# DYNAMIC RESPONSE OF RIGID FOUNDATIONS SUBJECTED TO VARIOUS TYPES OF SEISMIC WAVE

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## SUMMARY

The problem considered is the vibration of rigid massless foundations completely bonded to the surface of an elastic half-space. The authors attempt to evaluate not only the impedance functions but also the foundation input motions induced by various types of seismic wave using the Boundary Element Method. The effects of the existence of the adjacent foundation on the impedances and the input motions for seismic waves with various angles of incidence are mainly discussed. The results obtained indicate that the effects of the azimuth angle of incoming waves cannot be neglected.

## INTRODUCTION

An optimal aseismic design of structures requires a detailed understanding of the interaction effects between structures and the soil. The analyses of the dynamic Soil-Structure Interaction effects involve two different problems, i.e., the foundation excitation problem and the wave scattering problem. Many studies have been performed on the analyses of the dynamic response of a rigid foundation rested on an elastic half-space (Ref.1) and several studies have dealt with so-called Cross-Interaction problem of two foundations (Refs.2,3). While the main object of those studies is to determine the complex impedance functions corresponding to the foundation excitation problem, comparatively few studies (Ref.4) have dealt with the foundation input motions corresponding to the wave scattering problem. And the effects of the direction of incoming waves on the input motions still remain uncertain.

In this paper, the authors attempt to evaluate both the impedance functions of rigid massless foundations and the foundation input motions induced by the incident SH, SV, and Rayleigh waves by the use of the Boundary Element Method, which is suitable for the analyses of infinite media (Refs. 5,6). The method of analysis employed here is similar to the previous work (Ref.7). The discussion are focused on the impedance functions of two adjacent foundations and on the input motions for different seismic waves with various angles of incidence and azimuth.

## METHOD OF ANALYSIS

Considering the steady-state problem with time factor exp(iωt), the governing equation of an elastic, homogeneous, and isotropic medium is expressed in a Cartesian coordinate system $(X_1,X_2,X_3)$ with the Einstein summation convention for indices as follows:

$$\left(\lambda^* + \mu^*\right)u_{j,ji} + \mu^* u_{i,jj} + \rho\omega^2 u_i = 0 \tag{1}$$

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where $\lambda^*=\lambda(1+2hi)$ and $\mu^*=\mu(1+2hi)$ are the complex Lamé's constants ($h$ is a hysteretic damping facter), $\rho$ is the density of the medium, $\omega$ is a circular frequency, and $\mathbf{u}_i$ is the displacement vector. The basic formulation of BEM can be written as (Ref.6):

$$[c]^i\{u\}^i+\int_\Gamma [P^*]\{u\}\,d\Gamma-\int_\Gamma [U^*]\{p\}\,d\Gamma=\{w\}^i \tag{2}$$

where $[c]^i$ is a matrix which has constant values determined by the boundary configuration at the point $i$, $\{u\}^i$ is the displacement vector at the point $i$, $[U^*]$ and $[P^*]$ are the displacement and the traction matrices respectively due to a unit point load at the point $i$, $\{u\}$ and $\{p\}$ are the boundary value vectors of the displacement and the traction, respectively, $\Gamma$ is the integral domain, and $\{w\}^i$, which is named here "External Displacement Field", is necessary for the analyses of wave scattering problem.

By discretizing the integral domain into N elements, equation (2) is transformed into the following form for each point $i$ under consideration,

$$[c]^i\{u\}^i+\sum_{j=1}^{N}\left[\int_{\Gamma_j}[P^*][\Phi]^T d\Gamma\right]\{u\}_j-\sum_{j=1}^{N}\left[\int_{\Gamma_j}[U^*][\Phi]^T d\Gamma\right]\{p\}_j=\{w\}^i \tag{3}$$

where $\{u\}_j$ and $\{p\}_j$ are the displacement and the traction vectors of $j$-th element, respectively, and $[\Phi]^T$ is the interpolation function matrix. In case of using constant elements, $[\Phi]^T$ is equal to a unit matrix [I] . The whole set of equations for each point under consideration can be expressed in the matrix form as,

$$[H]\{u\}-[G]\{p\}=\{w\} \tag{4}$$

So far as the case of rigid foundations rested on an elastic half-space, the Green's functions only for the surface loading are necessary (Ref.7). Since these Green's functions satisfy the boundary condition on the soil surface, the integral domain in equation (2) is only the interface between the soil and the foundations.

In this paper, the impedance and foundation input motion of rigid foundations are calculated. Considering the matrix $[T]$ that transforms the displacement vector $\{U\}$ of a rigid foundation into the translational displacement vector $\{u\}$ , the relationship between the force $\{P\}$ and the displacement $\{U\}$ with respect to a rigid foundation is given by

$$\{P\}=[T]^T[A][G]^{-1}\Big([H][T]\{U\}-\{w\}\Big) \tag{5}$$

where $[A]$ is a diagonal matrix, the term of which represents the area governed by $i$-th element. In this equation, $\{w\}$ is set to be $\{0\}$ in the impedance problem, while in the foundation input motion problem, $\{P\}$ is assumed to be $\{0\}$ and $\{w\}$ takes the value of the free field motions due to incident and reflected wave at each element. The impedance matrix $[K]$ and the input motion vector $\{U_f\}$ are expressed as follows:

$$[K]=[T]^T[A][G]^{-1}[H][T] \tag{6}$$

$$\{U_f\}=[K]^{-1}[T]^T[A][G]^{-1}\{w\} \tag{7}$$

NUMERICAL RESULTS

In this paper, the problems of two square foundations with equal size are mainly discussed as illustrated in Fig.1. Numerical calculations are performed for various values of incidence angle $\theta$, azimuth angle $\phi$ and distance L between two foundations, and for various types of incoming seismic wave. Some of these results are presented in Figs. 2 to 6. In these figures, the hysteretic damping factor $h$ and the Poisson's ratio $\nu$ of an elastic half-space are set to be 0.02 and 1/3, respectively.

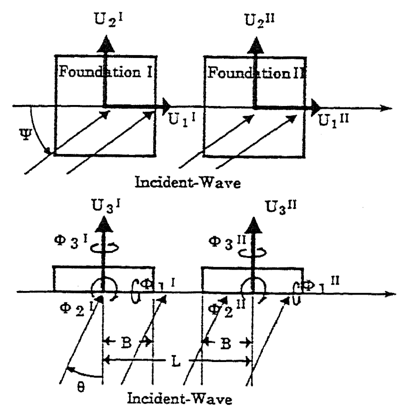


Fig.1 Model and Coordinate

Fig.2 shows the several components of the impedance functions for various values of the relative distance L/B between two foundations. In these figures, bold lines correspond to the

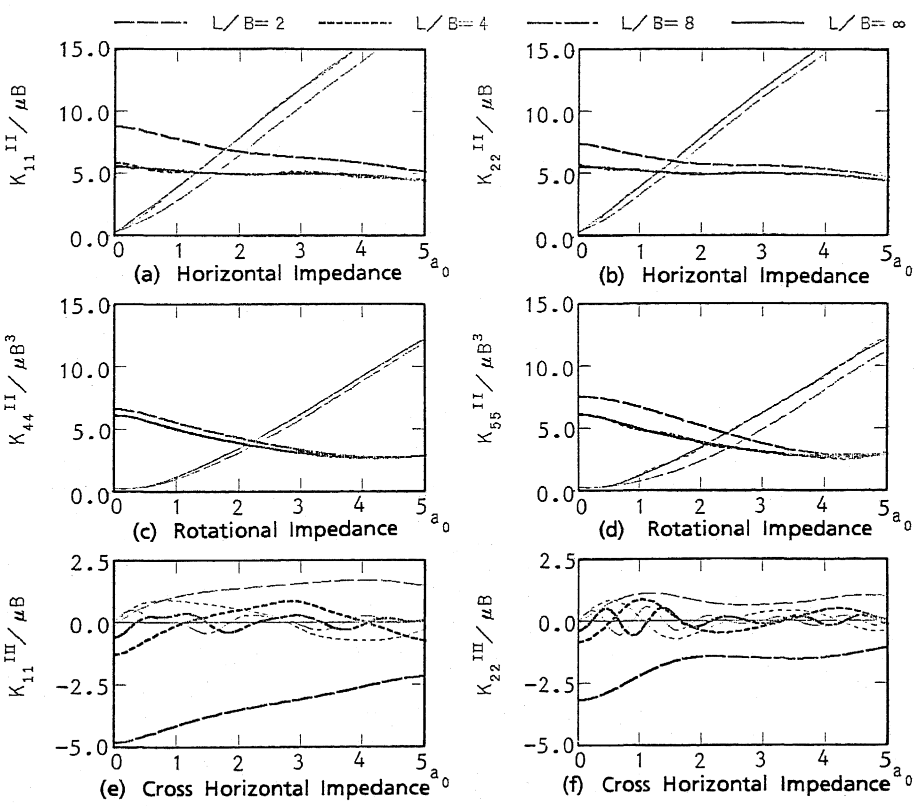


Fig.2 Impedance Functions of Two Foundations for Various Values of L/B

real part of the impedance and thin lines correspond to the imaginary part, which are plotted against the nondimensional frequency $a_0$ ($=\omega B/V_S$). The impedance functions are divided by $\mu B$ for translation and $\mu B^3$ for rotation to be expressed in the nondimensional representation. The impedances of a single foundation, in this case corresponding to L/B= ∞, are also presented in these figures except for Figs. 2 (e) and (f). It may be seen from Fig.2 that $K_{11}^{II}$ and $K_{55}^{II}$ are more affected by the adjacent foundation than $K_{22}^{II}$ and $K_{44}^{II}$. As the relative distance is greater, the period of fluctuation becomes shorter and the impedance functions converge to those of the single foundation. Therefore, the effects of the adjacent foundation may be negligible especially in the diagonal terms of the impedance matrix when L/B ≧ 4.0 . It should be concluded that the components of the impedance on the lateral direction of two foundations ($K_{11}^{II}$, $K_{11}^{III}$) are more influential than on the parallel direction ($K_{22}^{II}$, $K_{22}^{III}$), because the mechanism of wave transmission is different.

Fig.3 shows the amplitude characteristics of the foundation input motions due to the incident SH wave with the azimuth angle $\phi=0°$ and the incidence angle $\theta=30°$ for relative distance L/B=2.0, 4.0, 8.0 and ∞ . The values are nondimensionalized by the horizontal component of the free field displacement $2U_0$ at the center point of each foundation. Furthermore, three rotational components $\Phi_i$ ($i$=1,2,3) are multiplied by the half width B of the foundation. In this paper, the authors call the foundation subjected to incoming waves

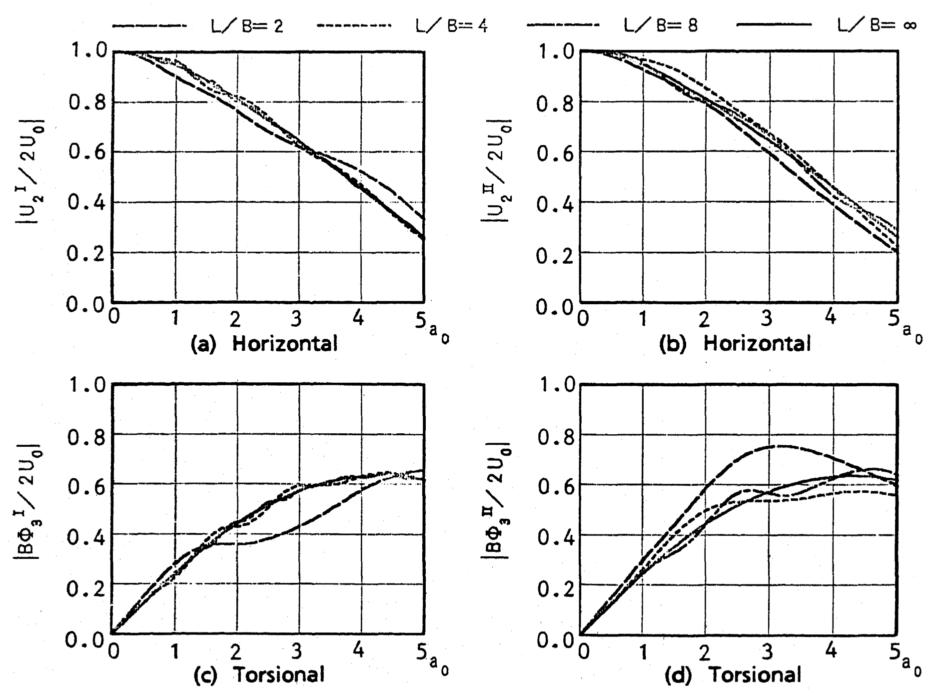


Fig.3 Foundation Input Motions due to SH wave for Various Values of L/B ($\theta$= 30°) ($\phi$= 0°)

first the "front" foundation and the other the "back" foundation. Figures of left side correspond to the front foundation and figures of right side to the back foundation. According as the distance of two foundations is greater, the cross-interaction effects become smaller and the period of the fluctuation of the input motions become shorter. The period may be shorter in the case of the front foundation than in the case of back foundation. From Figs.3 (c) and (d), the amount of the torsional input motion in the back foundation is considerably larger than that in the front foundation.

The foundation input motions due to incident SH waves with the incidence angles θ=30° for the distance L/B=2.0 are presented in Fig.4 for the azimuth

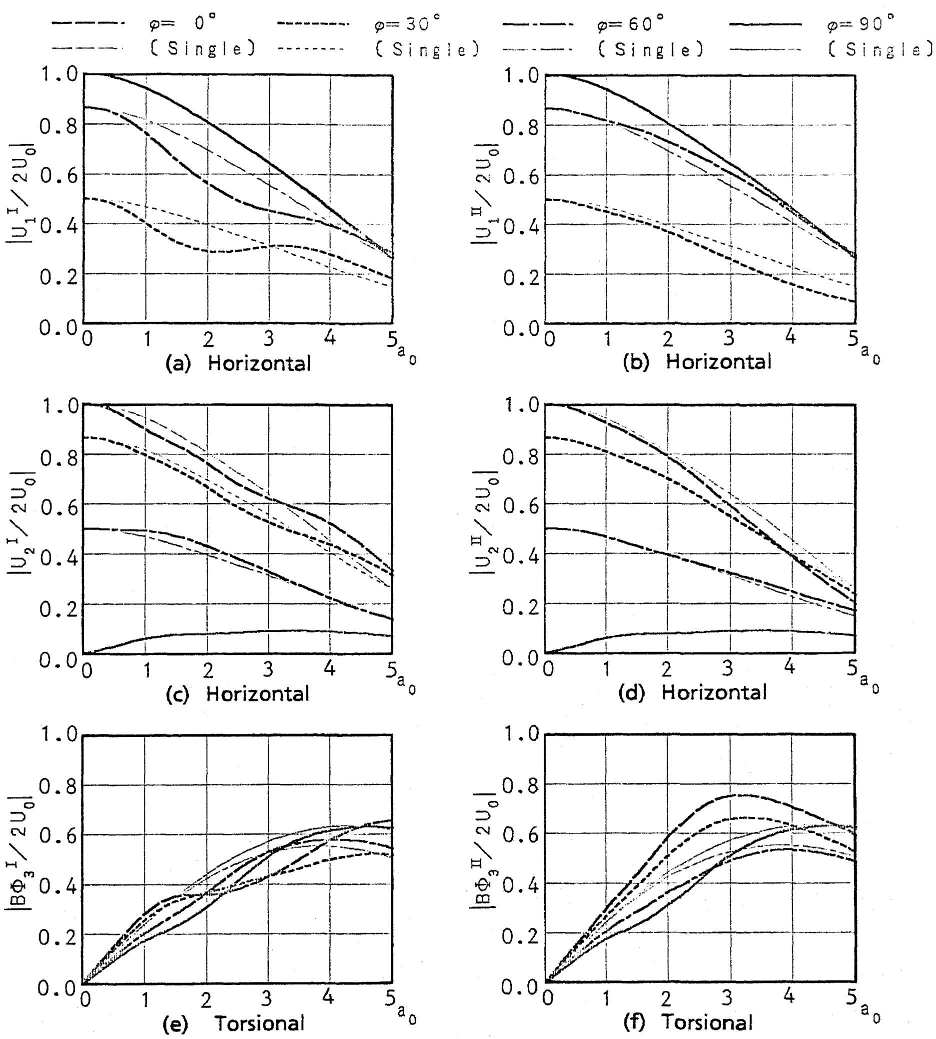


Fig.4 Foundation Input Motions due to SH Waves (L/B=2.0) for Various Values of Azimuth Angle $\phi$ (θ= 30° )

angle $\phi$=0°, 30°, 60° and 90°. In these figures, the results of a single foundation are presented by thin lines for comparison. It may be seen that even in the case of the single foundation, the azimuth angle of incoming seismic wave has a marked effect in the higher frequency domain. In the case of the back foundation, the amount of the torsional input motion is the largest for the azimuth angle $\phi$=0° . As for $U_1$, the fluctuation of the amplitude in the front foundation is larger than that in the back foundation, but its period is shorter. It should be noted that the effect of the existence of the adjacent foundation may be almost negligible for the case of $\phi$=90°.

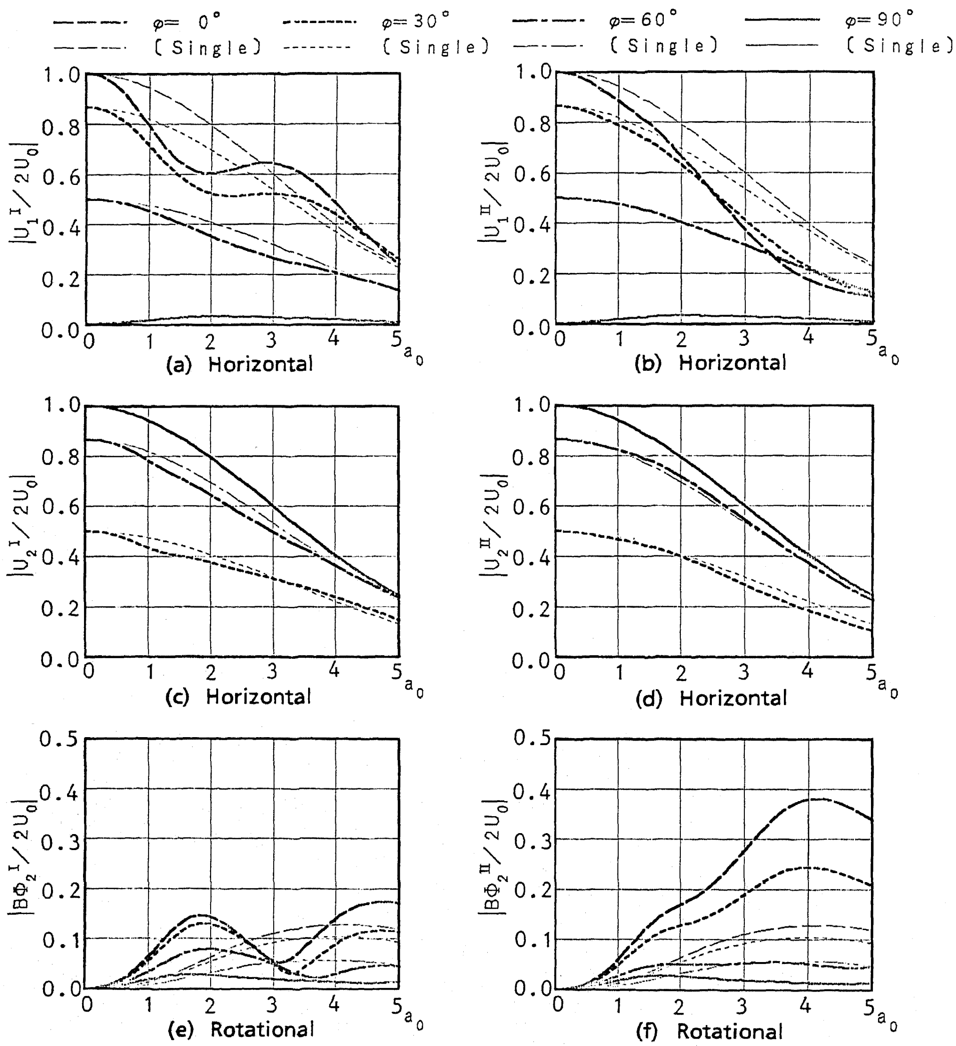


Fig.5 Foundation Input Motions due to SV Waves for Various Values of Azimuth Angle $\phi$ (L/B=2.0) ($\theta$= 30° )

Fig.5 shows the foundation input motions due to incident SV waves with incidence angle θ=30° for the distance L/B=2.0 . The characteristics of the foundation input motions by SV waves are such that the rocking, not the torsion, is generated and that the effect of the cross interaction appears more remarkably for $U_1$ and $\Phi_2$ than for $U_2$ and $\Phi_1$. It may be observed that both the influence of the distance and the period of fluctuation are similar to those for SH waves. But the fluctuations for SV waves are larger than those of SH waves in the front foundation. The rocking $\Phi_2$ with respect to $X_2$-axis in the back foundation is nearly twice as large as that in the front foundation. The effects of azimuth angle φ for SV waves are similar to those

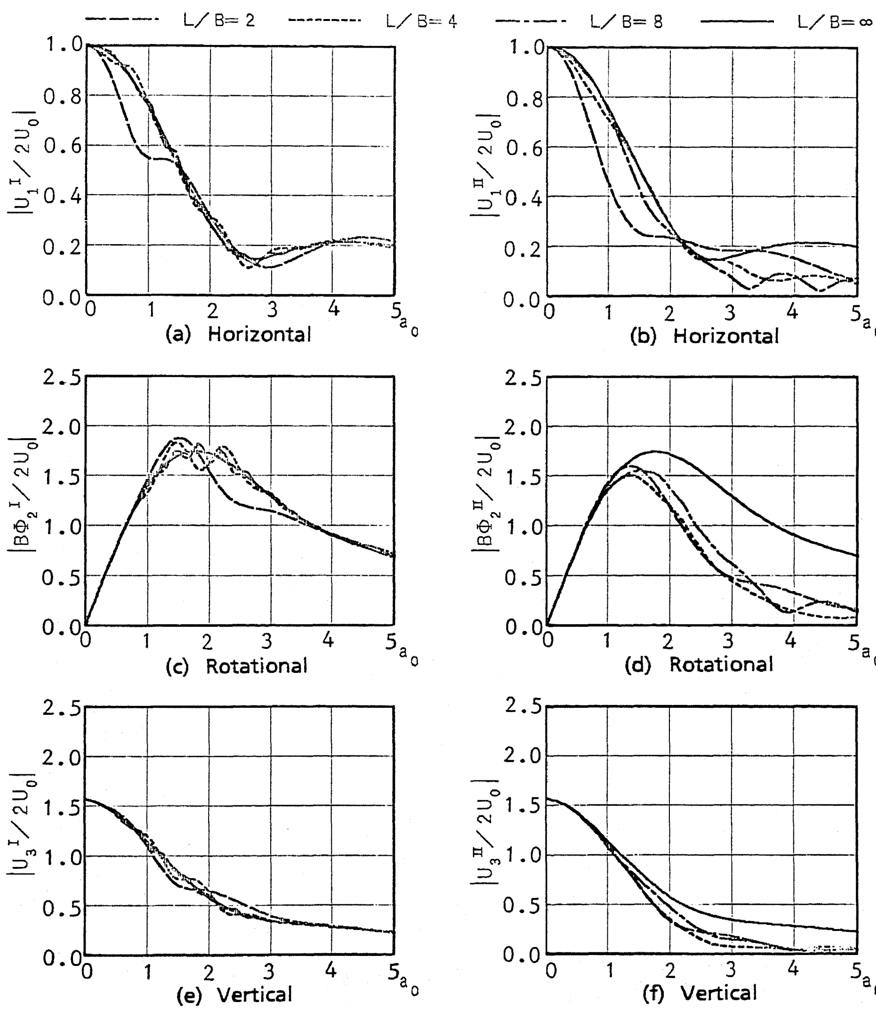


Fig.6 Foundation Input Motions due to Rayleigh Wave for Various Values of L/B (φ= 0°)

for SH waves and the effects of cross interaction for $\phi$=90° are also negligible. In the case of SH and SV waves, the amplitude characteristics of the input motions depend on the incidence angle $\theta$, however, those comments described above are qualitatively valid for another incidence angle except for $\theta$=0°.

The foundation input motions for Rayleigh waves with the azimuth angle $\phi$=0° are shown in Fig.6. The cases for Rayleigh wave do not have the parameter of the incidence angle $\theta$. It is found that large decrease appears in $U_1$ and $\Phi_2$ of the back foundation but that the fluctuations of the amplitude and its period are small in the front foundation in comparison with the back foundation. As a whole, the characteristics of the input motions for Rayleigh wave are similar to those for SV wave with deep incidence angle $\theta$.

## CONCLUSIONS

In this paper, both the impedance functions of rigid massless foundations and the foundation input motions induced by the obliquely incident SH, SV waves and Rayleigh waves are evaluated by three-dimensional Boundary Element Method. The results are summarized as follows. Even in the case of a single foundation, the azimuth angle of incoming waves has a significant effect in the higher frequency domain. In the case of two foundations, the effect of the cross interaction are most remarkable for azimuth angle $\phi$=0° but are negligible for $\phi$=90°. The input motions in the front foundation are considerably different from those in the back foundation except for azimuth angle $\phi$=90°. The effects of the cross interaction are serious in the case of the relative distance L/B= 2.0 but are not serious in the case of $L/B \geqq 4.0$. It cannot be concluded, however, that the effects of the cross interaction for larger value of L/B may be also negligible on the response of super-structures, since the effects of the cross interaction will be amplified on the response of them, especially at the resonant frequency.

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