ANALYSIS OF DYNAMIC BEHAVIOUR OF STRUCTURE
INCLUDING THE EFFECTS OF AN ADJACENT BUILDING

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SUMMARY

In this paper the dynamic behaviour of the structure-soil-structure system is analysed by GSM. The system equations are also given with due consideration to the relative displacements on the soil-structure interface. The dynamic behaviour will be greatly changed, due to the coupling effects between two buildings, if they stand very close to each other. It is especially true in such a case when a small building is near to a tall one. The changes of dynamic behaviour, however, will rapidly become smaller as the distance between them becomes longer.

INTRODUCTION

Up to now, in many papers dealing with the soil-structure interaction problem, only the coupling effects between one superstructure and the soil is considered, without taking the mutual coupling effects of adjacent buildings into consideration. As a matter of fact, the coupling takes place not only between the building and the soil, but also among the buildings through the underlying soil. In the simple soil-structure system, if the system is only vertically excited, the superstructure will solely vibrate up and down. However, it will not be so simple in the case of the structure-soil-structure system. When the problem is treated as a three dimensional one, the motion of the superstructure will have three components (vertical and horizontal translations and rocking), although the vertical component of displacement remains dominant and the other two are relatively smaller (Ref.1).

In this paper the dynamic behaviour of the structure-soil-structure system is analysed by using the general substructure method (GSM) and two buildings are assumed to be parallel to each other in their long directions. The system equations are presented, including the relative displacements on the soil-structure interface by inserting the so-called joint elements. The whole system is assumed to be linear because of the mathematical difficulties in working out the solution for the nonlinear system in frequency domain.

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MECHANICAL MODELS AND EQUATIONS OF MOTION

The equations of motion for the structure-soil-structure system to be analysed by GSM have been derived by J. A. Gutierrez (Ref. 2), and the mechanical model is shown in Fig. 1, in which the subscripts 1 and 2 refer to the 1st structure and the 2nd structure respectively. The equations of motion can be expressed as

$$\left\{\begin{bmatrix} \underline{\tilde{B}}_{bb1} & \underline{0} \\ \underline{0} & \underline{\tilde{B}}_{bb2} \end{bmatrix} + \begin{bmatrix} \underline{X}^{*}_{f11} & \underline{X}^{*}_{f12} \\ \underline{X}^{*}_{f21} & \underline{X}^{*}_{f22} \end{bmatrix}\right\} \begin{bmatrix} \hat{\underline{z}}_1 \\ \hat{\underline{z}}_2 \end{bmatrix} = -\begin{bmatrix} \underline{D}^{u}_{b1}\hat{\underline{A}}^{u}_{g1} \\ \underline{D}^{u}_{b2}\hat{\underline{A}}^{u}_{g2} \end{bmatrix} - \begin{bmatrix} \underline{D}^{v}_{b1}\hat{\underline{A}}^{v}_{g1} \\ \underline{D}^{v}_{b2}\hat{\underline{A}}^{v}_{g2} \end{bmatrix} \tag{1}$$

The meanings of the notations and super- or subscripts are given in Ref.2.

Eq. 1 shows that the problem of the structure-soil-structure system can be easily solved by using GSM through an equivalent soil-structure system with $(b_1 + b_2)$ degrees of freedom. In Eq. 1, $\underline{\tilde{B}}_{bb}$ is the term of forces acting on the superstructure, including elastic restoring force, damping force and inertia force; the term $\underline{X}_f$ is the contribution of the soil and the right hand of the equation is the loading terms.

If the relative displacements on the interface should be considered, it is necessary to insert the so-called joint elements into the interface as shown in Fig. 2. When the linear relation between shearing stresses and relative displacements is assumed (Ref. 3), the equation of motion of the soil, including the joint elements is as follows

$$\left\{ -\omega^2 \begin{bmatrix} \underline{m}_{ee} & \underline{0} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{m}_{ff1} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{0} & \underline{m}_{ff2} & \underline{0} & \underline{0} \\ \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} \end{bmatrix} + (1+i\eta_e) \begin{bmatrix} \underline{K}_{ee} & \underline{K}_{ef1} & \underline{K}_{ef2} & \underline{0} & \underline{0} \\ \underline{K}_{fe1} & \underline{K}_{ff11} & \underline{K}_{ff12} & \underline{0} & \underline{0} \\ \underline{K}_{fe2} & \underline{K}_{ff12} & \underline{K}_{ff22} & \underline{0} & \underline{0} \\ \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} \end{bmatrix} \right.$$

$$\left. + \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{K}_{L1} & \underline{0} & -\underline{K}_{L1} & \underline{0} \\ \underline{0} & \underline{0} & \underline{K}_{L2} & \underline{0} & -\underline{K}_{L2} \\ \underline{0} & -\underline{K}_{L1} & \underline{0} & \underline{K}_{L1} & \underline{0} \\ \underline{0} & \underline{0} & -\underline{K}_{L2} & \underline{0} & \underline{K}_{L2} \end{bmatrix} \right\} \begin{bmatrix} \hat{\underline{f}}_e \\ \hat{\underline{f}}_{f1} \\ \hat{\underline{f}}_{f2} \\ \hat{\underline{f}}_{L1} \\ \hat{\underline{f}}_{L2} \end{bmatrix} = \begin{bmatrix} \underline{0} \\ \underline{0} \\ \underline{0} \\ \hat{\underline{R}}_{L1} \\ \hat{\underline{R}}_{L2} \end{bmatrix} \tag{2}$$

in which subscript L means the joint elements, and subscripts 1 and 2 still refer to the 1st and the 2nd structures respectively; $\underline{K}_L$ is a diagonal stiffness matrix due to the joint elements, which can be obtained in the assembling process.

By some simple mathematical manipulation Eq. 2 can be split into

$$\underline{K}_{L1}\,(\hat{\underline{f}}_{L1} - \hat{\underline{f}}_{f1}) = \hat{\underline{R}}_{L1}$$

$$\underline{K}_{L2}\,(\hat{\underline{f}}_{L2} - \hat{\underline{f}}_{f2}) = \hat{\underline{R}}_{L2} \tag{3}$$

$$\begin{bmatrix} \underline{X}_{f11} & \underline{X}_{f12} \\ \underline{X}_{f21} & \underline{X}_{f22} \end{bmatrix} \begin{bmatrix} \hat{\underline{f}}_{f1} \\ \hat{\underline{f}}_{f2} \end{bmatrix} + \begin{bmatrix} \underline{K}_{L1}\hat{\underline{f}}_{f1} \\ \underline{K}_{L2}\hat{\underline{f}}_{f2} \end{bmatrix} - \begin{bmatrix} \underline{K}_{L1}\hat{\underline{f}}_{L1} \\ \underline{K}_{L2}\hat{\underline{f}}_{L2} \end{bmatrix} = \underline{0} \tag{4}$$

or

$$\begin{bmatrix} \hat{\underline{f}}_{f1} \\ \hat{\underline{f}}_{f2} \end{bmatrix} = \begin{bmatrix} \underline{J}_{11} & \underline{J}_{12} \\ \underline{J}_{21} & \underline{J}_{22} \end{bmatrix} \begin{bmatrix} \hat{\underline{f}}_{L1} \\ \hat{\underline{f}}_{L2} \end{bmatrix} \tag{5}$$

in which

$$\begin{bmatrix} \underline{J}_{11} & \underline{J}_{12} \\ \underline{J}_{21} & \underline{J}_{22} \end{bmatrix} = \begin{bmatrix} \underline{X}_{f11}+\underline{K}_{L1} & \underline{X}_{f12} \\ \underline{X}_{f21} & \underline{X}_{f22}+\underline{K}_{L2} \end{bmatrix}^{-1} \begin{bmatrix} \underline{K}_{L1} & \underline{0} \\ \underline{0} & \underline{K}_{L2} \end{bmatrix} \tag{6}$$

Substituting Eq. 5 into Eq. 3 leads to

$$\begin{bmatrix} \underline{X}^m_{f11} & \underline{X}^m_{f12} \\ \underline{X}^m_{f21} & \underline{X}^m_{f22} \end{bmatrix} \begin{bmatrix} \hat{\underline{f}}_{L1} \\ \hat{\underline{f}}_{L2} \end{bmatrix} = \begin{bmatrix} \hat{\underline{R}}_{L1} \\ \hat{\underline{R}}_{L2} \end{bmatrix} \tag{7}$$

where

$$\begin{aligned} \underline{X}^m_{f11} &= \underline{K}_{L1}(\underline{I}-\underline{J}_{11}), & \underline{X}^m_{f12} &= -\underline{K}_{L1}\underline{J}_{12} \\ \underline{X}^m_{f21} &= -\underline{K}_{L2}\underline{J}_{21}, & \underline{X}^m_{f22} &= \underline{K}_{L2}(\underline{I}-\underline{J}_{22}) \end{aligned} \tag{8}$$

and

$$\underline{X}^m_f(\omega) = \begin{bmatrix} \underline{X}^m_{f11}(\omega) & \underline{X}^m_{f12}(\omega) \\ \underline{X}^m_{f21}(\omega) & \underline{X}^m_{f22}(\omega) \end{bmatrix} \tag{9}$$

is called the modified impedance matrix of the soil.

By numbering schemes appropriately, the equations of motion of the superstructure can be written as

$$\left\{ -\omega^2 \begin{bmatrix} \underline{m}_{ss} & \underline{0} \\ \underline{0} & \underline{m}_{bb} \end{bmatrix}_1 + (1+i\eta_{s1}) \begin{bmatrix} \underline{K}_{ss} & \underline{K}_{sb} \\ \underline{K}_{bs} & \underline{K}_{bb} \end{bmatrix}_1 \right\} \begin{bmatrix} \hat{\underline{f}}_{s1} \\ \hat{\underline{f}}_{b1} \end{bmatrix} = -\underline{m}_{s1}\underline{1}^u_1\hat{A}^u_{g1} - \underline{m}_{s1}\underline{1}^v_1\hat{A}^v_{g1} + \begin{bmatrix} \underline{0} \\ \hat{\underline{R}}_{b1} \end{bmatrix} \tag{10}$$

and

$$\left\{ -\omega^2 \begin{bmatrix} \underline{m}_{ss} & \underline{0} \\ \underline{0} & \underline{m}_{bb} \end{bmatrix}_2 + (1+i\eta_{s2}) \begin{bmatrix} \underline{K}_{ss} & \underline{K}_{sb} \\ \underline{K}_{bs} & \underline{K}_{bb} \end{bmatrix}_2 \right\} \begin{bmatrix} \hat{\underline{f}}_{s2} \\ \hat{\underline{f}}_{b2} \end{bmatrix} = -\underline{m}_{s2}\underline{1}^u_2\hat{A}^u_{g2} - \underline{m}_{s2}\underline{1}^v_2\hat{A}^v_{g2} + \begin{bmatrix} \underline{0} \\ \hat{\underline{R}}_{b2} \end{bmatrix} \tag{11}$$

where

$$\underline{1}^u = (1\ 0\ 1\ 0\ \ldots)^T, \quad \underline{1}^v = (0\ 1\ 0\ 1\ \ldots)^T \tag{12}$$

$$\underline{m}_s = \begin{bmatrix} \underline{m}_{ss} & \underline{0} \\ \underline{0} & m_{bb} \end{bmatrix} \tag{13}$$

The continuous and equilibrium conditions between the superstructures and the joint elements are

$$\hat{\underline{r}}_{L1} = \hat{\underline{r}}_{b1}\ ,\ \hat{\underline{r}}_{L2} = \hat{\underline{r}}_{b2}\ ,\ \hat{\underline{R}}_{L1} = -\hat{\underline{R}}_{b1}\ ,\ \hat{\underline{R}}_{L2} = -\hat{\underline{R}}_{b2} \tag{14}$$

respectively. Substituting Eq. 14 into Eqs. 10 and 11 and making modal analysis similar to that in Ref. 2, we obtain

$$\left\{ \begin{bmatrix} \widetilde{\underline{B}}_{bb1} & \underline{0} \\ \underline{0} & \widetilde{\underline{B}}_{bb2} \end{bmatrix} + \begin{bmatrix} \underline{X}^{m*}_{f11} & \underline{X}^{m*}_{f12} \\ \underline{X}^{m*}_{f21} & \underline{X}^{m*}_{f22} \end{bmatrix} \right\} \begin{bmatrix} \hat{\underline{Z}}_1 \\ \hat{\underline{Z}}_2 \end{bmatrix} = - \begin{bmatrix} \underline{D}^u_{b1}\hat{A}^u_{g1} \\ \underline{D}^u_{b2}\hat{A}^u_{g2} \end{bmatrix} - \begin{bmatrix} \underline{D}^v_{b1}\hat{A}^v_{g1} \\ \underline{D}^v_{b2}\hat{A}^v_{g2} \end{bmatrix} \tag{15}$$

where

$$\begin{aligned} \underline{X}^{m*}_{f11}(\omega) &= \underline{\psi}^T_1 \underline{X}^m_{f11}(\omega)\underline{\psi}_1 \qquad & \underline{X}^{m*}_{f12}(\omega) &= \underline{\psi}^T_1 \underline{X}^m_{f12}(\omega)\underline{\psi}_2 \\ \underline{X}^{m*}_{f21}(\omega) &= \underline{\psi}^T_2 \underline{X}^m_{f21}(\omega)\underline{\psi}_1 \qquad & \underline{X}^{m*}_{f22}(\omega) &= \underline{\psi}^T_2 \underline{X}^m_{f22}(\omega)\underline{\psi}_2 \end{aligned} \tag{16}$$

and $\underline{\psi}$ is the modal matrix of the baseslab of the superstructure, and the others have the same meanings as those in Eq. 1. Eq. 15 is an extension of Eq. 1, considering the relative displacements on the interface.

NUMERICAL RESUITS

Here two examples of two identical buildings and two different buildings, at different distances, are illustrated, as shown in Fig. 3 and Fig. 4. The superstructures are R.C. frames widely used in China. The values of x are taken to be 3m, 9m, 15m and infinity, respectively*. "x=infinity" implies the soil-structure system. The soil is idealised as an isotropic and viscoelastic half-plane.

The parameters are given in Table 1. For the sake of simplicity the relative displacements on the interface are neglected because their influences on the responses of the superstructure are small (Ref. 3).

The inputs of the system are $\hat{A}^u_{g1}(\omega) = \hat{A}^u_{g2}(\omega) = e^{i\omega t}$ and $\hat{A}^v_{g1}(\omega) = \hat{A}^v_{g2}(\omega) = 0$. $\underline{X}^*_f(\omega)$ are shown in Fig. 5, in which only rigid horizontal translation and rocking of the baseslab are retained. In the figure, subscript 1 refers to horizontal direction and 2, rocking; supercript 1 refers to the 1st structure and 2, the 2nd structure. For example, X^{12}_{f12} means the horizontal force at the baseslab of the 1st structure due to the unity rocking of the baseslab of the 2nd structure. And

$$K_{st}=1.7649\frac{1-\nu}{(1+\nu)(3-4\nu)}E_e \quad \text{(Ref. 4)} \tag{17}$$

is the static stiffness coefficient of the soil. The amplifying factor Y/Y_0 of the top floor is shown in Fig. 6 - Fig. 8.

It can be seen that in the case of two different buildings the dynamic responses of the small one has an interesting jumping in the vicinity of F_{20}. It might be a result due to the dynamic response of the tall one having a resonant peak at F_{20} and the coupling effects of the two buildings through the soil. When the two buildings stand very close to each other, it is possible that the jumping value will be larger than the resonant peak value at F_{10}.

CONCLUSIONS

I. The dynamic behaviour will be greatly changed due to the coupling effects between two buildings, when they stand very close to each other. It is especially true when a small building is near to a tall one. In this case, attention should be paid to that the dynamic response of the small one will have two resonnant peaks at the corresponding resonant frequencies of the soil-structure system.

II. The effects of the adjacent buildings on the structural responses are mainly limited in a narrow region of the resonant frequency. The peak values will be increased for two different buildings and they will be reduced for two identical buildings.

III. The coupling effects between two buildings become smaller rapidly as the distance becomes larger.

Table 1 Parameters in calculation

Superstructure						Soil			
E_c $\frac{Kg}{cm^2}$	η_s	f_{20} Hz	f_{10} Hz	F_{20} Hz	F_{10} Hz	G_e $\frac{Kg}{cm^2}$	η_e	ν	C_s $\frac{m}{sec}$
3×10^5	0.04	1.469	2.106	0.45	0.90	200	0.1	1/3	100

Note: "f" refers to the 1st natural frequencies of the superstructures and "F" refers to the 1st natural frequencies of the corresponding soil-structure systems. The subscripts 10 and 20 refer to 10 storeys and 20 storeys, respectively.

* "x=3m, 9m" is unpractical, just for comparision.

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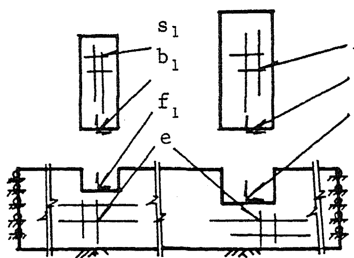

Fig.1 Model of GSM

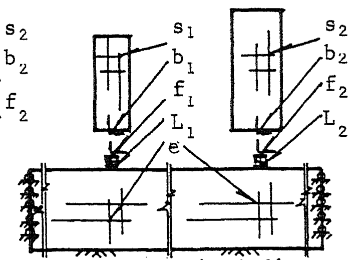

Fig.2 Model including rel. displacements

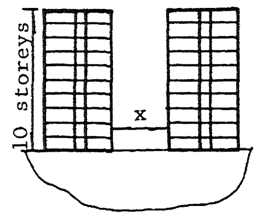

Fig.3 Example 1, case 1

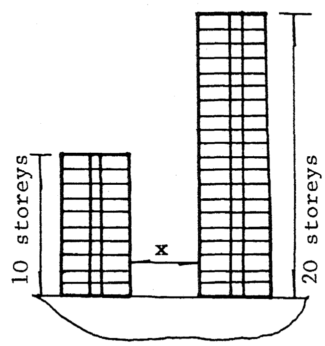

Fig.4 Example 2, case 2.

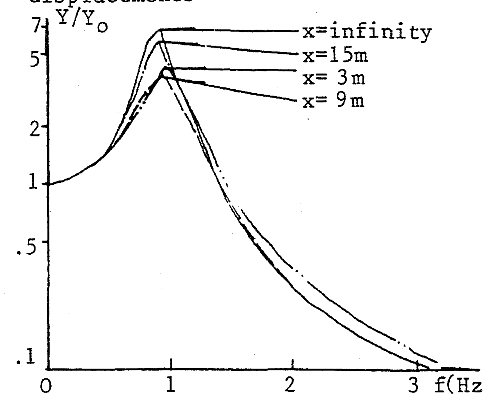

Fig.6 Amplifying factor for case 1.

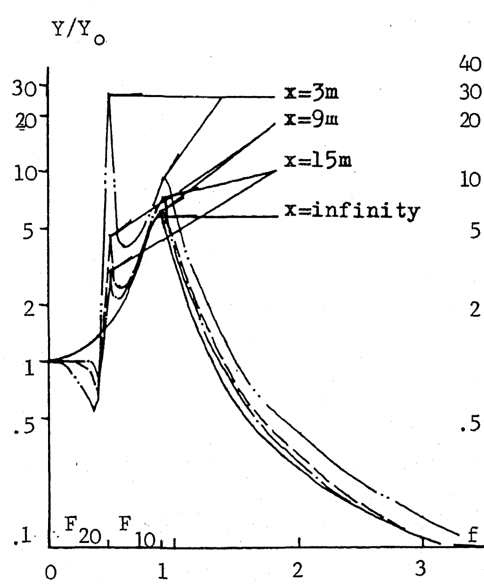

Fig.7 Amplifying factor of 10 storeys for case 2.

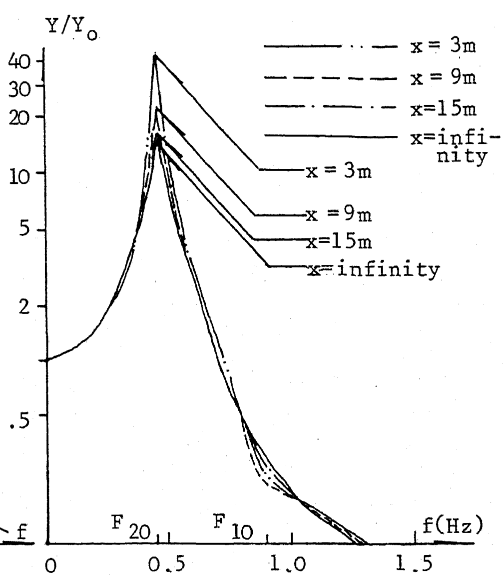

Fig.8 Amplifying factor of 20 storeys for case 2.

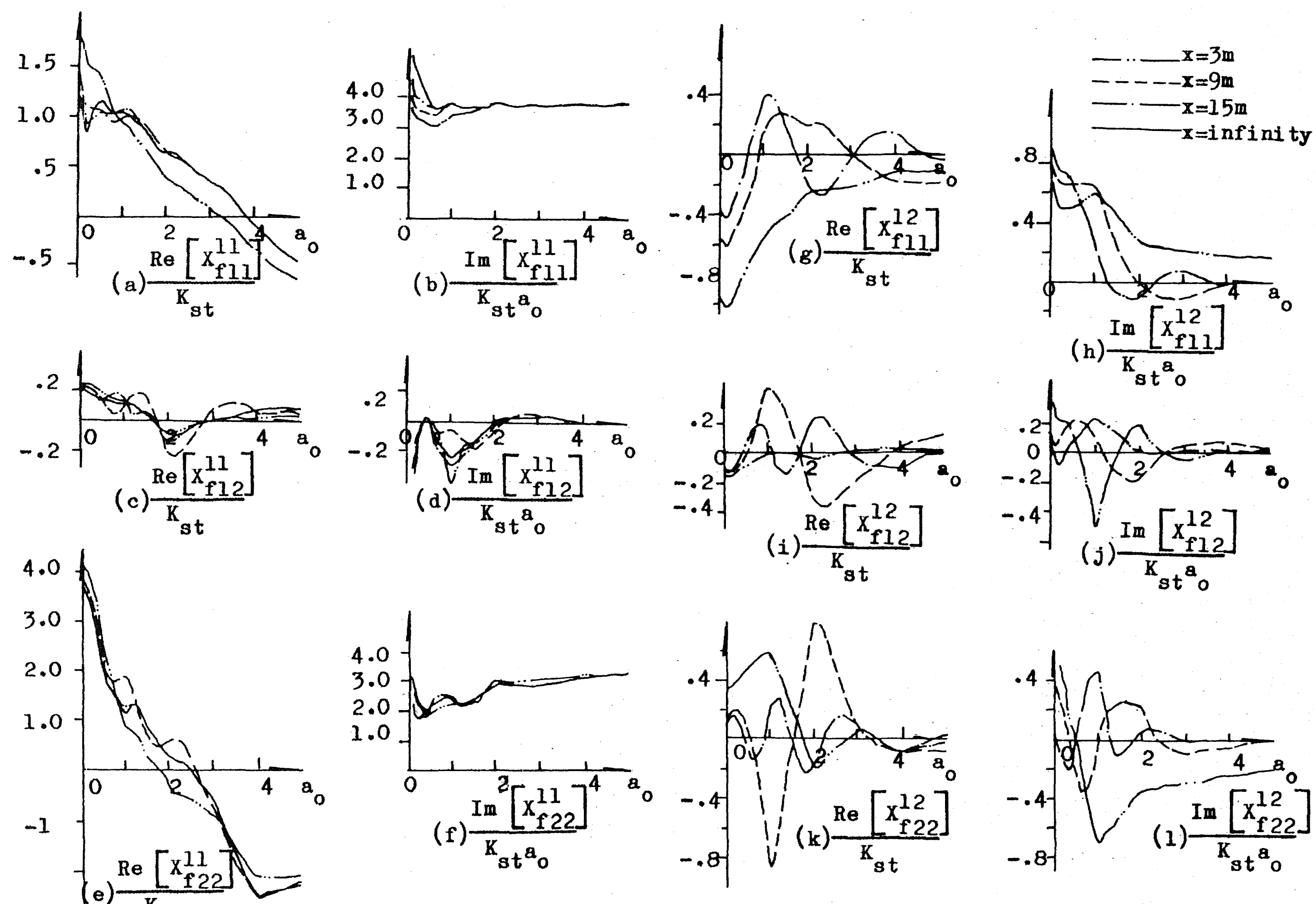

Fig. 5 Impedance matrix of soil $\underline{X}_f^*(\omega)$