

PARAMETRIC STUDY ON SOIL-STRUCTURE INTERACTION OF BRIDGES  
WITH SHALLOW FOUNDATIONS

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SUMMARY

The results obtained from a parametric study on soil-structure interaction of continuous girder bridges are described. Their shallow foundations are assumed either to be supported on the surface or to be embedded in a visco-elastic halfspace or a layer built-in at its base. Steady-state response for harmonic excitation and transient response for an artificial time history, both acting in the plane of the bridge, are investigated. An equivalent one-degree-of-freedom system is introduced and approximate equations describing its properties are presented. From these equations simple criteria are derived indicating when soil-structure interaction must be considered.

INTRODUCTION

The seismic analysis of continuous girder bridges is frequently performed neglecting the soil-structure interaction effects. Thus neither the reduction of the natural frequency of the dynamic system nor the radiation of waves to infinity nor the dissipation of energy in the soil by internal friction nor the change of motion at the base are considered. All these effects are more or less present in every soil-structure system. The aim of this study is to indicate when these effects are of practical importance and thus should be considered. In the following, only bridges with shallow foundations are considered. The free-field earthquake excitation is assumed to arise from vertically incident S-waves. The control motion is assumed to act at the free surface of the horizontal site. The influence of topography is thus not taken into account. Only the excitation acting in the plane of the bridge is considered.

SYSTEM AND METHOD OF ANALYSIS

The bridge whose columns can either be hinged or built-in at the girder can be modeled accurately as a three-degree-of-freedom system as shown in Fig. 1. This linear system has a mass  $m$  on the top of the massless column of height  $h$ . The lateral stiffness is  $k$ , the hysteretic damping ratio is  $\zeta$ . The fixed-base natural frequency is denoted as  $f_g$ . The model for a bridge with a built-in column has an additional boundary condition which prevents a rotation at the top of the column (Fig. 1c). This condition can be introduced because the girder is, in general, much stiffer than the column. The resulting model is statically indeterminate. The foundation with the mass  $m_0$  and the rotational inertia  $I_0$  is idealized as a rigid circular plate (of radius  $a$ ) which is embedded with depth  $e$  in a layer built-in at its base. The visco-elastic layer is characterized by the depth  $d$ , the shear-wave velocity  $c_s$ , the mass density  $\rho$ , the Poisson's ratio  $\nu$  and the hysteretic-damping ratio  $\zeta_g$ .  $u$  and  $u_0$  are the amplitudes of the total displacements at the top and at the foundation, respectively.  $\theta$  is the amplitude of the rotation of the foundation. The excitation is specified by the free-field motion of the ground surface with amplitude  $u_f$ . The frequency-dependent dynamic stiffness of the rigid circular foundation embedded in a layer built-in at its base is calculated using the approximate equations of Kausel et al. (Ref.1) in connection with the shear-cone models of

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Meek and Valetsos (Ref.2). The scattered motion for the soil with embedment is determined using the approximate equations of Kausel et al. (Ref.1). The energy dissipation of the soil by internal friction is taken into account using the correspondence principle. It is convenient to introduce the following dimensionless parameters to characterize the bridge-soil system: the mass ratio  $\bar{m} = m/(\rho a^3)$  and the dimensionless stiffness  $\bar{k} = kh^2 3(1-\nu)/(8c_s^2 \rho r^3 (1+2e/a))$ .  $\bar{k}$  represents the ratio between the horizontal static stiffness of the structure fixed at its base and the rocking stiffness of the foundation, non-dimensionalized with  $h^2$ . For the solution reported herein, the mass and rotational inertia of the foundation are considered to be negligible in comparison to that of the structure, if not indicated otherwise. The damping ratios are  $\zeta = .02$  and  $\zeta_g = .05$ ,  $\nu$  equals  $1/3$  and  $\bar{m} = 10$ . The response of the bridge-soil system is determined using the substructure method. The three coupled, second-order linear differential equations of motion are solved in the frequency domain. The response for transient excitation is calculated using the Fast Fourier Transform procedure.

## RESULTS

### Steady-State Response

First, the steady-state response for harmonic excitation is investigated. In Fig. 2, the absolute value of the structural distortion  $|\delta|$  (non-dimensionalized with  $|u_f|$ ) is plotted for  $\bar{k} = 0$  (rigid soil) and  $\bar{k} = 10$  (very soft soil) as a function of the dimensionless excitation frequency  $f/f_s$ . For the case of the built-in column,  $\delta$  refers to the shear force (and not the moment) in the column (Fig. 1c). The interaction effect is negligible for very small and very large ratios of  $f/f_s$ .

To gain further physical insight, an equivalent one-degree-of-freedom system is introduced. Its properties (natural frequency  $\tilde{f}$  and hysteretic-damping ratio  $\tilde{\zeta}$ ) are selected such that essentially the same response results as for the coupled system.  $\tilde{f}$  is defined as the excitation frequency for which the distortion reaches the maximum value  $\delta_{\max}$ , and the hysteretic-damping ratio is defined as  $\tilde{\zeta} = \tilde{f}^2 u_f / (2 \tilde{f}_s^2 \delta_{\max})$ . The distortion  $\delta$  for the equivalent one-degree-of-freedom system is plotted in the same figure as dots. The agreement with the coupled system is excellent, although the mass of the foundation is considered ( $m_o/m = 0.2$ ,  $I_o/mh^2 = 0.05$ ). The values  $\tilde{f}$  and  $\tilde{\zeta}$  capture the main effects of soil-structure interaction very well.

The next figures show the equivalent properties as a function of  $\bar{k}$ . In addition to these "exact" solutions for  $\tilde{f}$  and  $\tilde{\zeta}$  determined with the procedure described before the solutions using the following approximate equations are plotted as dots. For a bridge with a hinged column the following applies (Ref.3):

$$\tilde{f} = \sqrt{\frac{1}{1 + \bar{k} + \frac{(2-\nu)a^2(1+2e/a)}{(1-\nu)h^2(3+2e/a)}}} f_s \quad (1)$$

$$\tilde{\zeta} = \zeta \frac{\tilde{f}^2}{\tilde{f}_s^2} + \zeta_g \left(1 - \frac{\tilde{f}^2}{\tilde{f}_s^2}\right) + \frac{\tilde{f}^3}{\tilde{f}_s^3} \frac{\pi(2-\nu)^2 a^3}{8h^3(3+2e/a)} \sqrt{\frac{\bar{k}(1+2e/a)}{1-\nu}}^3 \sqrt{\frac{2}{3\bar{m}}} \quad (2)$$

The second and third terms in the denominator of Eq.(1) represents the rocking and translational stiffnesses of the foundation, respectively. The latter has a negligible influence on the results for  $h/a > 3$  (which is always the case for bridges) and can thus be omitted. The first term in Eq.(2) represents the con-

tribution of the structure's damping, the second that of the hysteretic damping of the soil and the third that of the radiation damping for the translation of the foundation (which is negligible for  $h/a > 5$  and can be disregarded). For the bridge with a built-in column the corresponding equations are (Ref.4):

$$\tilde{f} = \sqrt{\frac{1}{4 - \frac{36}{12 + \bar{k}} + \bar{k} \frac{(2-\nu)a^2(1+2e/a)}{(1-\nu)h^2(3+2e/a)}}} f_s \quad (3)$$

$$\tilde{\zeta} = \zeta + \frac{\tilde{f}^2}{f_s^2} \left[ \bar{k} \frac{(2-\nu)a^2(1+2e/a)}{(1-\nu)h^2(3+2e/a)} (\zeta_g + \frac{\pi \tilde{f}(2-\nu)a}{8f_s h} \sqrt{\frac{2\bar{k}(1+2e/a)}{3\bar{m}(1-\nu)}} - \zeta) + \frac{36\bar{k}}{(12+\bar{k})^2} (\zeta_g - \zeta) \right] \quad (4)$$

The influence of the translational stiffness of the soil is no more negligible. The first and the second terms in the denominator of Eq.(3) represent the coupled stiffnesses of the structure and of the foundation for rocking. The third term represents the translational stiffness of the foundation. The Eqs. (2) and (4) do not consider the radiation damping for rocking although this degree of freedom is more important than the translational one. This applies as the radiation damping for rocking is very small for small values of the dimensionless frequency  $a_0 = 2\pi fa/c_s$  (Ref.2). In figure 3,  $\tilde{f}/f_s$  and  $\tilde{\zeta}$  are plotted as a function of  $\bar{k}$  for a bridge with a hinged column resting on the surface of a halfspace. As expected, the fundamental frequency will decrease and the damping ratio will increase for decreasing  $c_s$  of the soil ( $\bar{k}$  increases). The asymptotic value for  $\tilde{\zeta}$  for very soft soils is very close to the hysteretic damping ratio of the soil. This means that practically no radiation of waves occurs. For such a soft soil the whole motion is caused by the rocking of the foundation. The corresponding dimensionless parameter  $a_0$  is very small ( $a_0 < 0.2$ ), for which the radiation damping for rocking is negligible (Ref.2). Figure 3 also shows the displacement amplitude  $\delta_f$  of the girder relative to the free field ( $\delta_f = u - u_f$ ), normalized with  $u_f$ .  $\delta_f$  is approximately equal to the relative motion of the girder with respect to the abutment. This value is important to determine the required gap between these two parts to prevent an impact. The softer the soil is, the more  $|\delta_f|/|u_f|$  decreases. But this does not mean that the gap can be designed in a conservative manner neglecting soil-structure interaction, as will be shown later. The results for the bridge with a built-in column are quite different (Fig. 4). The rotational motion of the foundation does not dominate any more. Because of the redistribution of the internal forces the translational motion becomes more and more important and cannot be neglected. This also applies to the associated radiation damping. A pronounced dependence of the results on the slenderness ratio  $h/a$  thus results in contrast to the bridge with a hinged column. A comparison of the results of Figs. 3 and 4 shows that a bridge with a hinged column is affected more by soil-structure interaction than one with a built-in column. When substituting the halfspace by a layer built-in at its base, no radiation of waves will occur for excitation frequencies below the fundamental frequency of the layer (Ref.1). The increase of the static rotational and translational stiffnesses of the foundation is also considered but it is, in general, small. For this reason, the results for a bridge with a hinged column will hardly change when a layer is present, as already for the halfspace only a small radiation damping is present. But for a bridge with a built-in column the results are again

quite different (Fig. 5 compared to Fig. 4). The ratio  $\bar{f}/f_s$  hardly depends on the site, in contrast to the damping ratio  $\tilde{\zeta}$  and to the relative displacement  $|\delta_f|/|u_f|$  which are affected strongly by radiation of waves. The embedment of the foundation leads, above all, to an increase of the stiffness of the soil. Therefore soil-structure interaction will be reduced. This effect is showed in Fig. 6, where the results are plotted for a bridge with a built-in column and the foundation is embedded in the halfspace ( $e/a = 1$ ). The consideration of the mass and the rotational inertia of the foundation, as well as the mass ratio  $\bar{m}$ , hardly influence the results (not shown). The agreement between the "exact" solution and that obtained using the approximate equations is excellent.

#### Earthquake Response

For the transient excitation of the bridges an artificial time history normalized to  $1\text{m/s}^2$  peak acceleration is used. The corresponding (site-independent) response spectra of the acceleration and the displacement as well as the corresponding design spectra of the US NRC (Regulatory Guide 1.60) are plotted in Fig. 7. The time histories of the total displacements and of the distortion for a bridge with a hinged column resting on a halfspace are shown in Fig. 8 for  $\bar{k} = 0$  and  $\bar{k} = 10$ . The distortion diminishes dramatically for soft soils. The translational motion of the foundation is hardly affected by soil-structure interaction. The results for a bridge with a built-in column plotted in Fig. 9 are completely different. Soil-structure interaction also modifies the translational motion of the foundation. The distortion does not decrease as much as for the bridge with a hinged column. In figure 10, the peak distortion  $\delta_{\max}$  and the peak relative displacement  $\delta_f \max$  are plotted for different fundamental frequencies of the fixed-base structure as a function of the dimensionless stiffness  $\bar{k}$  for a bridge with a hinged column lying on the surface of a halfspace. As expected, soil-structure interaction leads to a strong reduction in all cases. This is caused by the increase of the damping ratio  $\tilde{\zeta}$  and by the reduction of the fundamental frequency  $\bar{f}$  of the system, which leads to a reduced spectral acceleration (see Fig. 7a). On the other hand, the displacement  $\delta_f \max$  of the girder relative to that of the abutment increases. The decrease obtained for harmonic excitation (Fig. 3) is even overcompensated by the increase of the spectral displacement due to the reduction of the frequency (Fig. 7b), so that finally an increase results. The same results shown in Fig. 11 for a bridge with a built-in column are not affected so strongly. This confirms the statement made in connection with harmonic excitation. For the bridge with a hinged column the internal forces can be determined directly using the calculated distortion. This is not possible for a bridge with a built-in column. The distortion, as defined in Fig. 1c, can be directly used for the computation of the shear forces in the column. The bending moments at the ends of the column (normalized with  $m \cdot h$ ) are plotted in Fig. 12. The moment at the top is generally larger than that at the bottom because of the redistribution of the internal forces due to the introduction of the soil spring for rocking motion. It can be shown, that for an earthquake which is compatible with the design spectrum mentioned before, soil-structure interaction always leads to a reduction of the internal forces (even for the bending moment at the top).

#### CRITERIA FOR CONSIDERATION OF SOIL-STRUCTURE INTERACTION

The approximate equations presented above can be used to derive simple criteria. The assumption that a reduction of the fundamental frequency of the system of 5% when compared to that of the built-in bridge is tolerated leads

to the following criteria: soil-structure interaction must be considered if

$$c_s^2 \rho (1+2e/a)/(1-\nu) \leq 3.5 kh^2/a^3 \quad \text{for hinged column} \quad (5)$$

$$c_s^2 \rho (1+2e/a)/(1-\nu) \leq 0.8 kh^2/a^3 \quad \text{for built-in column} \quad (6)$$

This leads for an earthquake which is compatible with the design spectrum discussed above to a maximum error of about 10% in the shear force of the column. The maximum error for the relative displacement between the girder and the abutment will be about 5%. The results, the approximate equations and the criteria explained above are also valid for bridges with several columns, if they have the same type of connection (hinged or built-in) with the girder (columns with sliding bearings along the bridge axis are not considered), if they have approximately the same dimensions and if the soil conditions are uniform. In this case,  $k$  denotes the total horizontal stiffness of the fixed-base bridge divided by the number of columns.

#### CONCLUSIONS

- 1) The natural frequency  $\tilde{f}$  and the hysteretic-damping ratio  $\tilde{\zeta}$  of the bridge-soil system are representative quantities to judge soil-structure interaction effects.
- 2)  $\tilde{f}$  will always be smaller than the fixed-base fundamental frequency, while  $\tilde{\zeta}$  will be larger than the damping ratio of the fixed-base bridge (for  $\zeta_g > \zeta$ , which is usually satisfied).
- 3) These two effects lead to a distortion and internal forces in the column which are smaller than those computed neglecting soil-structure interaction effects. On the other hand, the displacement of the bridge-girder relative to the abutment,  $\delta_f$ , increases considering the interaction.
- 4) Bridges with hinged columns are affected more by the soil-structure interaction than those with built-in columns.
- 5) Simple approximate equations for the calculation of  $\tilde{f}$  and  $\tilde{\zeta}$  are presented which are accurate.
- 6) Simple criteria can be derived from these equations, indicating when soil-structure interaction must be considered.

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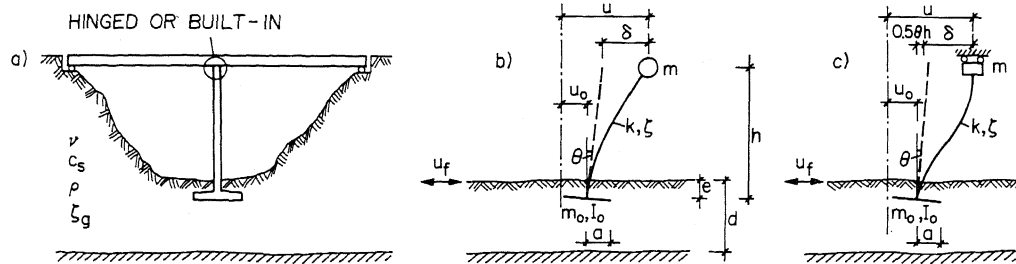


Fig. 1 a) Bridge with hinged or built-in column b) Model for bridge with hinged column c) Model for bridge with built-in column

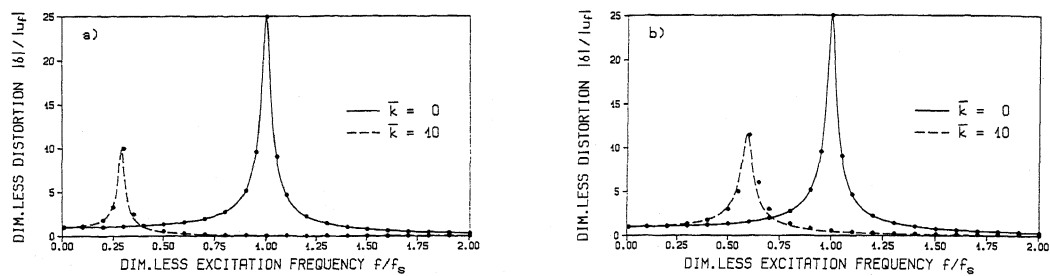


Fig. 2 Frequency-response curve for distortion, halfspace, no embedment,  $m_o=0.2m$ ,  $I_o=0.05mh^2$  a) Bridge with hinged column b) Bridge with built-in column

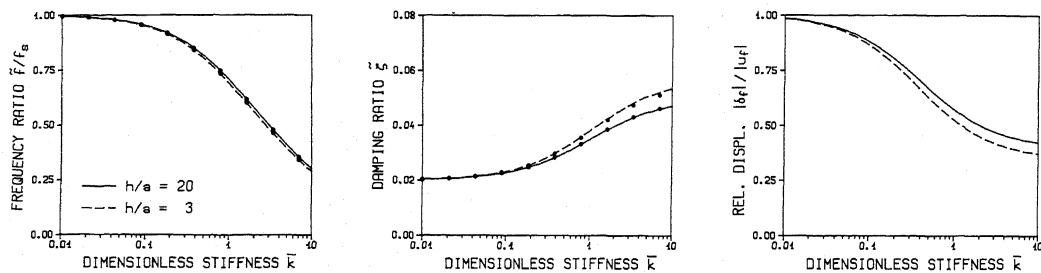


Fig. 3 Equivalent properties for bridge with hinged column, halfspace, no embedment,  $m_o=I_o=0$

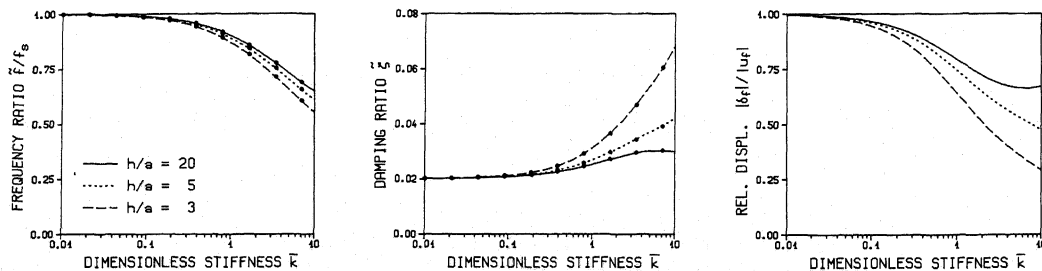


Fig. 4 Equivalent properties for bridge with built-in column, halfspace, no embedment,  $m_o=I_o=0$

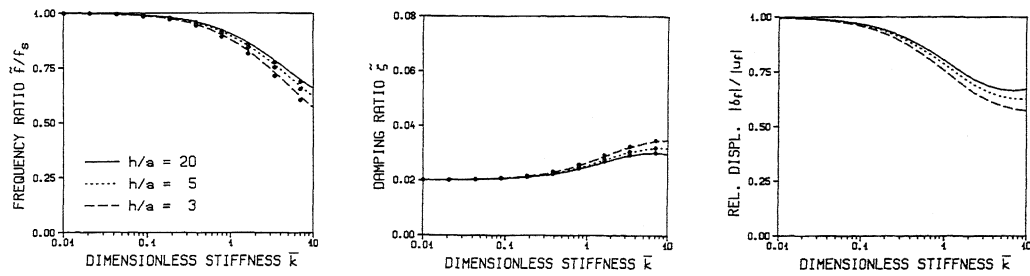


Fig. 5 Equivalent properties for bridge with built-in column, layer built-in  $d/a=2$ , no embedment,  $m_o=I_o=0$

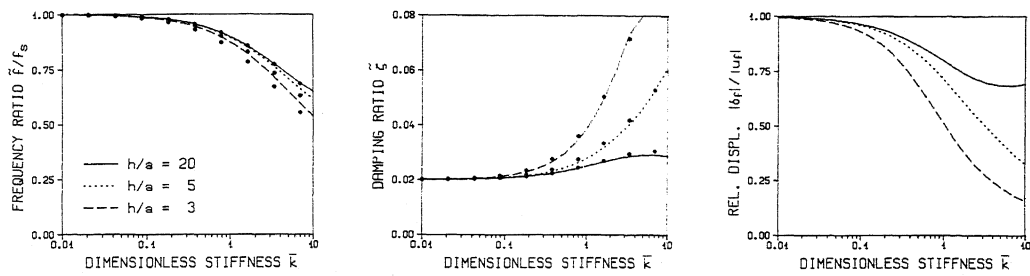


Fig. 6 Equivalent properties for bridge with built-in column, halfspace, embedded  $e/a=1$ ,  $m_o=I_o=0$

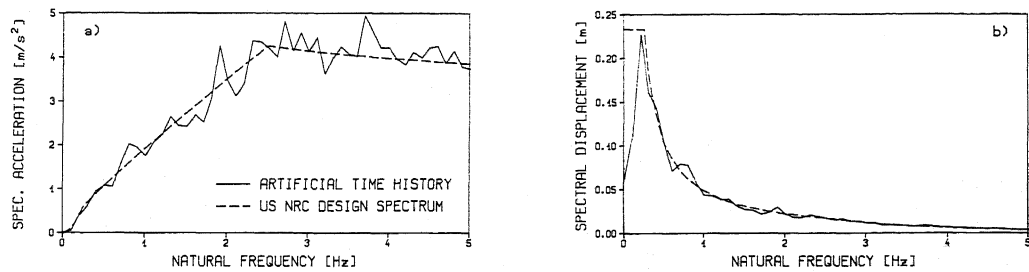


Fig. 7 US NRC design spectra and response spectra of artificial time history normalized with  $1 \text{ m/s}^2$ , damping ratio 0.02

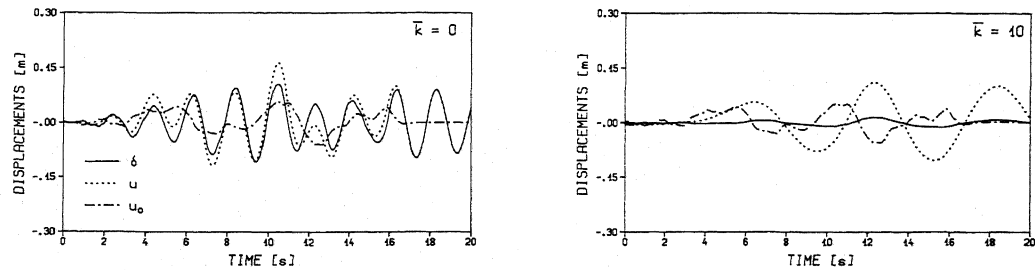


Fig. 8 Time history of total displacements and distortion for bridge with hinged column, halfspace, no embedment,  $m_o=I_o=0$

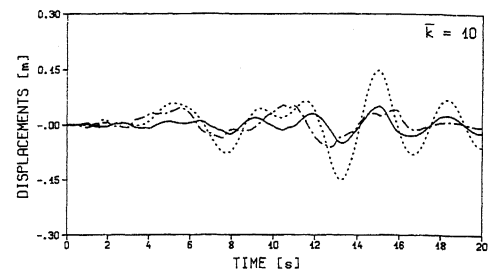
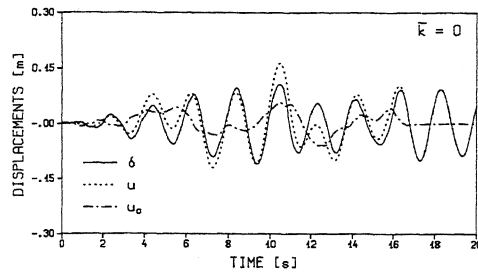


Fig. 9 Time history of total displacements and distortion for bridge with built-in column, halfspace, no embedment,  $m_o = I_o = 0$

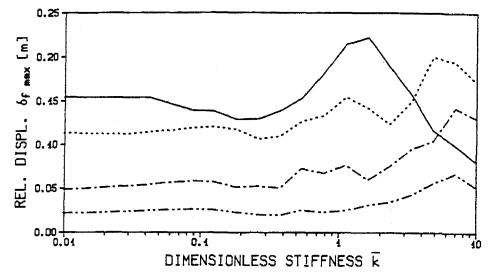
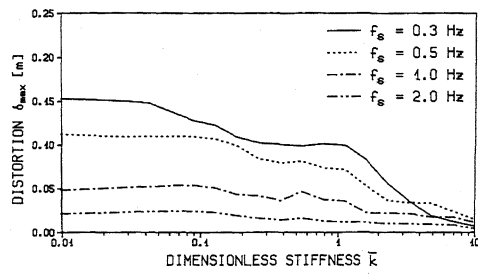


Fig. 10 Maximum distortion and relative displacement for earthquake excitation, bridge with hinged column, halfspace, no embedment,  $m_o = I_o = 0$

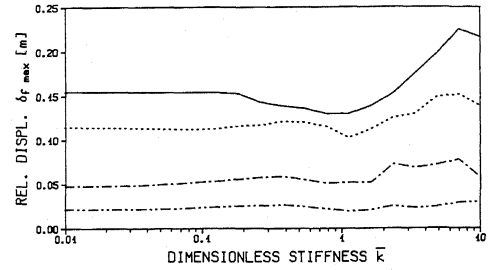
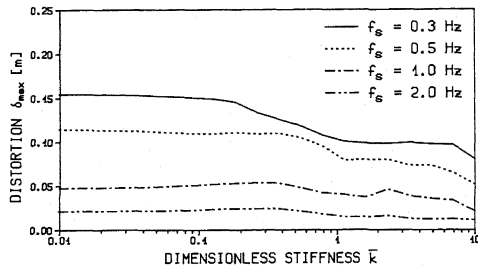


Fig. 11 Maximum distortion and relative displacement for earthquake excitation, bridge with built-in column, halfspace, no embedment,  $m_o = I_o = 0$

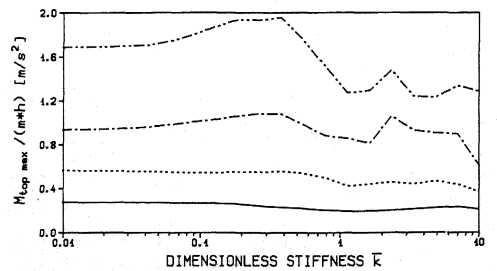
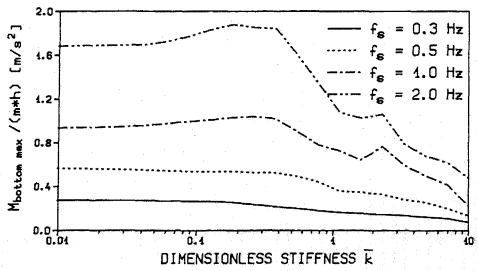


Fig. 12 Maximum bending moments at ends of column for earthquake excitation, bridge with built-in column, halfspace, no embedment,  $m_o = I_o = 0$