# VERTICAL SOIL-STRUCTURE INTERACTION WITH BASE MASS AND WAVE AVERAGING EFFECTS

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### SUMMARY

This paper presents a closed-form solution of the vertical response of an N-mass structure with a large base mass to wave averaging vertical seismic excitation. Response for different soil stiffness and structural characteristics are investigated. Numerical results are obtained by iteration for an idealized three-mass two-mode model of a nuclear power plant containment structure subjected to the El-Centro earthquake. The significance of the interaction effects are evaluated by comparing the free field response spectrum with the output acceleration response spectrum.

#### INTRODUCTION

In most seismic analyses of building structures, the earthquake free-field ground motion is assumed to be identical at all coordinate points at a given level beneath the structure. The foundation averaging phenomenon which is termed as the "tau effect" in the Diablo Canyon Nuclear Power Plant proceedings is often neglected in the seismic analysis of structures. This effect occurs when the ground motion varied over the contact area of the foundation. For some soil conditions when the wave lengths of the dominant frequencies in earthquake motions are comparable to the foundations, this "self-cancelling" effect acting as a filter can be very pronounced and will reduce the response level of the power plant structures. Rocking seismic input created from the vertical component of the propagating waves was not considered.

For the present investigation, the effect of the interaction between the vertical inertia forces and the vertical free-field ground motions which are caused by wave averaging effects are the principal considerations. By a semi-inverse method and applying the normal mode theory a closed-form solution of the vertical response of an N-mass structure with a large base mass to vertical seismic excitation is developed. Flexibility of the foundation is based on the response of the homogeneous elastic half-space to a time varying normal stress distributed over the surface of the boundary (Fig. 1). As a result, the vertical interaction

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equation is expressed in terms of the arbitrary base acceleration, the average effective seismic motions, the properties of the soil which are represented by the P- and S-wave velocities, and the properties of the structure which are described by the modal parameters. In particular, the amount of interaction was shown to be dependent on the modal characteristics of the structure, the ground stiffness and the relationship between the structural fixed-base frequencies and the frequency characteristic of the average ground motion.

## AVERAGE EFFECTIVE VERTICAL SEISMIC MOTIONS

Free field ground excitations used in this study are derived by assuming that the foundation is rigid, with propagating waves traversing the plane of the structural site. This approach tends to average the ground motion forcing the soil particles to displace as a rigid body. Also, by assuming that the seismic ground motion is a repeatable event over the period of time of analysis, the free-field vertical ground acceleration can be expressed in terms of the Fourier series. Specifically

$$\ddot{\mathbf{v}}_{\mathbf{F}}(\mathbf{x}, \mathbf{t}) = \sum_{n=1}^{N} \mathbf{A}_{n} \sin(\mathbf{k}_{n} \mathbf{t} + \mathbf{\theta}_{n} - \frac{\mathbf{k}_{n}}{\mathbf{b}})$$
 (1)

where b is the shear wave velocity, N is the number of harmonics,  $A_n$  are the amplitudes of the earthquake,  $\theta_n$  is the nth component phase angle and  $k_n$  are the circular frequencies.

Due to propagating seismic waves, the average vertical ground acceleration can be determined from

$$\ddot{\mathbf{v}}_{\mathbf{F}})_{\text{ave}} = \frac{1}{\mathbf{A}} \int_{-\mathbf{r}_{0}}^{\mathbf{r}_{0}} \ddot{\mathbf{v}}_{\mathbf{F}}(\mathbf{x}, \mathbf{t}) d\mathbf{A}$$
 (2)

where A is the contact area of the base mass. In terms of the wave number  $R_{\rm n}$ , equation (2) can be written as

$$\ddot{v}_{F}^{\prime}_{ave} = \sum_{n=1}^{N} \frac{2A_{n}}{R_{n}} \sin(k_{n}t + \theta_{n}) \sin \frac{1}{2} R_{n}$$
(3)

where  $R_n=4\pi r_0/\lambda_n$  is a parameter proportional to the ratio of structural x-dimension to the particular wave length which is related to the shear wave velocity by b = 1/2  $k_n\lambda_n$ .

# VERTICAL SOIL-STRUCTURE INTERACTION EQUATION

The vertical acceleration at the center of the foundation derived by Scavuzzo etc. (Ref. 1), includes the interaction effects between vertical inertia loads and vertical free-field ground motions. By using a semi-inverse method with the aid of Laplace transforms, the vertical soil-structure interaction equation for an N-mass structural model can be written as

$$\ddot{v}(t) = -\frac{1}{M_o} \int_{0}^{L} M_j \omega_j \int_{0}^{t} \ddot{v}(\xi) \sin \omega_j(\tau - \xi) d\xi$$

$$-\left(\frac{a}{2\pi r_o M_o}\right) \int_{0}^{L} M_j \omega_j \int_{0}^{t} \int_{0}^{t - \tau} \ddot{v}(\xi) \operatorname{Im}\left[g\left(\frac{b\tau}{r_o}\right)\right] \sin \omega_j(t - \tau - \xi) d\xi d\tau$$

$$-\frac{a}{2\pi r_o} \int_{0}^{t} \ddot{v}(t - \tau) \operatorname{Im}\left[g\left(\frac{b\tau}{r_o}\right)\right] d\tau + \frac{a\mu A}{b^2 M_o} \int_{0}^{t} \left[\ddot{v}_F\right]_{ave} - \ddot{v}(\tau) d\tau$$
(4

where  $M_{\text{O}}$  is the base mass, a is the P-wave velocity, b is the S-wave velocity,  $\mu$  is the shear modulus and  $M_{\text{j}}$  is the effective modal masses which are related to the concentrated masses  $m_{\text{j}}$  by the following expression:

$$\mathbf{M}_{j} = (\sum_{i}^{L} \bar{\mathbf{x}}_{ij}^{m})^{2} / \sum_{i}^{L} \bar{\mathbf{x}}_{ij}^{2} m_{i}, \tag{5}$$

 $\bar{x}_{ij}$  are the vertical mode shapes of the linear elastic N-mass structure. The function  $\text{Im}[g(\tau)]$  in equation (4) is defined as

$$Im[g(T)] = \begin{cases} 0 & 0 \le T < b/a \\ \frac{(\frac{1}{2} - T^2)^2 \sqrt{T^2 - (\frac{b}{a})^2}}{T[(\frac{1}{2} - T^2)^4 + T^4(T^2 - (\frac{b}{a})^2)(1 - T^2)]} & \frac{b}{a} \le T < 1 \\ \frac{\sqrt{T^2 - (\frac{b}{a})^2}}{T[(\frac{1}{2} - T^2)^2 - T^2 \sqrt{(T^2 - (\frac{b}{a})^2)(T^2 - 1)}]} & T > 1 \end{cases}$$
 (6)

Since  $\mathrm{Im}[g(T)]=0$  when  $\tau<\frac{\sigma}{a}$ , the second and third terms of the interaction equation (4) is zero during the time it takes a P-wave to travel a distance  $r_0$  (Ref. 1). When  $\tau>r_0/b$ , the time required the S-wave to travel a distance  $r_0$ ,  $\mathrm{Im}[g(T)]$  has the form given by the third of equations (6). By comparing response spectra at the fixed-base frequencies, the effects of interaction can be evaluated for a given structure with equation (4).

# RESPONSES TO THE VERTICAL COMPONENT OF THE EL-CENTRO EARTHQUAKE

Numerical results of this investigation are based on the idealized three-mass two-mode model of nuclear power plant shown in Fig. 2. Free-

field ground inputs used in this analysis are based on data from the vertical component of the El-Centro earthquake. By including the foundations average effects, the modified vertical ground motion calculated from equation (3) would have smaller amplitudes throughout the time history. This reduction is compared to the input motion in Fig. 3. Responses for different soil stiffness and structural characteristics are studied with two basic dynamic models:

- (i) A structural model has a fixed-base containment vessel frequency of 12 cps and a fixed-base internal frequency of 15 cps;
- (ii) A structural model has a fixed-base containment vessel frequency of 15 cps and a fixed-base internal frequency of 20 cps.

Each structure model is coupled to a ground having shear wave velocities of 152.4, 304.8, 609.6 and 914.4 m/s. Also, the ground density of 1600  $\rm Kg/m^3$  is used in this analysis.

For the seismic design of structure, it must be emphasized that only the spectrum values at the fixed base frequency are used to calculate the peak shock loads. This maximum load is proportional to the acceleration spectrum response evaluated at the natural frequency of the vibratory mode under consideration. The responses of vertical interaction effects can be examined by comparing the response spectra of the free-field motion to that of the foundation motion. The following expressions define the input and output undamped vertical response spectrum respectively:

$$S_{i}(\omega) = \left[\frac{\omega}{g} \int_{0}^{t} \ddot{v}_{F}(\tau) \sin\omega(t-\tau) d\tau\right]_{max \text{ over } t}$$
 (7)

$$S_{o}(\omega) = \left[\frac{\omega}{g} \int_{0}^{t} \ddot{v}(\tau) \sin \omega (t-\tau) d\tau\right]_{max \ over \ t}$$
 (8)

The spectrum responses determined from equation (4) for structural model number 1 with shear wave velocity of 914.4 m/s is shown in Fig. 4. Examination of the acceleration response spectrum curve indicates that large reduction in the free-field acceleration spectra values occurs at the center of the base mass due to the "tau" effect. This similar trend is also observed for structural model number 2 (Fig. 5). Peak vertical responses for all cases studied are summarized in Table 1. Comparison of the output response spectra for structural model number 1 with the corresponding spectra for model number 2 shows that increasing the fixed-base frequencies from 12 and 15 cps to 15 and 20 cps has very little effect on the output acceleration. Also, for all cases studied, the wave passage effect tends to smooth out the response spectrum curves.

# CONCLUSIONS

The results of this investigation show that vertical structure foundation interactions with wave averaging effect are significant for all conditions studied. The "tau" effect tends to smooth out the output response spectrum at the fixed-base frequencies and hence reduces seismic

loads. For a specified soil-structure combination, it should be noted that seismic analyses including this wave effect may also lead to rocking-input motions which can increase structural responses. Further examinations of the combined "tau" and rocking-input motion effects on vertical soil-structure interactions are needed in order to be able to predict the total responses of structures.

## REFERENCES

- 1. R. R. Little and R. J. Scavuzzo, "Vertical Soil-Structure Interaction of Nuclear Power Plants Subjected to Seismic Excitation", Nuclear Engr. & Design, 24 (1973) 203-213.
- P. C. Lam and R. J. Scavuzzo, "Lateral-Torsional Structural Response from Free-Field Ground Motion", Nuclear Engr. & Design, 65 (1981) 269-281.
- P. Cunniff and G. O'Hara, "Normal Mode Theory for 3-D Motion", NRL Report 6170, Naval Research Laboratory, Washington, D.C.
- 4. J. Wolf and P. Obernhuber, "Effects of Horizontal Waves in Soil-Structure Interaction", Nuclear & Engr. Design, 57 (1980), 221-244.
- 5. R. H. Scanlan, "Seismic Wave Effects of Soil-Structure Interaction", 3rd SMIRT, London, United Kingdom, 1975, paper K 2/1.
- 6. V. W. Lee, M.D. Trifunac and C.C. Feng, "Effect of Foundation Size on Fourier Spectrum Amplitudes of Earthquake Accelerations Recorded in Buildings", Soil Dynamics and Earthquake Engineering 1982, Vol. 1, No. 2.

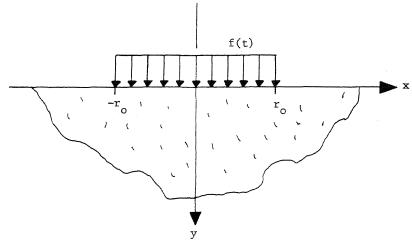


Fig. 1 A time dependent normal stress, f(t), is applied over a finite portion on the boundary of a 2-D elastic half-space

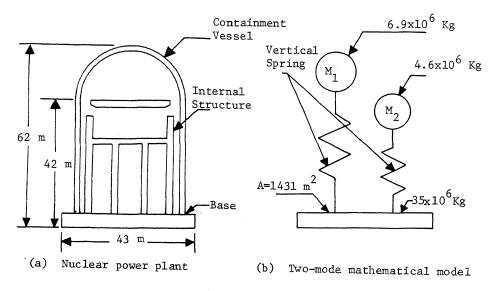
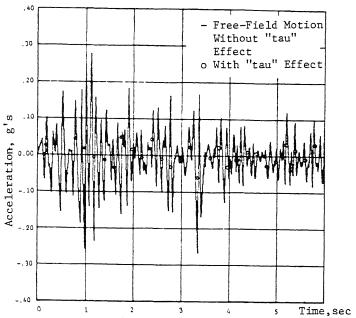
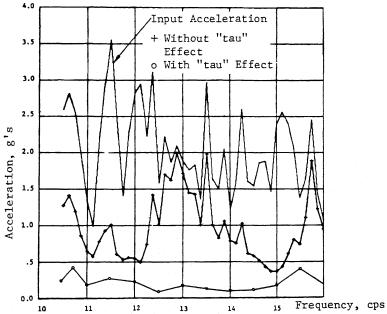


Fig. 2 Idealized three-mass two-mode nuclear power plant model



ig. 3 Comparison of the El-Centro earthquake with and Without foundation averaging effects



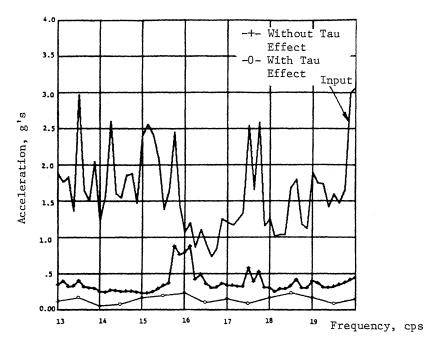


Fig. 5 Vertical structure response vs frequency (b = 304.8 m/s, structural model #2)

Table 1. Acceleration Spectra (g's) For The Structure Models Subjected To The El-Centro Earthquake Input

		Fixed-Base Frequencies					
Structure b		12 cps		15 cps		20 cps	
Description	(m/s)	g's(*)	g's(**)	g's(*)	g's(**)	g's(*)	g's(**)
Free-Field Motion		2.829		2.403		3.077	
Structure Model No. 1	152.4 304.8 609.6 914.4	.169 .376 .350 .550	.040 .096 .156 .186	.139 .244 .241 .368	.028 .089 .148 .160		
Structure Model No. 2	152.4 304.8 609.6 914.4			.092 .237 .282 .315	.029 .089 .141 .165	.130 .458 .470 .448	.038 .090 .137 .145

<sup>(\*)</sup> Without Wave Averaging Effect

<sup>(\*\*)</sup> With Wave Averaging Effect