APPROXIMATE THREE-DIMENSIONAL ANALYSES OF EMBEDDED STRUCTURES

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SUMMARY

In this paper studies are made to investigate the effect of viscous dashpots, which are introduced in a two-dimensional analysis to simulate energy dissipation in the third direction due to radiation, on the response of structures subjected to seismic waves. The analyses are made by the boundary element method combined with the substructure technique. A comparison among results obtained by two-dimensional, approximate three-dimensional and exact three-dimensional analyses leads to the conclusion that the existence of viscous dashpots produces a significant effect on the dynamic behavior of structures.

INTRODUCTION

The study of the soil-structure interaction for nuclear power plants and other massive structures is often conducted on the basis of two-dimensional representations due to the large effort involved in a realistic three-dimensional analysis. Previous works (Refs. 1 and 2) have shown that it is difficult to obtain a good estimation by a two-dimensional representation that approximates both the dynamic stiffness and the radiational damping over a reasonable range of frequencies. From this point of view a method of appoximate three-dimensional analysis of dynamic soil-structure interaction problems has been proposed, where the three-dimensional effect was achieved by the addition of viscous forces (dashpots) to account for wave propogation in the third direction (Ref. 3). Though these viscous dashpots have been widely used in the dynamic finite element analyses, the effect of them on the response of structures subjected to seismic waves seems not to have been discussed in detail.

The authors have already studied by the use of the boundary element method the effect of viscous dashpots introduced in a two-dimensional analysis on the dynamic response of structures through the analysis of surface foundations subjected to seismic waves (Ref. 4). In this paper, the study is extended to the case of embedded foundations. Foundation-soil systems considered and the method of analysis used are similar to those in the previous study (Ref. 4).

METHOD OF ANALYSIS

To study the effect of dashpots on the dynamic soil-structure interaction, embedded rigid foundations are selected as illustrated in Fig. 1. The supporting soil, to which dashpots are added, may or may not have a rigid rock at some depth. The analyses are made by the use of the boundary element method combined with the substructure technique. The formulation of the solution method is briefly summarized here.

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Substructure Formulation

Based on the assumed linearity of the model, the foundation-soil system can be partitioned into a set of simpler subsystems at the interface between the soil and the foundation, i.e., the soil subsystem and the foundation subsystem. The complete foundation-soil interaction problem is subdivided into four steps; i.e., evaluation of (1) the impedance matrix for massless rigid foundations, (2) the free-field motion of the soil in the absence of foundations (3) driving forces to keep foundations fixed and (4) the total motion. The boundary element method is applied in the first and the third steps while the well-known solution based on the theory of elastic wave propagation is used in the second step. The calculation in the fourth step will be achieved by a simple algebraic operation. Since viscous forces are proportional to the relative velocity between the soil under consideration and the far-field, dashpots are added in the first and the third steps.

Boundary Element Formulation

For an isotropic viscoelastic body, the displacement $\mathbf{u}$, in the presence of viscous forces, satisfy the equation:

$$(\lambda+\mu)\nabla\nabla\cdot\mathbf{u}+\mu\nabla^2\mathbf{u}+\rho\omega^2\mathbf{u}-i\omega\eta\mathbf{u}=0 \qquad \cdots (1)$$

where ρ is the density of the body, η is the viscous damping coefficient related to dashpots, and λ, μ are complex Lamé's constants. By the application of the method of weighted residuals with the help of an appropriate fundamental solution, the displacement vector $\mathbf{u}^i$ of a point i at the boundary of the domain under consideration can be expressed in terms of the boundary values:

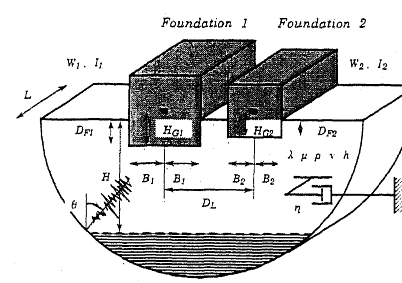
$$\mathbf{C}^i\mathbf{u}^i+\int_\Gamma \mathbf{p}^*\mathbf{u}\,d\Gamma=\int_\Gamma \mathbf{u}^*\mathbf{p}\,d\Gamma \qquad \cdots (2)$$

in which, $\mathbf{C}^i$ is the coefficient matrix depending on the geometry of the boundary, $\mathbf{u}^*$, $\mathbf{p}^*$ are the displacement and the traction vectors corresponding to the weighting field, and $\mathbf{u}$, $\mathbf{p}$ are the boundary values of the displacement and the traction, respectively.

To obtain a numerical solution the boundary is divided into so-called constant elements. Eq. (2) then becomes

$$\mathbf{Hu}=\mathbf{Gp} \qquad \cdots (3)$$

in which $\mathbf{H}$ and $\mathbf{G}$ are complex coefficient matrices. Introducing prescribed boundary values into this equation we can obtain unknown values which enable the evaluation of the impedance functions or the driving forces.



ρ	: Density
ν	: Poisson's ratio
μ, λ	: Lamé's constants
η	: Viscous damping coefficient
h	: Hysteretic damping factor
H	: Thickness of surface layer
θ	: Incidence angle of seismic wave
W_1, W_2	: Weight of foundation
I_1, I_2	: Moment of inertia of foundation
H_{G1}, H_{G2}	: Height of center of gravity of foundation
D_{F1}, D_{F2}	: Embedment depth of foundation
B_1, B_2	: Half width of foundation
L	: Depth of foundation
D_L	: Distance between two foundations

Fig. 1 Description of the Model

Fundamental Solutions

The fundamental solution for this

problem can be obtained by solving the equation

$$(\lambda+\mu)\nabla\nabla\cdot\mathbf{u}+\mu\nabla^2\mathbf{u}+\rho\omega^2\mathbf{u}-i\omega\eta\mathbf{u}+\mathbf{f}\delta^i=0 \qquad \cdots (4)$$

where δ is the Dirac delta function and f (=(p,q)) is the point source vector. To account for wave propagation in the third direction it can be assumed that

$$\eta=2\rho V_s/L \qquad V_s=\sqrt{\mu/\rho} \qquad \cdots (5)$$

in which Vs is the shear wave velocity of the medium and L is the width of a slice equal to the depth of the foundation. Eq. (4) can be solved by taking Fourier transforms with respect to x and z. The resulting solution, which corresponds to the Green's function for infinite domain, is as follows:

$$u_s(x,z)=\frac{i}{4\pi k^2}\left[\left\{\frac{\partial^2}{\partial x^2}H_0^{(2)}(hr)+\frac{\partial^2}{\partial z^2}H_0^{(2)}(kr)\right\}p+\frac{\partial^2}{\partial x\partial z}\left\{H_0^{(2)}(hr)-H_0^{(2)}(kr)\right\}q\right]$$

$$w_s(x,z)=\frac{i}{4\pi k^2}\left[\frac{\partial^2}{\partial x\partial z}\left\{H_0^{(2)}(hr)-H_0^{(2)}(kr)\right\}p+\left\{\frac{\partial^2}{\partial z^2}H_0^{(2)}(hr)+\frac{\partial^2}{\partial x^2}H_0^{(2)}(kr)\right\}q\right] \qquad \cdots (6)$$

in which, $H_0^{(2)}$ is the Hankel function of the second kind of order zero, and

$$h^2=(\omega/c_p)^2-i\omega/D_p、\ k^2=(\omega/c_s)^2-i\omega/D_s、\ c_p=\sqrt{(\lambda+2\mu)/\rho}、\ c_s=\sqrt{\mu/\rho}$$

$$D_p=(\lambda+2\mu)/\eta、\ D_s=\mu/\eta、\ r=\sqrt{x^2+z^2}、\ \alpha=\sqrt{\xi^2-h^2}、\ \beta=\sqrt{\xi^2-k^2} \qquad \cdots (7)$$

Since we consider a foundation embedded in a semi-infinite soil or in a finite layer overlying a rigid rock, it is convenient to find fundamental solutions for Eq. (4) satisfying boundary conditions corresponding to our problem. This is also done by the application of the Fourier transform method. We have for a semi-infinite space

$$u=\frac{1}{\pi\mu k^2}\int_0^\infty\frac{1}{F_H(\xi)}\Big[-p\frac{\xi^2}{\alpha}\{(2\xi^2-k^2)e^{-\alpha z}-2\alpha\beta e^{-\beta z}\}\{(2\xi^2-k^2)e^{-\alpha f}-2\alpha\beta e^{-\beta f}\}\cos(\xi x)$$
$$+q\xi\{-2\xi^2e^{-\alpha z}+(2\xi^2-k^2)e^{-\beta z}\}\{2\alpha\beta e^{-\alpha f}-(2\xi^2-k^2)e^{-\beta f}\}\sin(\xi x)\Big]d\xi+u_s(x,z-f)+u_s(x,z+f)$$

$$w=\frac{1}{\pi\mu k^2}\int_0^\infty\frac{1}{F_H(\xi)}\Big[p\xi\{(2\xi^2-k^2)e^{-\alpha z}-2\xi^2e^{-\beta z}\}\{(2\xi^2-k^2)e^{-\alpha f}-2\alpha\beta e^{-\beta f}\}\sin(\xi x)$$
$$-q\frac{\xi^2}{\beta}\{2\alpha\beta e^{-\alpha z}-(2\xi^2-k^2)e^{-\beta z}\}\{2\alpha\beta e^{-\alpha f}-(2\xi^2-k^2)e^{-\beta f}\}\cos(\xi x)\Big]d\xi+w_s(x,z-f)+w_s(x,z+f)$$

$$F_H=(2\xi^2-k^2)^2-4\alpha\beta\xi^2 \qquad \cdots (8)$$

The fundamental solution for a finite layer overlying a rigid rock is given by

$$u=\frac{1}{2\pi\mu}\int_0^\infty\frac{1}{F_R(\xi)}\Big[-p\{i\xi(A_He^{-\alpha z}+C_He^{\alpha z})+\beta(B_He^{-\beta z}-D_He^{\beta z}\}\cos(\xi x)d\xi-q\{-\xi(A_Ve^{-\alpha z}$$

$$
+ C_V e^{\alpha z}) + i\beta(B_V e^{-\beta z} - D_V e^{\beta z})\} \sin(\xi x) \Big] d\xi + u_s(x, z-f) + u_s(x, z+f)
$$

$$
w = \frac{1}{2\pi\mu} \int_0^{\infty} \frac{1}{F_R(\xi)} \Big[-p\{-i\alpha(A_H e^{-\alpha z} - C_H e^{\alpha z}) - \xi(B_H e^{-\beta z} + D_H e^{\beta z}\} \sin(\xi x) d\xi - q\{-\alpha(A_V e^{-\alpha z}
$$

$$
- C_V e^{\alpha z}) + i\xi(B_V e^{-\beta z} + D_V e^{\beta z})\} \cos(\xi x) \Big] d\xi + w_s(x, z-f) + w_s(x, z+f)
$$

$$
F_R = 8\alpha\beta\xi^2(2\xi^2 - k^2) + (\xi^2 - \alpha\beta)\{(2\xi^2 - k^2)^2 - 4\alpha\beta\xi^2\} \cosh\{(\alpha+\beta)H\}
$$

$$
- (\xi^2 + \alpha\beta)\{(2\xi^2 - k^2)^2 + 4\alpha\beta\xi^2\} \cosh\{(\alpha-\beta)H\} \qquad \cdots (9)
$$

where A_H, B_H, .. are the coefficients which are the functions of the exciting frequency, the elastic constants and the thickness of the finite layer. The expression of the traction which is not presented here may be obtained from the constitutive relations.

If we choose Eq. (8) or Eq. (9) as a fundamental solution no elements need to be placed on the boundary except on the foundation-soil interface.

NUMERICAL RESULTS AND DISCUSSION

A brief comment is first given of the verification of the present method of analysis. Then the effect of dashpots on the dynamic interaction of rigid foundations is discussed. Parameters used in the calculation are summarized in Table 1.

Verification of the Present Method

In order to verify the developments presented in the previous sections we consider a simple application, namely, compliance functions of a rigid strip on the surface of a half-space in plane strain (no viscous nor hysteretic damping). Fig. 2 is a plot of the normalized compliance function in horizontal translation. Also in the figure results presented by Luco et al. (Ref. 5) are plotted. Poisson's ratio of 0.25 is assumed. This figure clearly exhibits the validity of the present method.

Effect of Dashpots

Now we investigate the effect of dashpots. To this end we consider first a rigid foundation placed on the surface of a half-space. Fig. 3 shows the dependency of normalized impedance functions on the nondimensional frequency a_0 ($=\omega B/V_s$) while Figs. 4 and 5 provide the foundation input motion, which corresponds to the response of a massless rigid foundation due to a seismic wave, and the tranfer function against the far-field surface response. Exact three-dimensional results (Ref. 6) are also plotted in Figs. 3 and 5 as a comparison.

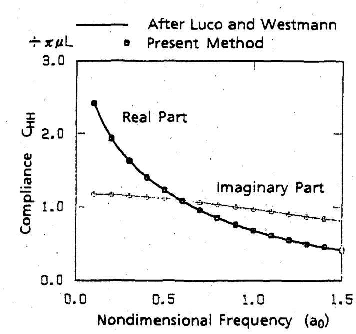
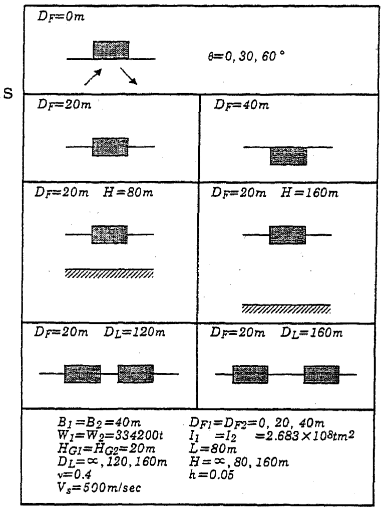


Fig. 2 Verification of the Present Method

It is found from these figures that the existence of viscous dashpots exerts a significant influence upon the dynamic characteristics of a rigid foundation. The real part of impedance functions moves upward and approaches to that of the exact three-dimensional solution. The change of the imaginary part, however, depends on the mode of vibration, i.e., translation or rotation. Unfortunately the imaginary part of the rocking impedance is shifted apart from the exact three-dimensional solution by adding dashpots. As a whole adding dashpots produces its desired effect especially upon the horizontal impedance. Foundation input motion is little influenced in turn by the addition of viscous dashpots. It may be understood as a result of less interaction effect due to the lack of the foundation mass. Finally it can be concluded that the response of the foundation due to a vertically incident SV wave is fairly improved. A relatively low peak level of the approximate three-dimensional solution may be resulted from the over-estimation of damping in the rocking impedance.

Table 1 Parameters Used in the Calculation



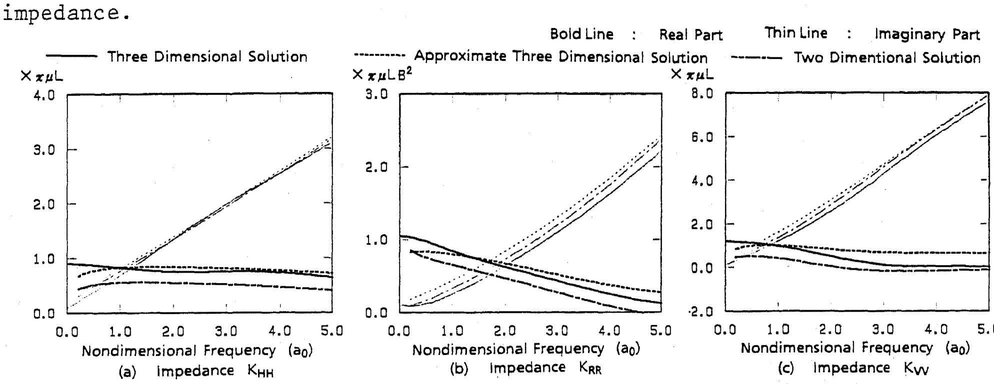


Fig. 3 Effect of Dashpots on the Impedance Function

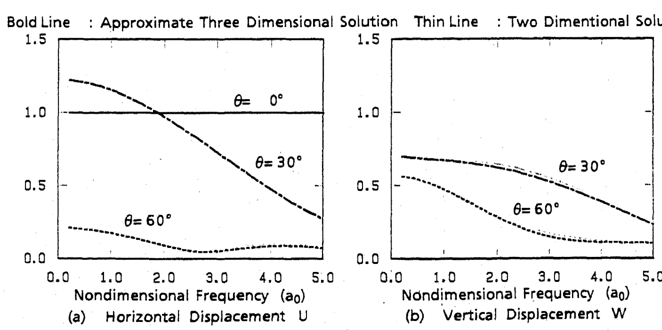


Fig. 4 Effect of Dashpots on the Foundation Input Motion

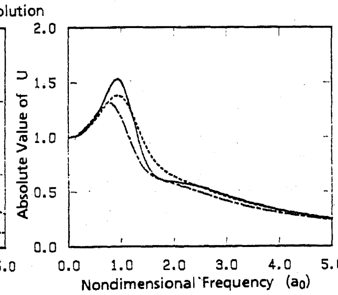


Fig. 5 Effect of Dashpots on the Transfer Function

Effect of Foundation Embedment

Next considered is the effect of foundation embedment. Analyses are made of a rigid foundation embedded in a half-space. Fig. 6 is a plot of the impedance functions and Fig. 7 shows the response of a foundation due to a vertically incident SV wave. Though the actual values of the impedance functions are changed by the addition of dashpots, a similar effect of embedment is seen in both cases. The imaginary part of the impedance grows conspicuously larger when the embedment increases, while the real part remains unchanged except the rotational impedance. The effect of embedment is significant for the foundation response, which is directly expected from the fact of the increase of the imaginary part of the impedance function.

Effect of Underlying Bed Rock

The effect of the assumed rigid rock underlying a finite layer is demonstrated on Figs. 8 and 9 in which the impedance function in horizontal direction and the response of a foundation due to a SV wave are depicted. It is clearly seen in these figures that the viscous damping dramatically reduces the fluctuation due to the assumed rigid rock and that the results for H/B=4 almost coincide with those for a half-space. This fact implies that viscous dashpots are effective in removing a bad influence of an inevitably assumed artificial bottom boundary in a finite element anlysis.

Effect of Adjacent Foundation

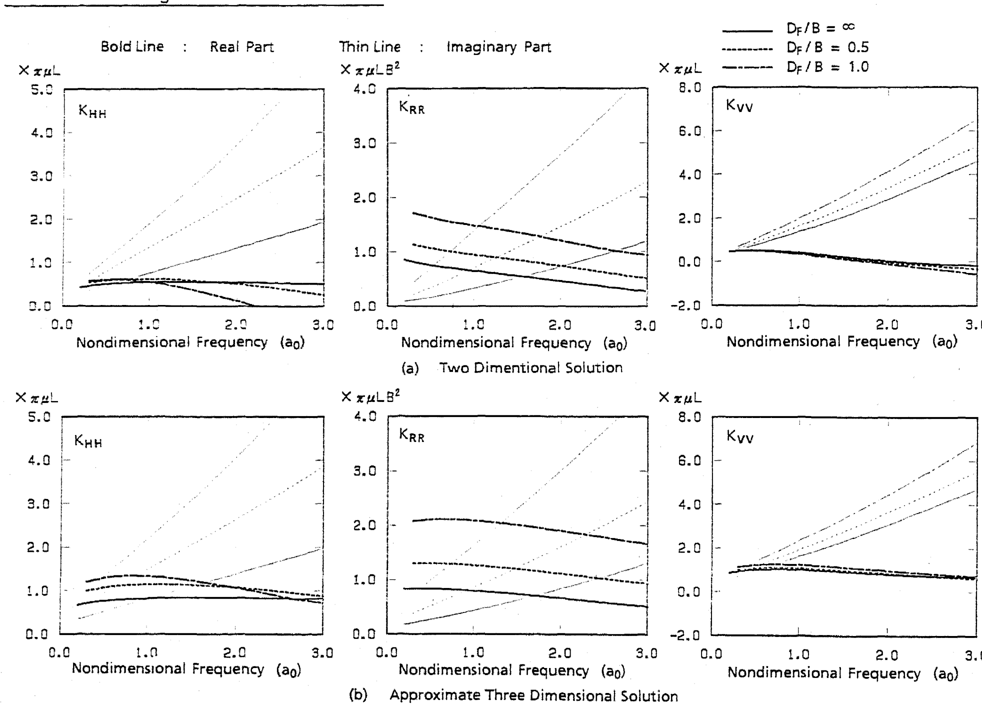
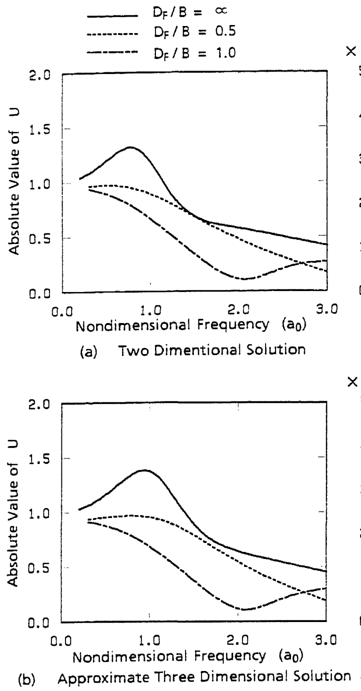
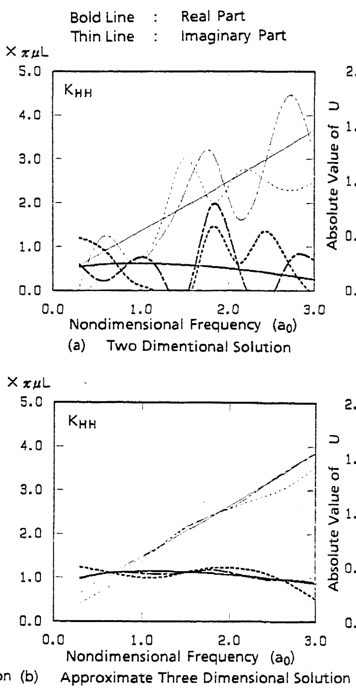


Fig. 6 Effect of Foundation Embedment on the Impedance Function





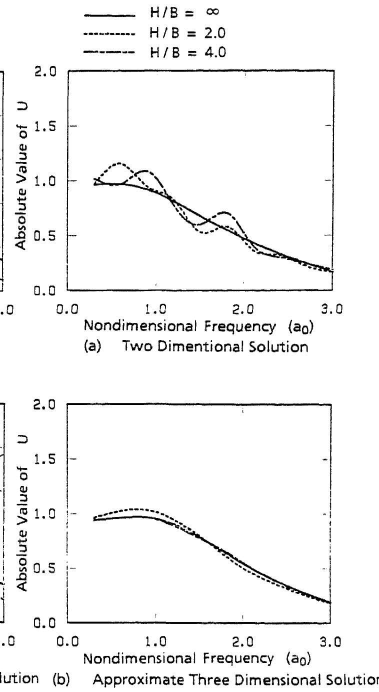


Fig. 7 Effect of Embedment (Transfer Function)

Fig. 8 Effect of Bed Rock (Impedance)

Fig. 9 Effect of Bed Rock (Transfer Function)

Attention is finally turn to the coupling effect due to the existence of an adjacent foundation. Two identical rigid foundations embedded in a half-space subjected to a vertically incident SV wave are considered. Diagonal terms of the impedance matrix are presented in Fig. 10. The response of foundations due to a vertically incident SV wave coincides with each other and is illustrated in Fig. 11. The fact that the effect of an adjacent foundation is prominant on the real part of the horizontal impedance and the imaginary part of the vertical impedance implies the great contribution of lateral soils to these components. Another remarkable feature observed is a strong fluctuation due to the existence of a second foundation and the fluctuation becomes more rapid in conjunction with decreasing amplitudes when the distance between two foundations increases.

CONCLUSIONS

1) The dynamic characteristics of a rigid foundation are fairly improved by adding dashpots to the ground in two dimension.

2) The imaginary part of the impedance grows larger when embedment increases, while the real part remains unchanged except the rotational impedance.

3) The fluctuation of the response due to the existence of an underlying rigid rock or a second structure is dramatically suppressed by adding dashpots.

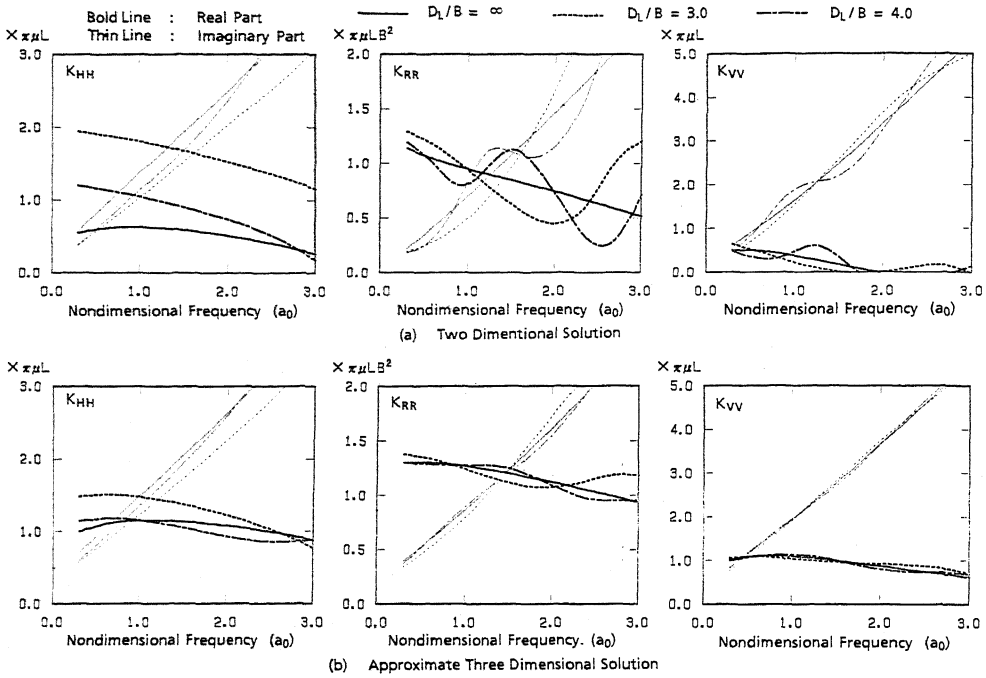


Fig. 10 Effect of an Adjacent Foundation on the Impedance Function

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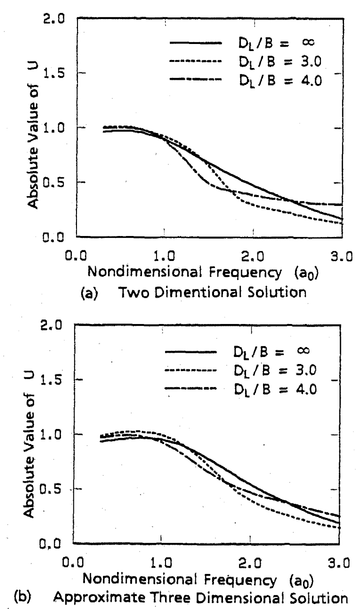

Fig. 11 Effect of Adjacent Foundation (Transfer Function)