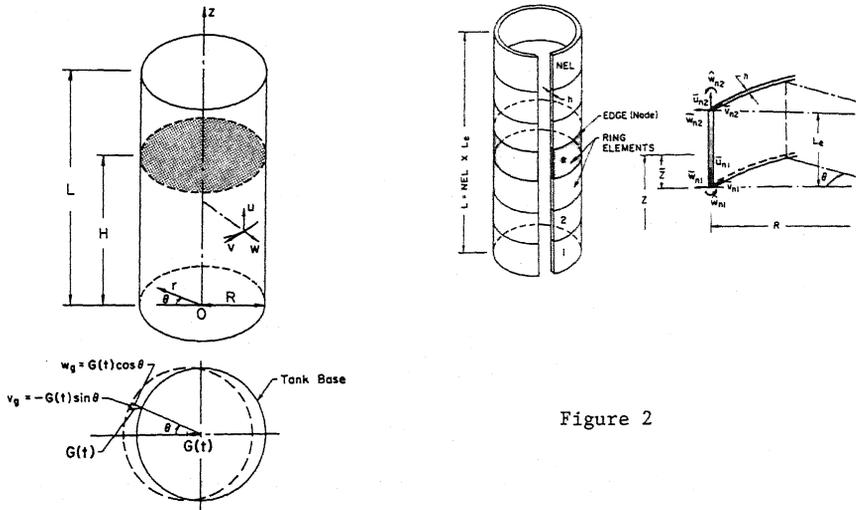


Figure 2

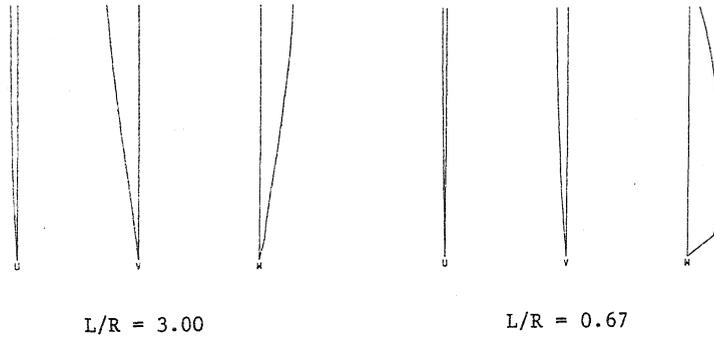


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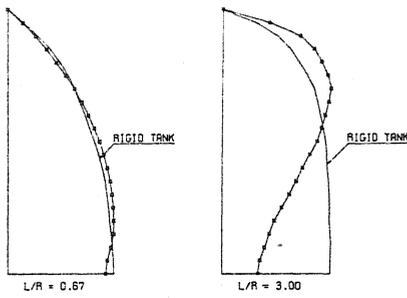


Figure 4

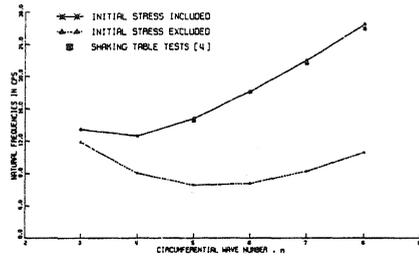


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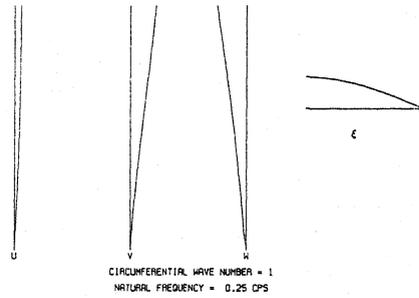


Figure 6

(Shell displacements are magnified 500 times)

Figure 7

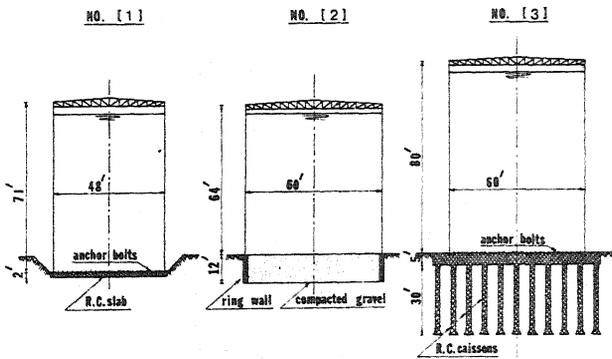
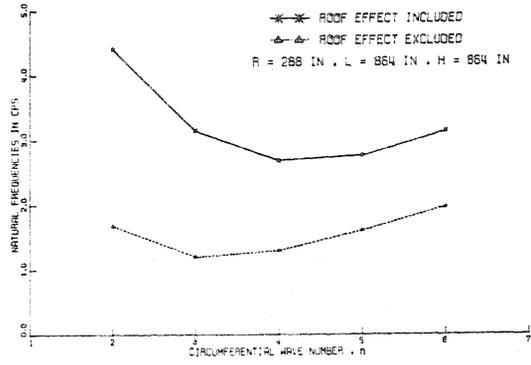
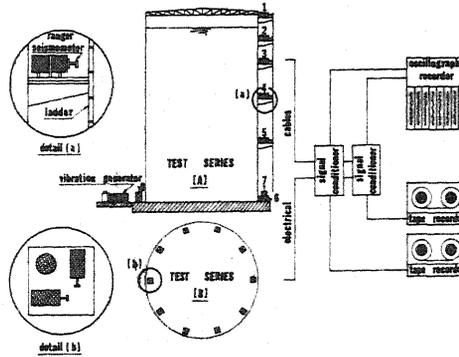


Figure 8

Figure 9



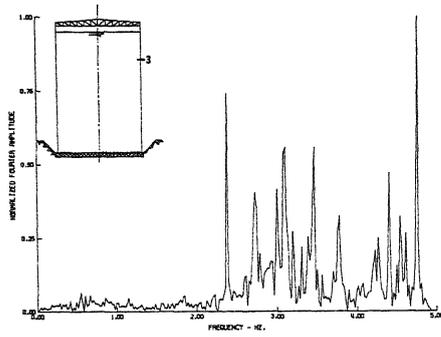


Figure 10

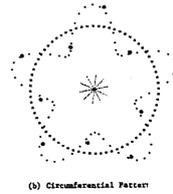
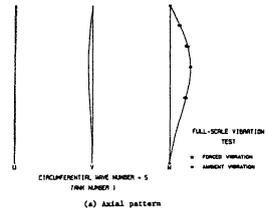


Figure 11

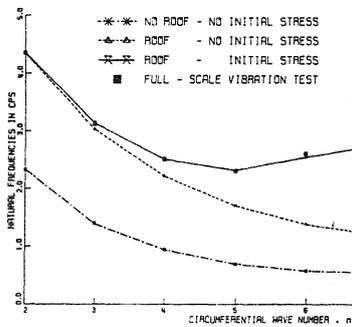


Figure 12

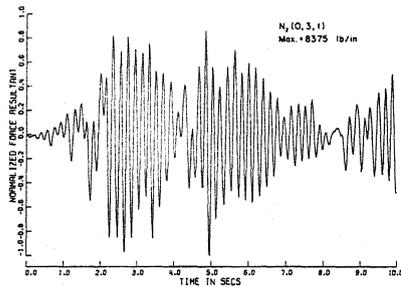
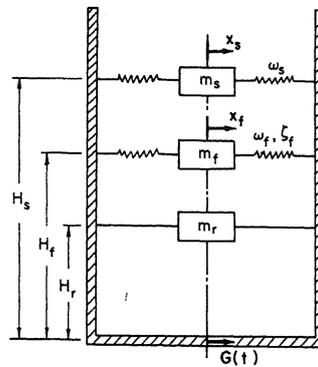


Figure 13



FLEXIBLE TANK

$$\text{Base Shear} = \sqrt{(m_r \ddot{G}_{\max})^2 + (m_f S_{Qf})^2 + (m_s S_{Qs})^2}$$

$$\text{Base Moment} = \sqrt{(m_r H_r \ddot{G}_{\max})^2 + (m_f H_f S_{Qf})^2 + (m_s H_s S_{Qs})^2}$$

Figure 14