THE EFFECT OF DAMPING CONSTANT ON THE ACCURACY OF SEISMIC DESIGN CALCULATIONS FOR SOILS AND FOUNDATIONS

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SUMMARY

This paper clarifies some ambiguity in the definition of equivalent viscous damping constant linearized from stress-strain hysteresis curve and confirms the validity of the usual procedure in determining the dynamic constants of soils. Throughout several dynamic response analyses of soil structures with different computer programs under the same given conditions this paper confirms too that same results are obtained from them if only the characteristics of the frequency dependency of damping constant which the computer programs possess are made uniform.

INTRODUCTION

The design equations for dynamic constants of soil structures consist usually of the secant moduli and the equivalent viscous damping constants defined for the stress-strain hysteresis curves obtained from seismic prospecting tests in fields, laboratory tests of resonant column type for low strain amplitude and dynamic simple shear or tri-axial compression tests for higher one (Ref. 1-6). Such expressions are widely applied to the seismic stability analyses for many soil structures and deposits. However, it is pointed out theoretically that the equivalent damping constants of both bi-linear and Ramberg-Osgood models have a maximum value of 15.9 % of critical damping (Ref. 8,9). In spite of that the actually measured damping constants of soils are said to have the maximum values of 20 % to 30 %. Moreover, there are some of ambiguity in the definition of above dynamic constants. In this paper the reason for the discrepancy appeared in the maximum values of above damping constants has been made clear and the validity of the usual procedure in determining the dynamic constants of soils has been confirmed to a certain extent of accuracy.

In the practical seismic design for every structure it is very important for us to confirm that we can obtain same results from every calculation with each of different computer programs under same given conditions. In this paper several dynamic response analyses of rockfill dams have been conducted with three computer programs of different numerical integration procedures and it has been clarified that the frequency dependency of the damping constant plays a leading role in producing the equivalent results with all computer programs.

STEADY STATE RESPONSES OF SINGLE-DEGREE-OF-FREEDOM OSCILLATORS

Solutions Based On The Method By Kryloff And Bogoriuboff

As for steady state responses of single-degree-of-freedom oscillators

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having hysteretic force-deflection relationships to sinusoidal excitation, an exact numerical solution for bi-linear system has been derived by Iwan (Ref. 10), and approximate analytical solutions on the basis of Kryloff-Bogoriuboff method have been derived by Caughey for bi-linear system (Ref. 7) and by Jennings for Ramberg-Osgood system (Ref. 9).

The equation of motion for the forced vibration of a mass m mounted on a spring of which restoring force is expressed by $F(x,\dot{x},t)$ is

$$m\ddot{x} + F(x,\dot{x},t) = f_0 \cos\omega t \tag{1}$$

Let

$$\frac{k}{m} = \omega_0^2,\ \omega_0 t = \tau,\ \frac{x}{x_y} = y,\ F_y = kx_y,\ \frac{\omega}{\omega_0} = \eta,\ \frac{f_0}{F_y} = f \tag{2}$$

where k represents the spring constant for initial or micro-displacement and x_y represents a yielding or a characteristic displacement. Substituting Eq.(2) into Eq.(1)

$$\frac{d^2y}{d\tau^2} + \frac{F(y,\dot{y},\tau)}{F_y} = f\cos\eta\tau \tag{3}$$

where $\dot{y}$ represents $dy/d\tau$. Solution of Eq.(3) for the steady state response is obtained as follows

$$y(\tau) = y_0\cos(\eta\tau+\phi_0) \tag{4}$$

$$X = \eta^2 - 1 = \frac{C(y_0)}{y_0} - 1 \pm\sqrt{\left(\frac{f}{y_0}\right)^2 - \left\{\frac{S(y_0)}{y_0}\right\}^2},\quad \tan\phi_0 = \frac{S(y_0)}{C(y_0)-y_0} \tag{5}$$

where $C(y_0)$ and $S(y_0)$ are derived with Kryloff-Bogoriuboff method and denoted in the following equations.

$$C(y_0) = \frac{1}{\pi}\int_0^{2\pi}\frac{F(y_0\cos\theta,\, y_0\sin\theta,\tau)}{F_y}\cos\theta d\theta \tag{6}$$

$$S(y_0) = \frac{1}{\pi}\int_0^{2\pi}\frac{F(y_0\cos\theta, y_0\sin\theta,\tau)}{F_y}\sin\theta d\theta \tag{7}$$

In resonance following equations to determine the resonant frequency and the maximum amplitude are derived from Eq.(5).

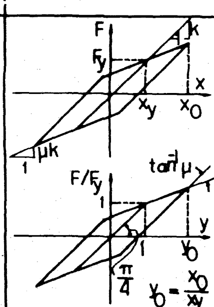
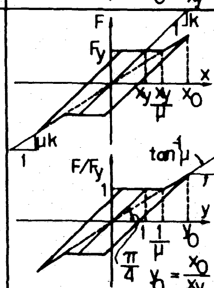
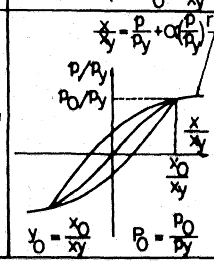
$$\eta_{res}^2 = \frac{C(y_0)}{y_0},\quad S(y_0) = f \tag{8}$$

Solution for a tri-linear system of which hysteresis loop is shown in Table 1 has been derived with above equations and is presented in Table 1 together with above mentioned solutions by Caughey and Jennings.

Solution For Strain Or Displacement-Dependent Equivalent Linear System

Let k_{eq} and c_{eq} be the equivalent spring constant and viscous

Table 1 Steady State Responses of Single-Degree-Of-Freedom Oscillators

SORT OF MODEL	HYSTERESIS LOOP	SORT OF SOLUTION	κ^2	h_{eq}	η^2_{res}	$X=\eta^2-1$
Bi-Linear		Equivalent Linear	$\frac{1+\mu(y_0-1)}{y_0}$	$\frac{2}{\pi}\frac{1-\mu}{1+\mu(y_0-1)}\frac{y_0-1}{y_0}$	$\frac{1+\mu(y_0-1)}{y_0}$	$-(1-\mu)\frac{y_0-1}{y_0} \pm\sqrt{\left(\frac{f}{y_0}\right)^2-\left(\frac{1-\mu}{\pi}\sin^2\theta^*\right)^2}$
		Kryloff & Bogoriuboff (After Caughey)			$\mu+\frac{1}{\pi}(1-\mu)\left(\theta^*-\frac{1}{2}\sin 2\theta^*\right)$	$-(1-\mu)\left[1-\frac{1}{\pi}\left(\theta^*-\frac{1}{2}\sin 2\theta^*\right)\right] \pm\sqrt{\left(\frac{f}{y_0}\right)^2-\left(\frac{1-\mu}{\pi}\sin^2\theta^*\right)^2}$
		Remarks	$\cos\theta^*=1-\frac{2}{y_0}$, $y_0\geq 1$			
Tri-Linear		Equivalent Linear	μ	$\frac{1}{2\pi\mu}(\sin^2\theta_1^*-\mu\sin^2\theta_2^*)$	μ	$-(1-\mu) \pm\sqrt{\left(\frac{f}{y_0}\right)^2-\left(\frac{\sin^2\theta_1^*-\mu\sin^2\theta_2^*}{\pi}\right)^2}$
		Kryloff & Bogoriuboff			$\mu+\frac{1}{\pi}\left[\left(\theta_1^*-\frac{1}{2}\sin 2\theta_1^*\right) -\mu\left(\theta_2^*-\frac{1}{2}\sin 2\theta_2^*\right)\right]$	$-(1-\mu)+\frac{1}{\pi}\left[\left(\theta_1^*-\frac{1}{2}\sin 2\theta_1^*\right) -\mu\left(\theta_2^*-\frac{1}{2}\sin 2\theta_2^*\right)\right] \pm\sqrt{\left(\frac{f}{y_0}\right)^2-\left(\frac{\sin^2\theta_1^*-\mu\sin^2\theta_2^*}{\pi}\right)^2}$
		Remarks	$\cos\theta_1^*=1-\mu-\frac{1}{y_0}$, $\cos\theta_2^*=-\frac{1}{\mu y_0}$, $y_0\geq\frac{1}{\mu}$			
Ramberg & Osgood		Equivalent Linear	$\frac{1}{1+\alpha P_0^{r-1}}$	$\frac{2\alpha}{\pi}\left(\frac{r-1}{r+1}\right)\frac{P_0^{r-1}}{1+\alpha P_0^{r-1}}$ $=\frac{2}{\pi}\left(\frac{r-1}{r+1}\right)(1-\kappa^2)$	$\frac{1}{1+\alpha P_0^{r-1}}\left(=\frac{P_0}{y_0}\right)$	$-\left(1-\frac{P_0}{y_0}\right) \pm\sqrt{\left(\frac{f}{y_0}\right)^2-\left[\frac{4}{\pi}\left(\frac{r-1}{r+1}\right)\left(1-\frac{P_0}{y_0}\right)\frac{P_0}{y_0}\right]^2}$
		Kryloff & Bogoriuboff (After Jennings)			$-\frac{2}{\pi}\frac{1}{y_0^2}\int_{y_0}^{-y_0}\frac{y\left(\frac{p}{p_y}\right)}{\sqrt{y_0^2-y^2}}dy$	$-\frac{2}{\pi}\frac{1}{y_0^2}\int_{y_0}^{-y_0}\left(y\frac{p}{p_y}\Big/\sqrt{y_0^2-y^2}\right)dy \pm\sqrt{\left(\frac{f}{y_0}\right)^2-\left[\frac{4}{\pi}\left(\frac{r-1}{r+1}\right)\left(1-\frac{P_0}{y_0}\right)\frac{P_0}{y_0}\right]^2}$

coefficient respectively and let

$$k_{eq}/k = \kappa^2, \quad h_{eq} = c_{eq}/(2\sqrt{m}\ k_{eq}) \tag{9}$$

then Eq.(3) becomes

$$\frac{d^2y}{d\tau^2} + 2h_{eq}\kappa\frac{dy}{d\tau} + \kappa^2 y = f\ \cos\eta\tau \tag{10}$$

Substituting Eq.(4) into Eq.(10) following equations are obtained.

$$y_0^2\ \{(\kappa^2 - \eta^2)^2 + 4h_{eq}^2\kappa^2\eta^2\} = f^2, \quad \tan\phi_0 = -\frac{2h_{eq}\kappa\eta}{\kappa^2 - \eta^2} \tag{11}$$

Equating the loss energy due to hysteresis loop to the one due to viscousity of the equivalent linear system, following equation is obtained.

$$2h_{eq}\kappa\eta = -S(y_0)/y_0 \tag{12}$$

The parameters κ^2 and h_{eq} in Eq.(11) correspond to dynamic constants of soils, that is, correspond to G/G_0 and h respectively which are represented by the functions mainly of strain amplitude. In practical seismic designs of soil structures the linear equation of motion such as Eq.(10), for an example of the simplest simulation of general structure, is solved iteratively untill the dynamic constants and the response strain amplitude become to be consistent with the design equations of them in every part of the structure. In the present simplified simulation the solution for steady state response of the equivalent linear system to sinusoidal excitation can be obtained analytically from Eqs.(11) and (12) without any iterative procedure and is expressed in similar form to Eq.(5) as follows

$$X = \eta^2 - 1 = (1-2h_{eq}^2)\kappa^2 - 1 \pm \sqrt{\frac{f^2}{y_0^2} - \frac{S(y_0)^2}{y_0^2}(1-h_{eq}^2)} \simeq \kappa^2 - 1 \pm \sqrt{\frac{f^2}{y_0^2} - \frac{S(y_0)^2}{y_0^2}} \tag{13}$$

The equivalent viscous damping constant h_{eq} is specified for the resonant condition and can be expressed as follows

$$h_{eq} = -\frac{S(y_0)}{2\kappa^2 y_0} \tag{14}$$

The resonant frequency and the maximum amplitude are derived from Eq.(13).

$$\eta_{res}^2 = \kappa^2, \quad S(y_0) = f \tag{15}$$

Considerations

From dynamic simple shear or tri-axial compression tests, removing the permanent strain from stress-strain relationship in the latter cases, hysteretic stress-strain relationships are obtained for most soils. The hysteresis loop is linearized by the secant modulus defined with extreme point of the loop and the critical damping ratio defined as follows

$$h = \frac{1}{2}\cdot\frac{1}{2\pi}\cdot\frac{\Delta W}{W} \tag{16}$$

where ΔW represents the area inside the hysteresis loop, that is, the loss energy and W represents the triangular area, that is, potential energy due to elastic deformation in the spring linearized by above secant modulus. Jacobsen proposed to adopt " *Work Area Under Skelton*" as W recognizing that there is an ambiguity about the definition of W in the non-linear system. In soils the skelton is scarcely known so that it is approximated by above secant line.

In the usual procedure determining equivalent viscous damping ratio, k is adopted for the equivalent spring constant k_{eq}, that is, κ in Eq.(8) is taken to be 1, and so the value of W is considerably larger than the one defined in soils. For an example both of W are shown in Fig. 1. Because of this the maximum value of h_{eq} defined in a way for soils becomes about 4 times larger than the one defined in usual procedure and is $2/\pi$ in elasto-plastic cases. This is the reason why the measured damping constants in soils have the maximum values of 20 % to 30 %. Comparing Eq.(15) with Eq.(8), however, it is noted that the maximum response amplitudes with the equivalent viscous damping constants defined in any different ways coincide exactly with the one in the solution by the method of Kryloff and Bogoriuboff if only the loss energy of the hysteretic stress-strain or force-deflection relationship is same. The discrepancy caused of different definitions for equivalent linear systems occurs in resonant frequencies and the response characteristics have to be investigated.

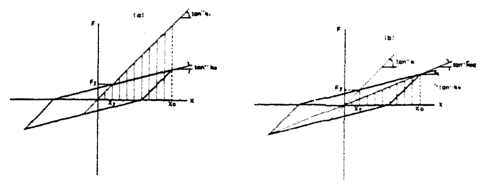


Fig. 1 Potential Energy

The solutions for the steady state responses of the equivalent linear systems linearized from three hysteresis loops mentioned before in the way for soils to sinusoidal excitation have been derived with Eqs.(13),(14) and (15). These results are shown in Table 1. The frequency response curves of above solutions are plotted for several values of the force parameters f in Fig. 2, and those of the solutions by Kryloff-Bogoriuboff method are also superimposed in Fig. 2. From the agreement of both results for each of three hysteresis loops it is concluded that the steady state responses of non-linear structures can be described by the linear equation of motion with the strain or displacement-dependent equivalent linear dynamic constants defined in the way for soils.

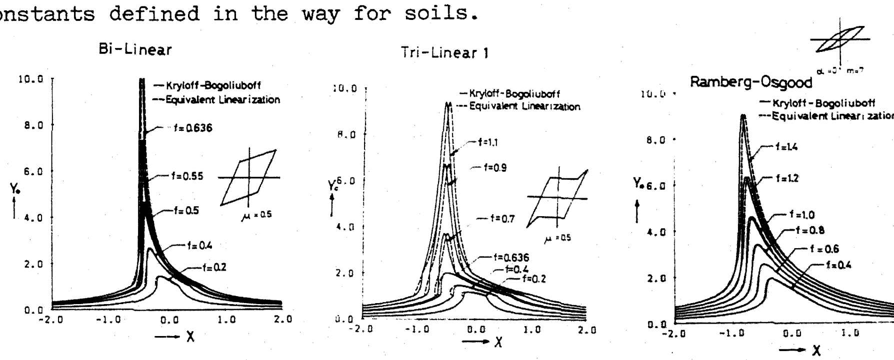


Fig. 2 Frequency Response Curves

EXAMINATIONS ON THE CHARACTERISTICS OF SEVERAL COMPUTER PROGRAMS

Comparison Of Dynamic Responses Of A Fill Dam By Three Computer Programs

With the computer program NODAL developed by the author et al. many numerical experiments of dynamic response analyses had been conducted on a typcal rockfill dam of 150 meter in the height in which sine waves possessing various acceleration amplitudes and periods were employed as earthquake motions (Ref.11). The dynamic values of materials applied in the calculations have the characteristics of parabolic strain amplitude dependency proposed by Hardin and Drnevich (Ref.6). The computer program NODAL has a function of dynamic response analysis for visco-elastic body with step-by-step integration procedure. Giving perfectly same cross section, FEM idealization and dynamic constants of materials as those of above numerical experiments some of above dynamic response analyses has been conducted with the computer programs FLUSH and QUAD 4 (Refs. 12, 13). An example of calculated results is shown in Fig. 3. The tendency of calculated results agrees well to each other in all cases. The absolute response values except for those in the crest and in the case of the short period coincide nearly in all cases. In the case of short period, however, there yields the discrepancy between the results by above three programs, that is, the calculated acceleration becomes smaller in the order of FLUSH, NODAL and QUAD 4. A major cause for the discrepancy is supposed to come from the frequency dependency of damping constant of which characteristic is different in every computer program.

Frequency Dependency Of Damping Constant

The computer program FLUSH is a complex response finite element program and main damping consists of complex moduli. Therefore the damping constant is independent of frequency. The computer program QUAD 4 is a direct integration finite element program and has damping matrix proposed by Rayleigh. The computer program NODAL has a system with visco-elastic damping such as Maxwell body, Voigt body, Zener body and four element body. Rayleigh damping and the visco-elastic damping have the characteristics of frequency dependency. These characteristics have been analyzed and the relationships between the damping constants and the natural angular velocity have been derived.These results are summarized in Table 2. The ratio of damping constant of the n-th mode to the one of fundamental mode has been related to the ratio of the n-th natural angular velocity to the fundamental one. These relationships are plotted in Fig. 4. From above results it is noted that Rayleigh damping utilized in QUAD 4 increases as natural frequency increases so that the calculated responses

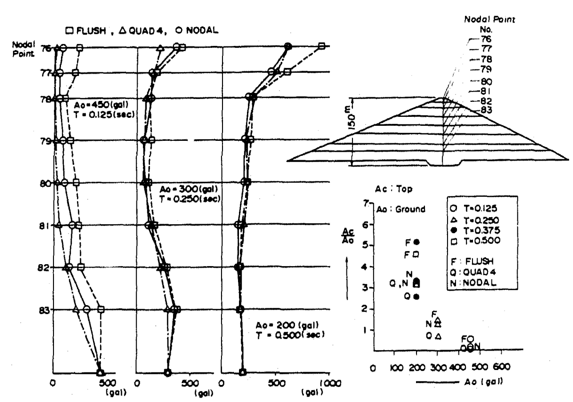


Fig. 3 Calculated Results by 3 Computer Programs

Table 2 Frequency Dependency of Damping Constants

Models	Stress ~ Strain $\tau \sim \gamma$	n-th Damping Constant h_n	$\frac{h_n}{h_1}$
Complex	$\tau=(G+j\cdot\eta_0)\gamma$	$\frac{\eta_0}{2G}$	1
Voigt	$\tau=(1+\tau_v\cdot D)\gamma$ $\tau_v=\eta/G,\ D=d/dt$	$\frac{1}{2}\cdot\tau_v\cdot\omega_n$	$\frac{\omega_n}{\omega_1}$
Maxwell	$\tau=\frac{\tau_m\cdot D}{1+\tau_m\cdot D}\gamma,\ \tau_m=\frac{\eta}{G}$	$\frac{1}{2}\cdot\frac{1}{\tau_m}\cdot\frac{1}{\omega_n}$	$\left(\frac{\omega_n}{\omega_1}\right)^{-1}$
Zener	$\tau=\frac{G\cdot G_1}{G+G_1}\cdot\frac{1+\tau_k\cdot D}{1+\tau_z\cdot D}\gamma$ $\tau_k=\eta_1/G_1,\ \tau_z=\eta_1/(G+G_1)$	$\frac{1}{2}\cdot\frac{\tau_z^{-1}-\tau_k^{-1}}{\omega_n^2+\tau_z^{-1}\tau_k^{-1}}\cdot\omega_n$	$\frac{1+\frac{\tau_z^{-1}\tau_k^{-1}}{\omega_1^2}}{\left(\frac{\omega_n}{\omega_1}\right)^2+\frac{\tau_z^{-1}\tau_k^{-1}}{\omega_1^2}}\cdot\left(\frac{\omega_n}{\omega_1}\right)$
Rayleigh	$[\tau]=\{[C]\cdot D+[K]\}[\gamma]$ $[C]=\alpha[M]+\beta[K]$ $=h\omega_1[M]+\frac{h}{\omega_1}[K]$ (QUAD4) [M]: Mass Matrix [K]: Stiffness Matrix ω_1: Angular Velocity of 1st Mode	$\frac{X_n^T[C]X_n}{2\sqrt{M_n^*\cdot X_n^T[K]X_n}}=$ $\frac{h}{2}\left[\left(\frac{\omega_n}{\omega_1}\right)^{-1}+\left(\frac{\omega_n}{\omega_1}\right)\right]$ $M_n^*=X_n^T[M]X_n$ $M_n^*\omega_n^2=X_n^T[K]X_n$	$\frac{1}{2}\cdot\left\{\left(\frac{\omega_n}{\omega_1}\right)^{-1}+\left(\frac{\omega_n}{\omega_1}\right)\right\}$

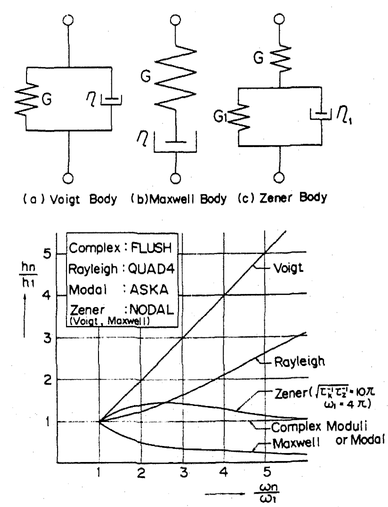


Fig. 4 Frequency Dependency of Damping Constants

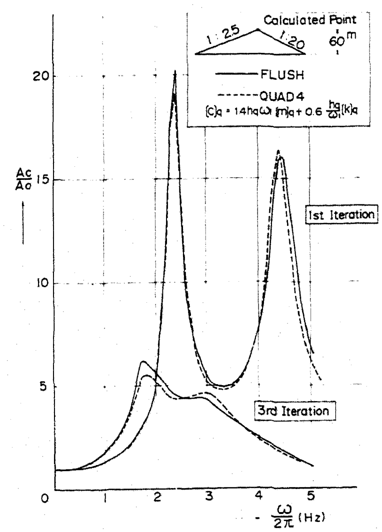


Fig. 5 Variation of Response due to Iteration of Analysis

become small in the shorter period and that the Zener damping utilized in NODAL increases at begining and decreases after peak point. These characteristics coincide with the results shown in Fig. 3.

With FLUSH and QUAD 4 some of dynamic response analyses have been conducted on a fill dam of which height is 60 meter. The damping matrix of QUAD 4 is modified so as to make the frequency dependency of damping be even as shown in Fig. 5. Giving same initial dynamic constants dynamic response analyses against an accelerogram have been iterated several times with both computer programs. The calculated frequency response curves vary with the times of iteration according to the given design equations of dynamic constants as shown in Fig. 5. From the agreement of frequency response curves by both computer programs, adding to the results in Fig. 3, it is concluded that same results are to be obtained from every dynamic response analysis conducted with different computer programs under same given conditions if only the characteristics of the frequency dependency of damping constant which the computer programs possess are made uniform.

CONCLUSIONS

Throughout above investigations the effect of damping constant on the accuracy of seismic design calculation for soils and foundations has been clarified and the reliability on the usual procedure of seismic response analysis utilized in practical seismic design of soil structures has been confirmed.

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