SEISMIC DESIGN OF GRAVITY RETAINING WALLS

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SUMMARY

Factors affecting the choice of a suitable safety factor for use with the Richards-Elms method are investigated: errors in the use of a sliding block to represent a retaining wall and associated backfill, near-randomness in time-histories of earthquake ground motion and uncertainty in strength parameters. A systematic approach for treating these uncertainties is developed. While some factors should be studied further, recommendations are presented based upon best available information.

INTRODUCTION

Gravity retaining walls typically are designed by a static analysis using seismic coefficients. Typical values for these coefficients range up to 0.2, and almost always are smaller than a coefficient corresponding to the peak accelerations which are thought possible for the region in which a wall is to be situated. This is a perfectly sensible approach to design, since a gravity wall represents a ductile system (providing there is no liquefaction in the supporting soil or backfill) and permanent displacements of inches are usually acceptable. However, considerable uncertainty has existed concerning the selection of coefficients and associated safety factors.

Richards and Elms (Ref. 1) have used Newmark's sliding block concept to develop the first systematic approach to the design of such walls. Their method involves the following steps:

1. Select values of peak acceleration Ag and peak velocity V to characterize the earthquake ground motion.
2. Select the maximum allowable permanent displacement d_L.
3. Find the resistance factor N (where Ng is the acceleration at which the wall will begin to slip) such that the actual permanent displacement d_R will just equal d_L. They recommend use of the equation

$$d_L = d_R = 0.087 \ \frac{V^2}{Ag} \ \left(\frac{N}{A}\right)^{-4} \quad \text{or} \quad \frac{N}{A} = \left(\frac{0.087V^2}{d_L Ag}\right)^{0.25} \qquad (1)$$

which is a conservative envelope for displacements computed using recorded ground motions.

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4. Use N to evaluate the static-plus-dynamic active earth pressure against the wall and the inertia force acting on the wall. For evaluating the earth pressure, they recommend the Mononobe-Okabe equation (Ref. 2).
5. Find the weight of wall required to balance these forces.

Richards and Elms recommend increasing this computed weight by a factor of 1.5, to cover shortcomings in the method. They also recomended that walls be designed so as to yield by sliding rather than in tilting.

The following sections of this paper investigate a number of the sources of error and uncertainty in the Richards-Elms analysis. The final sections draw together the several results so as to provide recommendations for an improved approach to design. The overall aim is to retain the simplicity of the Richards-Elms method while placing the choice of a safety factor on a sound basis. The presentation of necessity has been condensed; full details may be found in Ref. 3.

UNCERTAINTY ASSOCIATED WITH GROUND MOTIONS

There can be enormous differences in the computed permanent displacement of a sliding block depending upon the ground motion recording used as input to the analysis. Previous studies (Ref. 4) have yielded displacements varying by a factor of 20, all for a given A, V and N. As mentioned, Eq. 1 is intended to give nearly an upper bound.

To study this matter further, a suite of 14 earthquake recordings was selected, each with two components of motion (Ref. 5). In order to reduce problems arising when scaling of such records to large peak accelerations, all had peak accelerations greater than 0.15g. Further, to minimize the role of duration as a variable, the magnitudes of the causative earth- were restricted to the range 6.3-6.7. (Actually, 2 of the 14 records were caused by larger earthquakes.) When scaling these records, each component was scaled by the same factor, so as to preserve the relationships among them. Four values of permanent slip of a block (with one-way sliding) were computed for each record, using both sides of each component of acceleration. Thus, scatter in computed permanent displacements reflects the effect of different orientations of a wall at a site as well as variations from site-to-site and earthquake-to-earthquake.

The mean values of the permanent displacements, computed ignoring the vertical component of ground acceleration, are fitted well by:

$$d_R = \frac{37V^2}{Ag} e^{-9.4\ N/A} \tag{2}$$

The coefficients of variation (CoV) range from about 0.6 at smaller N/A to about 1.4 at larger N/A. At the larger N/A, the scatter results principally from different orientations of the direction of slip at a site. At the lower N/A, variations among sites and earthquakes are more important. At each N/A, the distribution of slips is approximately lognormal, although some deviation in the lower tail occurs at the larger N/A.

Including vertical accelerations in the computation causes the mean slip to increase, the increase being a function of N/A and of the peak acceleration to which the records were normalized. For N/A = 0.5, the increase in the mean ranges from 1.03 for A = 0.3 to 1.17 for A = 0.7. At N/A = 0.7, the corresponding increases are 1.09 and 1.41. There is considerable influence of vertical acceleration upon the slip computed from any one component of input motion. However, overall the CoV for slips is almost identical whether or not vertical accelerations are included, indicating correlation between vertical accelerations and other factors causing scatter in the permanent displacements.

It may be noted that these results are generally applicable to any problem in which sliding block analysis is used to predict permanent displacements or deformations.

UNCERTAINTY IN RESISTANCE

Various factors influence the resistance N: the unit weight of the concrete and of the backfill, the friction angle ϕ for the backfill, the friction angle ϕ_b at the base of the wall, the wall friction angle δ and the geometry and dimensions of the wall and backfill. It is reasonable to assume that uncertainties in unit weights and dimensions are small compared to those in the friction angles. If the three friction angles are assumed to be independent of each other, then the standard deviation for N, σ_N, can be determined from:

$$\sigma_N^2 = \left(\frac{\partial N}{\partial \phi}\right)^2 \sigma_\phi^2 + \left(\frac{\partial N}{\partial \phi b}\right)^2 \sigma_{\phi b}^2 + \left(\frac{\partial N}{\partial \delta}\right)^2 \sigma_\delta^2 \tag{3}$$

where σ_ϕ, $\sigma_{\phi b}$ and σ_δ are standard deviations (SD) for the three friction angles. The partial derivatives may be found by implicit differentiation of the equations for the dynamic equilibrium of the wall (Ref. 5), and are evaluated at the mean values of ϕ, ϕ_b and δ.

The values of these derivatives depend upon the values of the friction angles and upon the geometry of the retaining wall system. However, the sum of the squared derivatives in the first two terms in Eq. 3 is more-or-less constant for reasonable ranges of these properties. Using $\sigma_\phi = \sigma_{\phi b} = 2^\circ$, the first two terms correspond to σ_N of about 0.03.

The derivative in the third term varies more with changes in parameters and geometry. Moreover, uncertainty in σ_δ is greater than for the other angles; it is reasonable to assume $\sigma_\delta = 5^\circ$. When this term is included, σ_N becomes as much as 0.06.

MODEL ERRORS

Kinematic Constraints

As indicated in Fig. 1, if slip develops between a wall and its supporting soil, the failing wedge of backfill must experience a vertical acceleration. This is true whether or not there is vertical acceleration in the input ground motion. This kinematically-required vertical acceleration, which was not considered by Richards and Elms, influences the thrust exerted by the soil on the wall, and thus the amount of slip that occurs. This effect causes a reduction in the amount of slip (Refs. 5,6). Calculations for typical cases predict reductions ranging upward to 75%, but more typically 30-40% or less.

The "two-block model" depicted in Fig. 1 is quite a reasonable representation for what is actually observed in model tests upon shaking tables. Moreover, it has been found that inclusion of this effect is necessary to obtain reasonable agreement between predicted permanent displacements and those measured in model tests (Ref. 7). There is no doubt that the effect is real.

However, there are difficulties in the way of predicting the expected reduction. The reduction varies with the slope of the wall and the backfill, and is somewhat affected by the average values of the friction angles; these factors complicate the practical evaluation of the reduction. Moreover, the inclination of the failure surface through the backfill theoretically varies with time so as to be always at the critical slope for the forces existing at each time. Practically, it has been observed in model tests that the inclination of the failure plane remains constant, but uncertainty remains as to how to analytically determine this inclination. (Ref. 3,7).

All in all, it is reasonable to account for the mean effect of kinematic constraints by multiplying the displacement predicted by the sliding block model by of 0.65. Errors arising from the decision not to use a more complex model, and from uncertainties in the modelling, may be accounted for by assigning a standard deviation of 0.2.

Elastic Backfill

All the models discussed to this point are rigid plastic; that is, the backfill is assumed not to deform until slip begins. Other analyses for the dynamic earth pressures upon walls have assumed linearly deformable backfill of unlimited strength. Actual behavior must lie between these two extremes. In order to investigate this aspect of the problem, a special finite element model (Fig. 2) was developed (Ref 8). This mesh has slip elements (shown as heavy lines) but is otherwise linear, and thus provides an improved representation of actual behavior.

Including deformability increases the computed slip, although overall the behavior is much as predicted by the "two block" model. The amount of increase is a function of the predominant frequency of the input motion in relation to the fundamental frequency of the backfill. Some results are given in Fig. 3. The increase is associated with an amplification of acceleration up through the backfill, and thus becomes larger as the frequency ratio approaches unity. Deformability of the backfill also becomes more important as N/A increases.

Figure 3 might be used to develop correction factors for predicted slip, as a function of N/A and the frequency ratio. However, in the interest of simplicity a constant correction ratio of 3 is suggested, inasmuch as the frequency ratio will typically be about one-half or less. A SD of 2 is assigned to cover the spread in actual values.

Tilting

Little work has been done concerning tilting. Results obtained using a finite element mesh similar to that of Fig. 2 have emphasized the complexity of the stress distribution between backfill and wall. As shaking progresses, the location of the resultant thrust varies considerably, often being below the lower third point. A preliminary study (Ref. 9) suggests that permanent displacement at the top of a tilting wall is approximately 1.5 times the slip calculated by a sliding block model. A SD of 0.75 is assigned to account for uncertainties in this estimate.

COMBINED EFFECTS

Considering the factors discussed in the previous sections, the random quantity d_R may be represented by the equation:

$$d_R = \frac{37V^2}{Ag} e^{-9.4N/A} \; M \; Q \tag{4}$$

where N is now a random variable with mean $\bar{N}$ and standard deviation σ_N, M is a model error term with mean $\bar{M}$ and standard deviation σ_M, and Q (mean of unity and standard deviation σ_Q) reprsents the uncertainty associated with the details of ground shaking. The mean value of d_R is:

$$\bar{d}_R = \frac{37V^2}{Ag} e^{-9.4\bar{N}/A} \; \bar{M} \tag{5}$$

To evaluate the uncertainty in d_R, Eq. 4 is best rewritten as:

$$\ln d_R = \ln \left(\frac{37V^2}{Ag}\right) - 9.4 \frac{N}{A} + \ln M + \ln Q \tag{6}$$

Then, assuming independence of the various terms:

$$\sigma^2_{\ln d_R} = \left(\frac{9.4}{A}\right)^2 \sigma^2_N + \sigma^2_{\ln M} + \sigma^2_{\ln Q} \tag{7}$$

Table 1 summarizes previously stated results concerning the several means and standard deviations. The model error term M is itself a product of several factors, which (assuming independence among them) may be combined using rules from probability theory.

Using the numbers from Table 1 in Eq. 5, the mean slip becomes:

$$\bar{d}_R = \frac{130V^2}{Ag} e^{-9.4N/A} \quad \text{or} \quad \frac{N}{A} = -\frac{1}{9.4} \ln \left(\frac{d_R Ag}{130V^2}\right) \tag{8}$$

After several further operations upon the SD in Table 1, Eq. 7 yields values of SD for ln d_R from about 1.3 to nearly 3. At A = 0.2, the first term on the right hand side of Eq. (7) is dominant, and the scatter is the greatest. At A = 0.7, this term is negligible compared to the 2nd and 3rd terms, and the scatter is least.

DESIGN

As a result of these many considerations, it is recommended that Eq. 8 replace Eq. 1 in the Richards-Elms design procedure. It is further recommended that a safety factor F_c be applied to the allowable slip D_L, so as to account for scatter in the expected motions and errors and uncertainties in the use of such a simplified method. Thus $\bar{d}_R = d_L/F_c$. A suitable value for F_c may be derived by assuming that d_R is lognormally distributed (a reasonable assumption in view of results of calculations and of the form of Eq. 4) and using the SD of ln d_R determined above. For example, if one wishes to be 95% confident that the allowable displacement will not be exceeded, then F_c = 3.8 should be used. This confidence level can be used to determine the resistance N to be used as a basis for design as:

$$\frac{N}{A} = 0.66 - \frac{1}{9.4} \ln \frac{d_L Ag}{V^2} \tag{9}$$

Use of this equation for typical values of A and V and a limiting slip d_L = 100 mm yields values of N/A ranging from 0.55 to 0.65. These values are somewhat greater than those obtained from Eq. 1, but now it should no longer be necessary to apply any safety factor to the weight of wall required for dynamic equilibrium. Thus the overall result generally is a saving in the weight of wall in contrast to that required by the Richards-Elms procedure. For example, the weights of wall required by this approach range from 1.1 times to 1.4 those found in step 5 of the Richards-Elms approach (instead of the factor of 1.5 suggested). The lower factor applies for A = 0.2, and the factor increases with increasing A. If one were to use a larger allowable displacement, or to accept a lower level of confidence that a stated allowable displacement will not be exceeded, the procedure will lead to a smaller required N/A and hence a lighter wall. Of course, the weight of wall must not be less than that required for satisfactory performance under static loads.

FINAL COMMENTS

This paper has a given systematic but partial analysis of the errors and uncertainties associated with the Richards-Elms procedures for the seismic design of gravity retaining walls. Further work will be required to arrive at still firmer recommendations, especially study of tilting and field and/or further model tests.

ACKNOWLEDGEMENTS

The work described in this paper has been supported by the National Science Foundation under Grant No. PFR-7919666, and also by the U.S. Army

Engineer Waterways Experiment Station. Many students have contributed to this work and their efforts are gratefully acknowledged.

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Table 1

SUMMARY OF MEANS AND STANDARD DEVIATIONS

Random Variable	Mean	SD
Ground motion term Q	1	0.6-1.4 as function of N/A
Resistance parameter N	$\overline{N}$	0.03 to 0.06
Model error term M		
Ignoring 2-block effect	0.65	0.2
Ignoring vert. accel.	1.2	0.2
Deformable backfill	3.0	2.0
Tilting	1.5	0.75
Combined Model Error	3.51	3.56

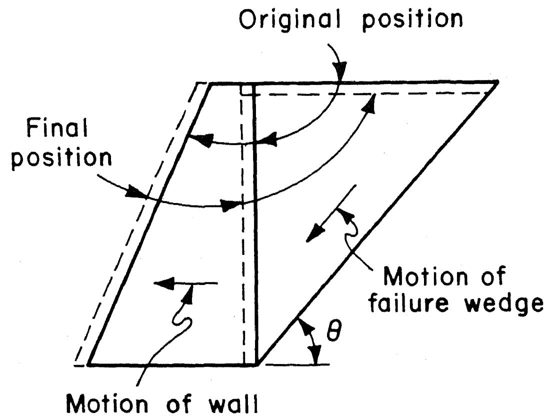
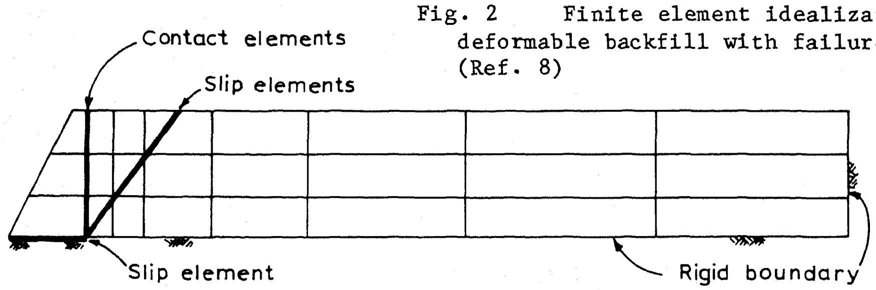


Fig. 1 Motion of backfill during sliding of wall

Fig. 2 Finite element idealization of deformable backfill with failure plane (Ref. 8)



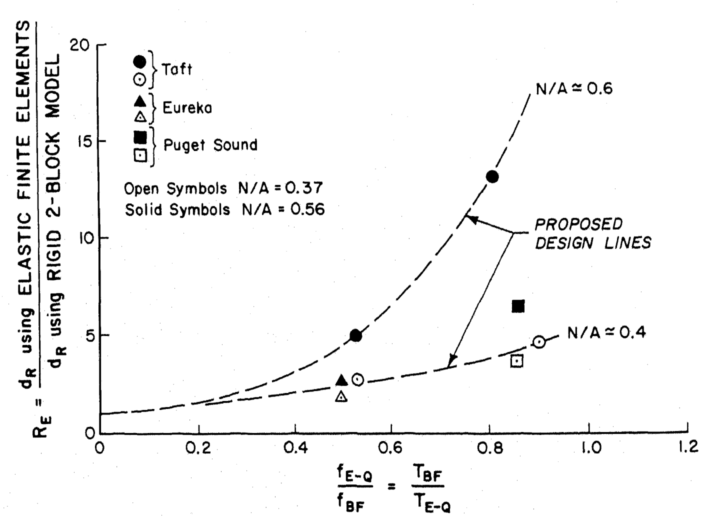


Fig. 3 Increased displacement caused by deformability of backfill (after Ref. 8)