NONLINEAR UPLIFT BEHAVIOR OF SOIL-STRUCTURE SYSTEM WITH FREQUENCY-DEPENDENT CHARACTERISTICS

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SUMMARY

An analytical method is presented in this paper for the nonlinear uplift response of the soil-structure system with frequency-dependent soil-foundation characteristics expressed as the form of Dynamical Ground Compliance. The proposed method is essentially an iterative method combining the evaluation of nonlinear uplift correction forces in the time-domain with the pseudo-linear response calculation in the frequency-domain. The method has the advantage of proper consideration on the energy loss from the foundation as well as the frequency-dependent properties of the soil-foundation. Some example problems are demonstrated and several features of uplift effects are discussed.

INTRODUCTION

Nonlinear soil-structure interaction effects due to the foundation slab uplift induced by large seismic forces have recently acquired much interest in aseismic designs of *relatively stiff* structures such as nuclear power plant structures. Uplift effects have been studied through the mathematical model with the soil-foundation replaced by a frequency-independent spring-dashpot (Ref.1). However, the dynamic properties of the soil-foundation depend on the exciting frequency, which may be significant in the interaction behaviors of a relatively stiff structure and the surrounding soil. This paper presents a new analytical method for the nonlinear uplift response of the soil-structure system with frequency-dependent soil-foundation characteristics expressed as the form of *Dynamical Ground Compliance* (D.G.C.) by the senior author (Ref.2).

D.G.C. is defined in the frequency-domain as the ratio of the displacement of a foundation on a semi-infinite soil medium to a harmonic exciting force on the foundation and corresponds to the inverse of the complex stiffness of the soil-foundation system, the imaginary part of which is associated with the *energy radiation* caused by the wave propagation. Therefore the method presented in this paper has the advantage of proper consideration on the energy loss from the foundation as well as the *frequency-dependent* properties of the soil-foundation.

Although the frequency-domain approach is usually employed in the earthquake response analyses of frequency-dependent systems, this approach is not applicable directly to nonlinear problems because of the principle of superposition inherent in the Fourier transform. The proposed method involves an iterative procedure consisting of the evaluation of nonlinear correction forces due to the uplift in the time-domain and the pseudo-linear response calculation in the frequency-domain considering nonlinear forces as equivalent accelerations applied on the foundation.

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FORMULATION OF EQUATIONS FOR UPLIFT RESPONSE

The equation of motion for the soil-structure system of Fig.1 subjected to a horizontal ground excitation can be written as

$$[M_O]\{\ddot{U}\} + [C_O]\{\dot{U}\} + [K_O]\{U\} + \{R\} = -\{m_O\}a_G \tag{1}$$

where $\{U\}=\{U_S,\ U_B\}^T$, which represents relative displacements (relative to the ground displacement) of the super-structure and the foundation. The displacement of the foundation $\{U_B\}$ contains the horizontal translation (swaying) and the rotation (rocking). $[M_O]$ is the mass matrix. $[C_O]$ and $[K_O]$ are the damping and stiffness matrices and do not include those terms which correspond to the damping and elastic forces of the frequency-dependent soil-foundation, which are represented collectively by the term $\{R\}=\{O,R_B\}^T$. $\{m_O\}$ is the inertia mass and a_G is the ground acceleration. The dots denote differentiation with respect to time. Since the nonlinear effects considered in this paper are restricted to the foundation uplift, $[M_O]$, $[C_O]$ and $[K_O]$ remain constant while the term of the foundation reactions $\{R_B\}$ is treated as nonlinear.

When no uplift occurs, the relationship between $\{U_B\}$ and $\{R_B\}$ can be expressed in the frequency-domain through the D.G.C. matrix $[C(\omega)]$ which is dependent on the exciting circular frequency ω, that is,

$$\{\tilde{U}_B\} = [C(\omega)]\{\tilde{R}_B\}\ . \tag{2}$$

The symbol ~ denotes the Fourier transform with respect to the time parameter. In order to take account of nonlinear effects due to the uplift, let $\{R_B\}$ be represented in the following form.

$$\{R_B\} = \{R_B^P\} + \{R_B^C\}\ ,\ \{\tilde{R}_B^P\} = [C(\omega)]^{-1}\{\tilde{U}_B\}\ ,\ \{R_B^C\} = [M_C]\{a_C\}\ . \tag{3a-c}$$

$\{R_B^P\}$ is related to the foundation displacement $\{U_B\}$ through the linear (neglecting the uplift) compliance matrix in the frequency-domain and is defined as pseudo-reactions while $\{R_B^C\}$ correction forces. For convenience, $\{R_B^C\}$ may be written as the product of an inertia mass matrix $[M_C]$ and an acceleration vector $\{a_C\}$. The conceptual illustration of Eqs.(3a-c) is shown in Fig.2.

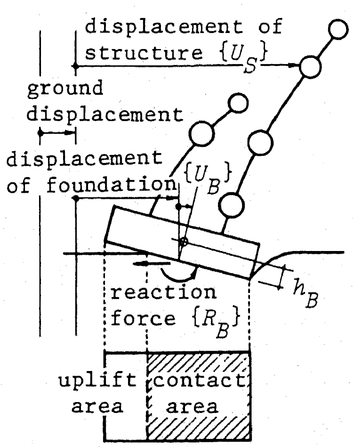

Fig.1 Soil-Structure System with Uplift

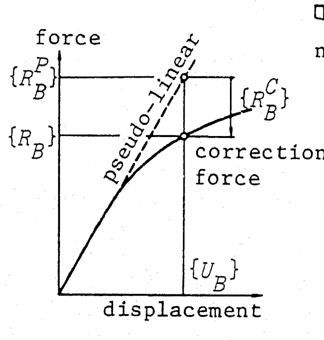

Fig.2 Conceptual Illustration of Nonlinear Forces

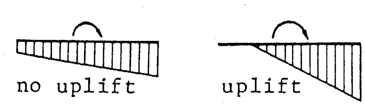

Fig.3 Vertical Stress Distribution

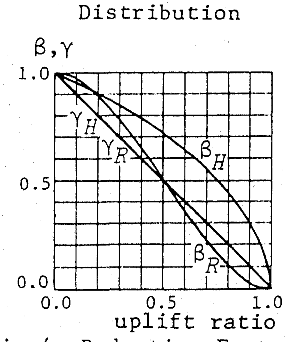

Fig.4 Reduction Factors

It should be noted that the linear stiffness in Fig.2 cannot be specified explicitly in the time-domain. $[M_C]$ and $\{a_C\}$ are expressed as

$$[M_C] = \begin{bmatrix} m_B & 0 \\ 0 & m_B h_B \end{bmatrix} \ , \ \{a_C\} = \begin{Bmatrix} a_H \\ a_R \end{Bmatrix} \tag{4a,b}$$

where m_B is the foundation mass and h_B is the height of the gravity center of the foundation from the base end. The subscripts H and R are concerned with the degrees of freedom of the horizontal translation and the rotation of the foundation, respectively.

From Eqs.(1),(3) and (4), the equation of motion in the frequency-domain

$$[H]\{\tilde{U}\} = - \{m_O\}\tilde{a}_G - \{m_H\}\tilde{a}_H - \{m_R\}\tilde{a}_R \tag{5}$$

is obtained, where

$$[H] = - \omega^2[M_O] + j\omega[C_O] + [K_O] + \begin{bmatrix} 0 & 0 \\ 0 & C(\omega)^{-1} \end{bmatrix} , \tag{6}$$

$\{m_H\}$ is the vector which has only nonvanishing component m_B corresponding to the swaying of the foundation while $\{m_R\}$ has only nonvanishing $m_B h_B$ corresponding to the rocking, and j is the imaginary unit. From the expression of Eq.(5), a_H and a_R will be suitably called *equivalent accelerations* acting on the foundation if the uplift occurs. Introducing the transfer function $\{\tilde{u}_O\}$ and auxiliary transfer functions $\{\tilde{u}_H\}$ and $\{\tilde{u}_R\}$ defined by

$$\{\tilde{u}_O\} \equiv -[H]^{-1}\{m_O\} \ , \ \{\tilde{u}_H\} \equiv -[H]^{-1}\{m_H\} \ , \ \{\tilde{u}_R\} \equiv -[H]^{-1}\{m_R\} \tag{7a-c}$$

and substituting into Eq.(5) lead to the following expression :

$$\{\tilde{U}\} = \{\tilde{u}_O\}\tilde{a}_G + \{\tilde{u}_H\}\tilde{a}_H + \{\tilde{u}_R\}\tilde{a}_R \ . \tag{8}$$

EQUIVALENT ACCELERATION

The reaction force of the foundation will consist of the elastic $\{R_{BK}\}$ and damping $\{R_{BC}\}$ forces, that is,

$$\{R_B\} = \{R_{BK}\} + \{R_{BC}\} = [K_B]\{U_B\} + [C_B]\{\dot{U}_B\} \tag{9}$$

where $[K_B]$ and $[C_B]$ are the nominal stiffness and damping matrices of the soil-foundation. Let $[K_{BO}]$ and $[C_{BO}]$ be the stiffness and damping matrices of the soil-foundation when no uplift occurs, and it is assumed that $[K_B]$ and $[C_B]$ will be given by the modification of $[K_{BO}]$ and $[C_{BO}]$ through reduction matrices $[\beta]$ and $[\gamma]$ as follows :

$$[K_B] = [\beta][K_{BO}] \ , \ [C_B] = [\gamma][C_{BO}] \ . \tag{10}$$

Representing the pseudo-reaction $\{R_B^P\}$, similar to Eq.(9), as the sum of the elastic $\{R_{BK}^P\}$ and damping $\{R_{BC}^P\}$ terms gives

$$\{R_B\} = [\beta]\{R_{BK}^P\} + [\gamma]\{R_{BC}^P\} \ . \tag{11}$$

By substituting Eqs.(3a) and (11) into Eq.(3c), the equivalent acceleration may be written in terms of pseudo-reactions and reduction matrices

$$\{a_C\} = [M_C]^{-1}\left(([\beta]-[I])\{R^P_{BK}\} + ([\gamma]-[I])\{R^P_{BC}\} \right) \tag{12}$$

where $[I]$ denotes the identity matrix.

Here, it is assumed that the reduction matrices will be expressed as

$$[\beta] = \begin{bmatrix} \beta_H(r) & 0 \\ 0 & \beta_R(r) \end{bmatrix} , \quad [\gamma] = \begin{bmatrix} \gamma_H(r) & 0 \\ 0 & \gamma_R(r) \end{bmatrix} , \tag{13a,b}$$

$$\beta_H(r) = \sqrt{1-r} \ , \ \beta_R(r) = (1+2r)(1-r)^2 \ , \ \gamma_H(r) = \gamma_R(r) = 1-r \tag{14a-d}$$

where subscripts H and R refer to the swaying and rocking of the foundation. First, the reduction matrices are diagonal and depend only on the uplift ratio r which is the ratio of the area separating from the supporting soil to the whole area of the foundation slab. Second, the swaying stiffness reduction factor β_H is proportional to the square root of the contacting area of the foundation, which is based on the consideration that the shear coefficient in static problems is proportional to the inverse of the square root of the foundation area (Ref.3). The rocking stiffness reduction factor β_R is estimated on the supposition that the vertical compressive stress distribution at the soil-slab interface changes linearly as shown in Fig.3. Third, as to the damping, nonlinear properties due to the uplift are less clear and have been neglected in the most of previous works. In the demonstrative example, this paper considers that the damping reduction factors γ_H and γ_R are proportional to the foundation area contacting with the soil. These reduction factors of Eqs.(14a-d) are plotted in Fig.4.

ITERATIVE PROCEDURE

The uplift response calculation based on the foregoing formulation leads to an iterative procedure where the whole time history of the foundation response is needed in every iteration cycle for the calculation in the frequency-domain. The procedure of the response calculation consists of the following two steps as sketched in Fig.5.

Step 1 : *Transfer functions* In addition to the complex transfer function $\{\tilde{u}_O\}$ given by Eq.(7a), the uplift analysis in the frequency-domain makes use of auxiliary functions $\{\tilde{u}_H\}$ and $\{\tilde{u}_R\}$ of Eqs.(7b,c) which are complex responses to the harmonic excitation of equivalent accelerations.

Step 2 : *Foundation response iteration* Let the prefix denote the number of each iteration cycle. In the nth iteration. $_n\{U_B\}$ is calculated by substituting the former $_{n-1}\{a_C\}$ into the right-hand side of Eq.(8), then $_n\{R_B\}$ is obtained by Eqs.(3a-c). The present equivalent acceleration $_n\{a_C\}$ is evaluated through Eqs.(12) to (14) and $_{n+1}\{U_B\}$ is again calculated. If the difference between $_{n+1}\{U_B\}$ and $_n\{U_B\}$ is not satisfactory small, n+1st iteration cycle will start.

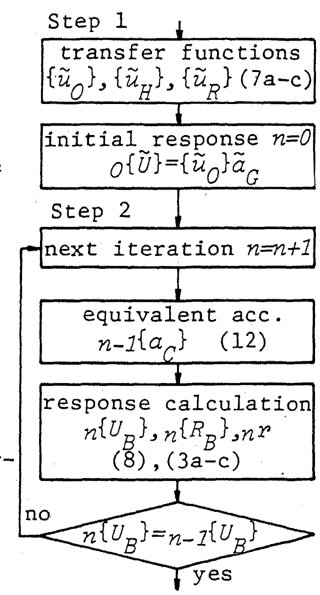

Fig.5 Iterative Procedure

EXAMPLE PROBLEM

Some features of the proposed method and uplift behaviors are illustrated by earthquake response analyses of the soil-structure model shown in Fig.6. The super-structure is treated as a shear type lumped spring-mass model, and the soil-foundation is represented by a two-degree-of-freedom model (swaying and rocking) with the dynamic characteristics based on D.G.C.. Fundamental properties of the model are also given in Fig.6. The following two cases of the supporting soil condition are considered : the *soft soil* site (elastic modulus $E=20t/cm^2$, shear wave velocity V_s=580m/sec) and the *stiff soil* site ($E=200t/cm^2$, V_s=1830m/sec). Three different earthquake accelerograms, El Centro 1940 NS, Taft 1952 EW and Golden Gate Park 1957 EW are used as the ground excitation with three levels of the peak acceleration, A1(500gals), A2 (700gals) and A3(900 gals).

Transfer property

Amplitudes of displacement transfer functions are shown in Fig.7(a) for the soft soil site and in Fig.7(b) for the stiff soil site. The ratio of the fundamental frequency of the soil-structure coupled system to that of the super-structure is 0.77 in the soft soil site and 0.97 in the stiff soil site, which roughly indicates that the soft soil model (the super-structure is relatively stiff) may involves more significant effects resulting from the soil-structure interaction. Another index for the interaction is the nondimentional frequency defined as $a_1=\omega_1 b/V_s$, where ω_1 is the fundamental circular frequency of the coupled system and b is the half length of the foundation in the direction of excitation. These models have a_1=0.97 (soft soil) and 0.39 (stiff soil). It is noted that the stiff soil model has greater amplitudes of the super-structure and smaller amplitudes of the foundation than the soft soil model. The dominant frequency of the foundation is about 3Hz in the soft soil site while about 7.5Hz in the stiff soil site.

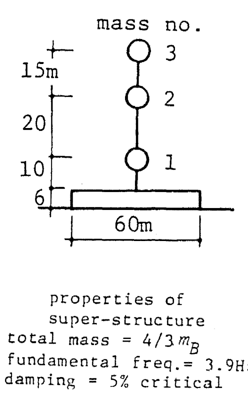

Fig.6 Example Model

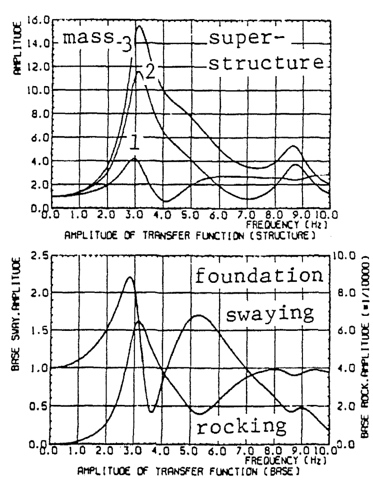

Fig.7(a) Transfer Property (Soft Soil Model)

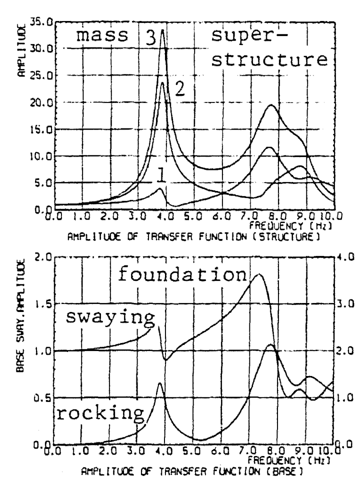

Fig.7(b) Transfer Property (Stiff Soil Model)

Comparison of D.G.C. model and constant soil-foundation model

Nonlinear uplift response analyses are carried out for two models, the D.G.C. model which has soil-foundation properties based on D.G.C. and the constant model which has conventional constant soil-foundation properties. In the latter model, the soil-foundation stiffness is approximately given by the equivalent value of D.G.C. corresponding to a_1 and the damping is tentatively chosen 10% of critical. In order to discuss the intrinsic features of the stiffness reduction due to the uplift, the quotient is introduced of the force response to displacement response of the foundation in the frequency-domain, which corresponds to the complex stiffness. The real part of this complex stiffness associated with the foundation rocking response is plotted in Figs.8 and 9 where the dashed line indicates the original rocking stiffness of the soil-foundation. Comparisons between the D.G.C. model and the constant model may be summarized as follows. In the case of the stiff soil site, the reduction patterns of the soil-foundation stiffness due to the uplift show a great resemblance between two models in the whole frequency range as might be expected, whereas the difference in the higher frequency is notable when no uplift occurs. In the case of the soft soil site, the constant model has distinct reduction at particular frequencies while the D.G.C. model has less distinct reduction.

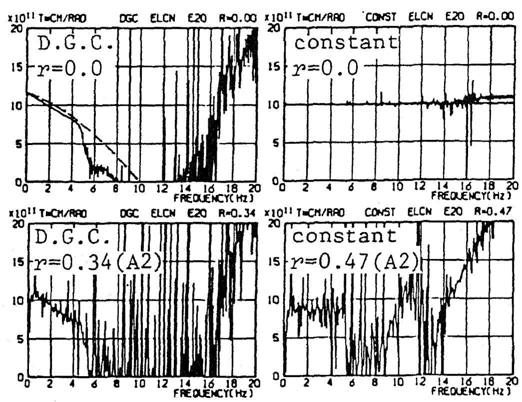

Fig.8 Response Rocking Stiffness (Soft Soil, El Centro)

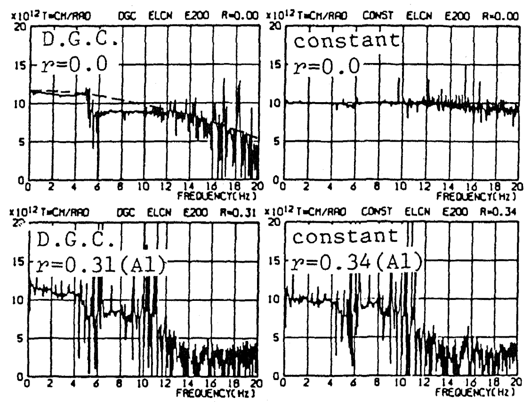

Fig.9 Response Rocking Stiffness (Stiff Soil, El Centro)

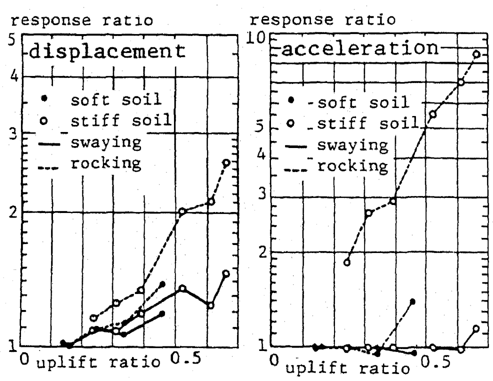

Fig.10 Ratio of Nonlinear Response to Linear Response

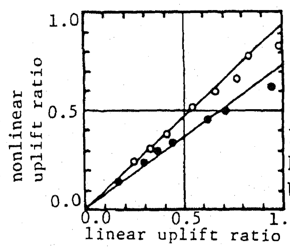

Fig.11 Relation of Nonlinear and Linear Uplift Ratio

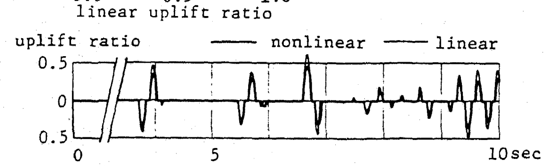

Fig.12 Time History of Uplift Ratio (Soft Soil, Taft, A2)

Comparison of nonlinear and linear responses of D.G.C. model

Results of the nonlinear analysis including the uplift are compared with the linear results neglecting the uplift in Figs.10 to 13. Fig.10 shows the ratio of the maximum nonlinear response to the maximum linear response of the foundation versus the maximum nonlinear uplift ratio. Though the rocking rotation and acceleration are very small in the stiff soil site, they increase as the uplift ratio increases, which may have some effects on the super-structure if the rocking dominates the response. Fig.11 illustrates the relation of the maximum uplift ratio between two analyses and Fig.12 gives the time history of the uplift ratio. The uplift ratio by the nonlinear analysis is about 75% of that by the linear analysis in the soft soil site and about 95% in the stiff soil site. It is shown in Fig.13 that the maximum force response by the linear analysis is approximately 5 to 15% greater than that by the nonlinear analysis. Therefore, in the cases considered the linear analysis neglecting uplift effects provides conservative results.

Convergence of the iterative method

Convergence of the proposed iterative method is not proved mathematically but examined numerically through the example problems. The maximum rotation of the foundation at each iterative step devided by the maximum resulting from the linear analysis is sketched in Fig.14. The results indicate that the convergence of the iterative calculation will depend on the uplift ratio and the method seems to be applicable until the overturn occurs. Uplift problems which have maximum uplift ratio less than 0.7 need at most 10 times of the iteration. As the iterative calculation is restricted to the foundation response only, that is two degrees of freedom, 20 times of the iteration (the uplift ratio is greater than 90%) may be also practical.

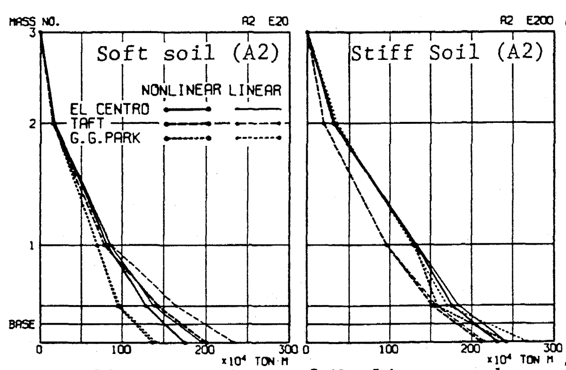

Fig.13 Comparison of Nonlinear and Linear Responses of Overturning Moment

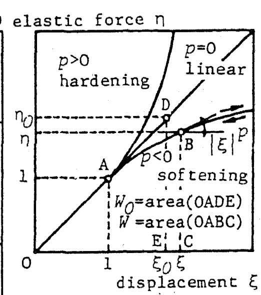

Fig.15 Nonlinear Elastic Stiffness

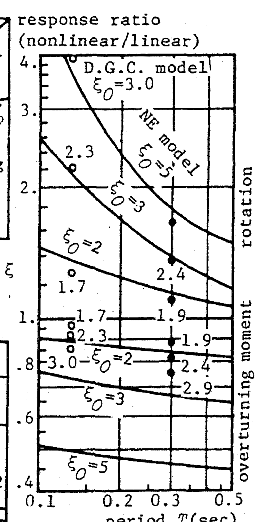

Fig.16 Comparisons of NE model and D.G.C. model (El Centro)

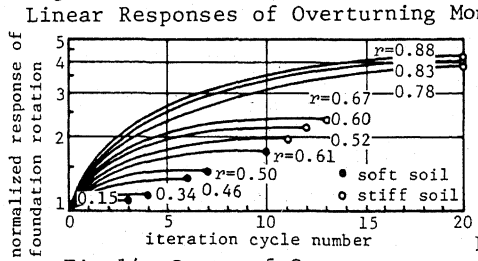

Fig.14 State of Convergence

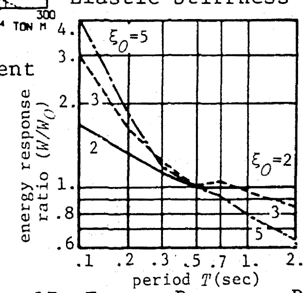

Fig.17 Energy Response Ratio of NE model (El Centro)

ENERGY EQUIVALENCE

Conservative results of force responses in the linear analysis neglecting uplift effects are often explained by the concept of energy equivalence with respect to the maxima of linear and nonlinear responses. In order to examine energy equivalence in uplift problems, another seismic response analysis of a nonlinear elastic single-degree-of-freedom model (NE model) with conventional frequency-independent properties is made (Ref.4). The nondimentional force (η) - displacement (ξ) relationship is shown in Fig.15, where the positive nonlinear parameter ($p>0$) gives a hardening-type stiffness while the negative parameter ($p<0$) a softening-type. In a rough estimate, NE model with $p=-1.5$ can represent a nonlinear stiffness of the uplifting soil-foundation. Comparisons of the response ratio (nonlinear/linear) of the D.G.C. model with that of the NE model are demonstrated in Fig.16, where T is the natural period of the NE model or the rocking-dominant period of the D.G.C. model, and ξ_0 is the excess ratio in the linear response as defined in Fig.15. Since there is close agreement in the response ratio between two models, the NE model provides a good lead to study energy equivalence of uplift responses. From the energy response ratio of the NE model with $p=-1.5$ plotted in Fig.17, it may be noticed that the energy equivalence method is applicable to the range of T=0.3 to 0.7sec. The energy method may underestimate the response in the range of T<0.3sec while may overestimate in the range of T>0.7sec, which is more remarkable as ξ_0 gets larger.

CONCLUSIONS

A new method has been presented for the nonlinear uplift response analysis of the soil-structure system with frequency-dependent soil-foundation characteristics based on D.G.C.. The proposed method is an iterative method combining the frequency-domain and the time-domain calculations. The features of the uplift effects have been illustrated by the example problems and it has been concluded that in the cases considered reduction patterns of the soil-foundation stiffness show a remarkable difference between the proposed D.G.C. model and the conventional model when the soil-structure interaction is significant and the linear analysis neglecting uplift effects provides conservative results. The convergence of the iterative method has been exemplified through numerical examples, and a brief discussion has been devoted to a study of energy equivalence associated with maximum responses.

REFERENCES

1. Kennedy,R.P., S.A.Short, D.A.Wesley and T.H.Lee, Effect on Non-Linear Soil-Structure Interaction due to Base Slab Uplift on the Seismic Response of a High Temperature Gas-Cooled Reactor (HTGR), Nuclear Eng. and Design 38, pp.323-355, 1976.
2. Kobori,T., R.Minai, T.Suzuki and K.Kusakabe, Dynamical Ground Compliance of Rectangular Foundation on a Semi-Infinite Elastic Medium (In Japanese), Annuals of the Disaster Prevention Research Inst., Kyoto Univ., No.10A, pp.283-314, 1967.
3. Barkan,D.D., Dynamics of Bases and Foundations, McGraw-Hill Book Co., pp.34-40, 1962.
4. Hisatoku,T. and T.Nagase, Seismic Responses of a Nonlinear Elastic SDOF System (In Japanese), Summaries of Tech. Papers, 1983 Annual Meeting, Archit. Inst. Japan, pp.1023-1024, 1983.