PROBABILISTIC SEISMIC STABILITY OF A COHESIONLESS SLOPE OF LIMITED EXTENT

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SUMMARY

This paper describes a model for predicting the probability of a failure of a slope of limited extent of saturated cohesionless soil due to seismically induced pore pressures. The probability of failure is defined as the probability of a factor of safety less than one. This probability is evaluated at the end of each cycle of loading as a function of the cumulative distribution functions of pore pressures, earthquake acceleration and soil parameters. A simulation algorithm is used to solve for the probability of failure. An example of application to a hypothetical site is presented.

PORE PRESSURES IN SLOPES OF LIMITED EXTENT

Most methods available to predict pore pressure development under seismic loading assume 1) that the soil is under a K_o state of stress and 2) that the earthquake generated shear stresses are the same at every location at the same depth in the deposit (one dimensional response). In a slope of limited extent, the soil elements near the face are not under K_o stresses and ground motions will vary spatially due to reflection of the shear waves from the free surface in non-vertical directions. A limited number of experimental investigations of the pore pressure response of non K_o consolidated samples under cyclic loading have been conducted (2,4,5,9,10). The general conclusion is that pore pressure develops at a slower rate in samples under initial static shear than in sample under no initial static shear, as shown in table 1 (10). Based upon these studies the following method was developed to evaluate pore pressure generation within saturated slopes of limited extent.

Consider the site shown on Figure 1 subjected to an earthquake. It is reasonable to assume that at some distance away from the slope (point F) the deposit behaves as a level ground deposit of thickness H_2. This distance corresponds to the free field. Seismically induced pore pressures in the free field may be evaluated using the methods developed for horizontal ground deposits. At the face of the slope there will be total drainage and extremely high initial static shear stresses. As a consequence it may be assumed that no pore pressure will develop near the face of the slope. It then may be stated that the pore pressure will

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vary smoothly from a maximum value in the free field to zero at the face of the slope. Furthermore it seems reasonable to assume that this variation will be a monotonic decrease towards the free surface.

It is proposed in this paper to evaluate the pore pressure at any location in the slope as a function of the pore pressure in the free field modified by a transfer function based upon the initial static shear stress on the horizontal plane. Using the notation in figure 1, the transfer function $T(x,y)$ is defined as follows:

$$T(x_o, y_o) = 0$$
$$T(x_{fo}, y_o) = 1$$
$$T\ (x_o \leq x \leq x_{fo}, y_o) \leq 1$$

Where y_o is the plane of interest, x_o the abscissa of the free surface on that plane and x_{fo} the abscissa of the free field for that particular plane. Note that x_{fo} will be different for each plane considered. Ideally three steps should be performed to evaluate $T(x,y)$:

1. A finite element analysis of the slope must be performed to evaluate the initial static stresses.

2. A series of tests should be performed in the laboratory to evaluate the pore pressure response of the material under those initial static shear stresses.

3. Information from points 2 and 3 are combined to evaluate the effect of initial static shear on the liquefaction resistance and to derive $T(x,y)$.

For the analysis described herein, in lieu of laboratory test data, pore pressure generation was assumed inversely proportional to the initial static shear stress on the horizontal plane.

PROBABILISTIC ANALYSIS

A probability of failure can be defined as the probability of occurrence of the event FS   1.0, or as the probability of the demand (D) exceeding the capacity (C).

$$P_f = P[\ FS \leq 1.\ ] \quad = \quad P[\ C \leq D\ ] \qquad (2)$$

Any conventional stability analysis can be used to evaluate C and D. In this paper the ordinary method of slices is chosen to analyze a potential failure surface in a homogeneous slope of limited extent under seismic loading. Using the notation shown in Figure 1 the FS is evaluated as:

$$FS = \frac{\Sigma(W_i \quad \cos \alpha_i - U_i) \tan \phi_i}{\Sigma\ W_i \cdot \sin a_i} \qquad (3)$$

The previous equation does not explicitly include seismically induced pore pressures and does not account for inertia forces. To account explicitly for the seismic pore pressures the term U_i in equation 3 may be written as

$$U_i = U_{is} + U_{ie} \tag{4}$$

Where U_{is} is the steady state pore pressure and U_{ie} is the seismically induced pore pressure.

The inertia force is given by

$$FI = a \, \Sigma \, W_i \tag{5}$$

where a is the acceleration on the potential failure mass of weight ΣW_i. In the ordinary method of slices, the capacity and demand terms C and D are moments with respect to the selected center of the potential failure surface and the contribution to the driving moment from FI, M(FI), is given by

$$M(FI) = \frac{R_{acc}}{R} \cdot a \, \Sigma \, W_i \tag{6}$$

where R_{acc} is the moment arm of FI about the center of the failure surface and R the radius of the failure surface.

Combining equations 2 to 6 the probability of failure of one potential failure surface is given by:

$$P_f = P \, [\, FS \leq 1 \,] \; = \; p \left[\frac{\Sigma \, (W_i \, . \, \cos \alpha_i - U_{ih} - U_{ie}) \, \tan \phi_i}{\Sigma \, W_i \, . \, \sin \alpha_i + aR_{acc}/R\Sigma W_i} \leq 1 \right] \tag{7}$$

where FS is considered to be a random variable. The probability of failure as given by equation 7 is a function of the earthquake time history and is thus a function of time. The probability of failure can be evaluated at every cycle during loading to see the effect of duration on the reliability of the slope.

The probability of failure of the slope is defined as the highest probability of failure of all potential failure surfaces. This definition has been adopted by a number of other researchers (7,10). Equation 7 is solved for one particular potential failure surface at a time. To evaluate overall slope stability the process must be carried out for a large number of surfaces.

The complexity of equation 7 is such that it cannot be solved by direct integration. The alternative solution adopted in this work is a simulation method. The method requires the knowledge of the distribution of each random variable or parameter in equation 7. A set of values is generated and the factor of safety is computed according to equation 7. The operation is repeated a large number of times. The total number of times where the FS is less than one is counted and normalized by the total number of simulations to yield a frequency of occurrence of FS less than one. The probability of failure of the given surface is set equal to this frequency of occurrence of the event "FS less than one." A complete description of the method to find the probability of the slope is shown in Reference 3. For the purpose of introducing random variables into the simulation model the inverse CDF method was used in this paper.

EXAMPLE OF APPLICATION

A hypothetical site consisting entirely of medium dense sand is used to demonstrate the application of the proposed methodology. The deposit has a 2.5 to 1 slope at the ground surface and is 70 feet high and 175 feet long. The water table is horizontal 15 feet below the top of the deposit.

A stabilizing force is exerted by the 55 feet of water against the face of the slope and a term accounting for that effect is included in the expression for the FS and probability of failure. Pore pressure response of the sand is taken to be that of the Crystal Silica Sand as defined by Martin et al (6). The probability of failure of the slope is evaluated by performing the operations listed in Figure 2 once the random variables and their respective distributions are identified. All the geometrical quantities can be reasonably considered to be deterministic quantities. Inertia force, pore pressures and soil parameters are treated as random variables.

a) Distribution

The probability distribution function of the seismic pore pressure in the field was determined using a model developed by Chameau (1). The probability distribution function of pore pressure may then be derived from that in the free field using the transfer function discussed earlier. A Rayleigh distribution is selected in this study to model the amplitudes of the randomly arriving peaks of the earthquake time history.

b) Results

Figure 2 shows the results for the analysis of 5 potential failure surfaces within the slope. It may be seen that the initial conditions represented by cycle 1 are dramatically different for the 5 surfaces. For the curve closest to the face (BD = 32.5 ft, where BD is the distance from top of slope to the beginning of the slip surface) the probability of failure is the greatest, on the order of 42%. This behavior shows a trend similar to that of the static factor of safety within the slope. Figure 3 shows the variation of the static factor of safety versus distance from the apex of the slope. Although it is a deterministic quantity which does not incorporate the effect of ground acceleration of pore pressure, the static factor of safety shows that a curve closer to the face of the slope is initially closer to the state "FS less than 1" than a curve far away. However, as previously discussed, greater pore pressures will be induced by seismic excitation along the failure surfaces farthest from the face of the slope, hence they may become more critical during the course of the seismic event.

This trend becomes evident when one follows the evolution of the probability of failure of two potential failure surfaces curve 'a' (BD = 32.5 ft) and curve 'b' (BD = 112.5 ft) with duration. The probability of failure for curve 'a' starts at about 0.44 after the first cycle and goes to about 0.88 after 40 positive zero crossings. The probability of failure for curve 'b' on the other hand, starts at about 0.02 and jumps to 0.88 after 25 positive zero crossings. After 40 cycles there is almost 100% chance of failure along surface 'b'. At about 21 positive zero crossings the two curves yield the same probability of failure: about 70%. So, for the first 20 positive zero crossings curve 'a' yields a higher probability of failure than curve 'b'. After the 21st positive zero crossing curve 'b' yields the higher risk of failure. The difference in behavior between curves 'a' and 'b' is directly related to the development of pore pressures. The initial higher risk of failure of curve 'a' is rapidly offset by the greater potential for high pore pressure to develop away from the face of the slope in a region crossed by curve 'b'.

If one is only interested in the risk at the end of the earthquake then only the failure surface yielding the highest probability of failure will be considered. In our case if we assume a duration of 16 to 20 cycles (positive zero-crossings), then there is a 72-78% chance of failure along a curve located at 32.5 feet away from the apex. For a duration of 40-50 cycles there is a 98-100% chance of failure along a curve located at 112.5 feet away from the apex.

SUMMARY

A theoretical model that accounts for strength reduction due to excess pore pressure development has been developed to evaluate the probability of failure of a slope of limited extent subjected to seismic loading. The probability of failure defined as the probability that the factor of safety is less than one.

The factor of safety of the slope is evaluated using the ordinary method of slices. Free field excess pore pressures induced by seismic loading are calculated using a probabilistic pore pressure model developed by Chameau (1). Excess pore pressures in the vicinity of the slope are calculated based upon the free field pore pressures and a transfer function which depends on the initial static shear stress on the horizontal plane. The influence of initial static shear in the excess pore pressure development is determined based on experimental studies performed by Vaid and Finn (10). A simulation algorithm is used to evaluate the probability of failure at the end of each cycle of loading.

Results show that the critical failure surface progresses away from the face of the slope during seismic loading. This progression is due to the increase in excess pore pressure with distance from the face that is predicted based upon the initial level of static shear.

The primary theoretical limitation of the model is the uncertainty with respect to pore pressure development under a non-K_o initial stress status. More information on the dynamic behavior of cohesionless soils subject to the non-K_o initial stress status is needed. To demonstrate the validity of the model for practical problems. case histories of the

failure of slopes of limited extent due to seismically induced pore pressure are required.

ACKNOWLEDGEMENT

The work described in this paper was funded by the National Science Foundation Earthquake Hazard Mitigation Program, contract NSF CEE 81-10616.

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TABLE 1

Effect of Non-K_0 Condition on Liquefaction resistance, (After Vaid and Finn, 1979)

Initial Static Shear Stress Ratio τ/σ_0'	Number of cycles N
0.0	5
0.04	14
0.093	25
0.192	290

Table 2

Site and Loading Conditions

Height: 70'   Length: 175'   Height of Water: 55'

γ = 120 pcf

$\phi'_{mean} = 30^o$ $\phi'_{min} = 25^o$ $\phi'_{max} = 35^o$

PGA = 0.20g

Duration = 80 positive zero crossings

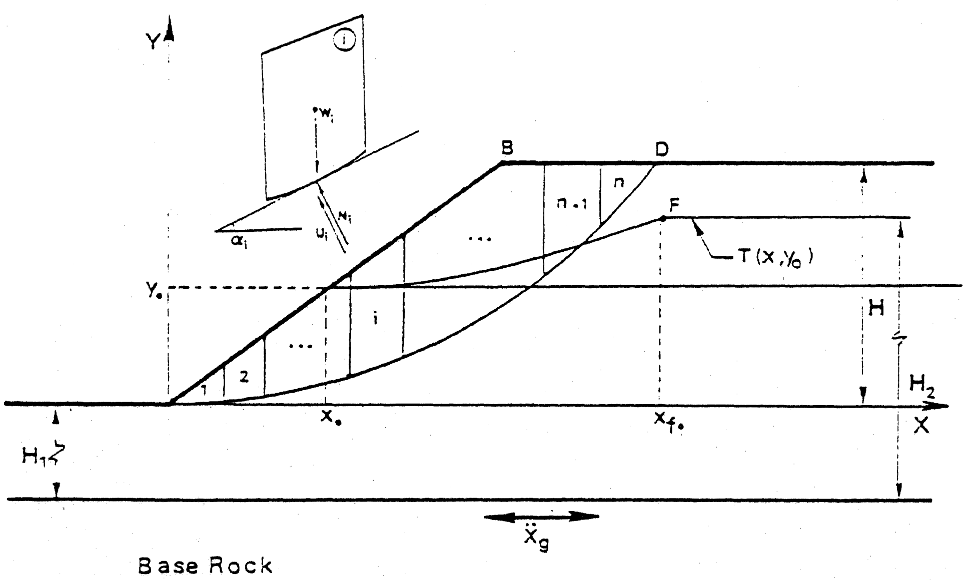

Figure 1 - Typical Site under Study

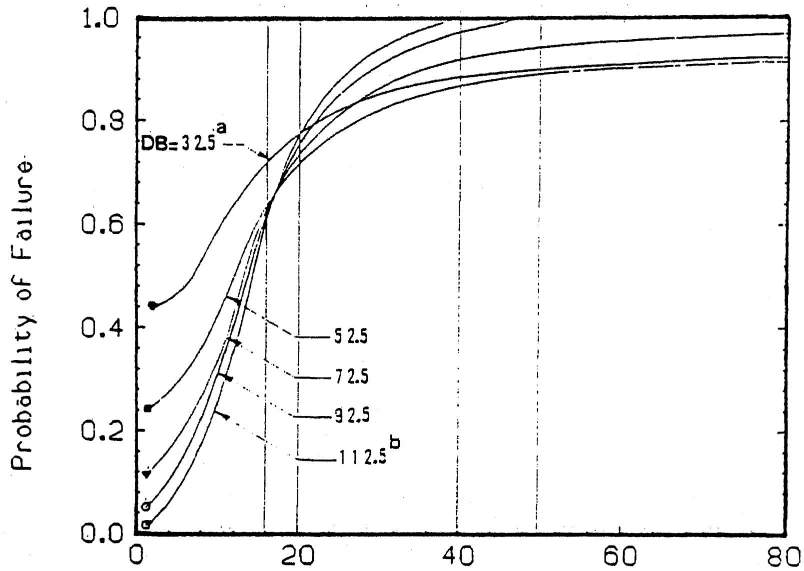

Figure 2 - Probability of Failure - Hypothetical Site

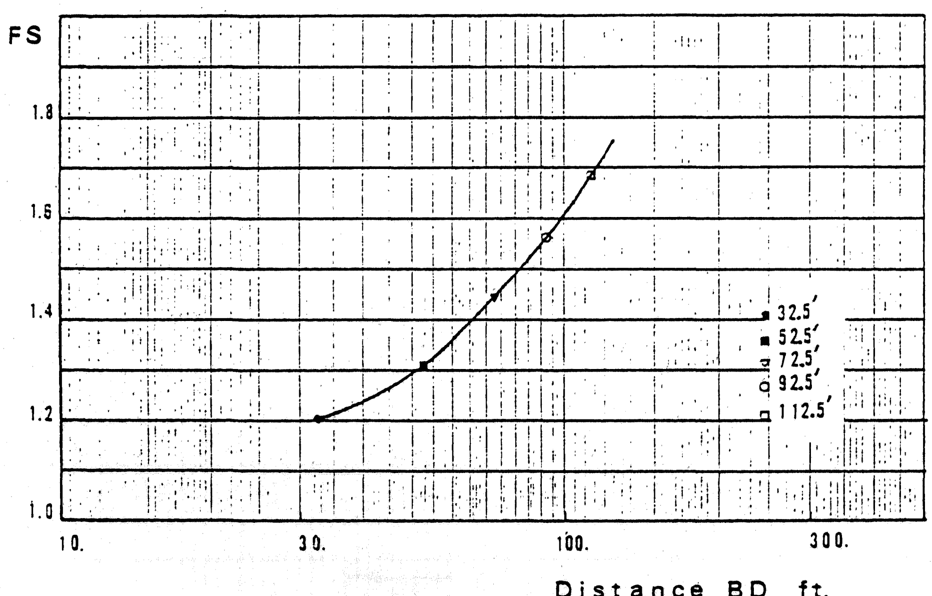

Figure 3: Factor of Safety versus BD