SIMPLIFIED DYNAMIC MODEL FOR SOIL-STRUCTURE INTERACTION PROBLEMS

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SUMMARY

A simplified dynamic model in which the interaction forces corresponding to each mode of vibration are represented by a set of frequency independent stiffness and damping coefficients is developed. Such coefficients are determined from steady state analysis by conserving the average workdone and the average rate of energy dissipated by the interaction forces.

INTRODUCTION

The subject of forced vibration of footings on a semi-infinite elastic half-space have been studied by using harmonic excitation of rigid base. As the interaction forces thus obtained are frequency-dependent, the transient analyses of structure-foundation system are carried out in the frequency domain (Refs. 1, 2) or by modified excitation method (Refs. 3,4). These methods are complicated and time consuming. Parmelee et al (Ref. 5) approximated the frequency-dependent coefficients by average values so that the transient analyses of structure foundation system can be carried out in the time domain. Recently, Balendra et al (Refs. 6,7) obtained the frequency-independent coefficients more accurately, through transient analysis of plane strain problem, by conserving the average work done and average rate of energy dissipated by the interaction forces. The purpose of the present study is to obtain the frequency-independent stiffness and damping coefficients using the steady-state analysis.

INTERACTION FORCES

When a massless rigid disc of infinitesimal thickness situated on the surface of a homogeneous, isotropic and linearly elastic half-space is subjected to harmonic excitation, the corresponding interaction forces are dependent on the following parameters: radius of the rigid base plate r; mass density ρ, shear modulus μ and Poisson's ratio ν of the elastic soil medium, frequency of excitation ω, which can be expressed in terms of a dimensionless frequency factor $a = \frac{\omega r}{V_s}$, where the shear-wave velocity, $V_s = \sqrt{\frac{\mu}{\rho}}$. For translation, u_b, and rocking, ϕ, motions shown in Fig. 1,

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the respective interaction forces are given by

$$P^*(t) = K_T \cdot u_b(t) + C_T \dot{u}_b\ (t) \tag{1}$$

$$Q^*(t) = K_R \cdot \phi\ (t) + C_R \dot{\phi}\ (t) \tag{2}$$

where K_T, K_R are the stiffness coefficients, while C_T, C_R the damping coefficients. The stiffness coefficients reflect the flexibility of the foundation medium while the damping coefficients reflect the dissipation of energy within the medium as the waves propagate to infinity.

FREQUENCY DEPENDENT STIFFNESS AND DAMPING COEFFICIENTS

Bycroft's Solution

The frequency dependent stiffness and damping coefficients are given in Ref. 8 for $\nu = 0$ and $0 \leqslant a \leqslant 1.5$ as

$$K_T = \rho V_s^{\ 2}\ r \frac{f_{1H}}{f_{1H}^2 + f_{2H}^2} \quad ; \quad C_T = -\frac{\rho V_s\ r^2}{a} \frac{f_{2H}}{f_{1H}^2 + f_{2H}^2} \tag{3a}$$

$$K_R = \rho\ V_s^2\ r^3 \frac{f_{1R}}{f_{1R}^2 + f_{2r}^2} \quad ; \quad C_R = -\frac{\rho\ V_s\ r^4}{a} \frac{f_{2R}}{f_{1R}^{\ 2} + f_{2R}^2} \tag{3b}$$

where $f_{1H} = -0.2188+0.05992a^2-0.007917a^4+0.0005852a^6-0.00002693a^8+..$

$f_2H = 0.1248a-0.02366a^3+0.002299a^5-0.0001319a^7+0.000004866a^9-..$

$f_{1R} = -0.3730-0.3113a-0.1782a^2+0.4880a^3-0.8498a^4+0.671a^5-0.1820a^6+..$

$f_{2R} = -0.01660a-0.1117a^2+0.3680a^3-0.2596a^4+0.1024a^5-0.02275a^6+..$

Solution of Veletsos and Wei

The frequency dependent stiffness and damping coefficients are given by

$$K_T = \frac{8\rho\ V_s^2\ r}{2 - \nu} \left(\frac{f_{11}}{f_{11}^2 + g_{11}^2}\right) \quad ; \quad C_T = \frac{8\rho\ V_s\ r^3}{a\ (2 - \nu)} \left(-\frac{g_{11}}{f_{11}^2 + g_{11}^2}\right) \tag{4a}$$

$$K_R = \frac{8\rho\ V_s^2\ r^3}{3(1 - \nu)} \left(\frac{f_{22}}{f_{22}^2 + g_{22}^2}\right) \quad ; \quad C_R = \frac{8\rho\ V_s\ r^4}{3a\ (1 - \nu)} \left(-\frac{g_{22}}{f_{22}^2 + g_{22}^2}\right) \tag{4b}$$

The values of the dimensionless flexibility coefficients f and g are given in Ref. 9 for the range $0 \leqslant a \leqslant 10.0$ and $0 \leqslant \nu \leqslant 0.5$.

FREQUENCY INDEPENDENT STIFFNESS AND DAMPING COEFFICIENTS

In order to use the direct time-domain analysis for transient response, Parmelee et. al. (Ref. 5) obtained frequency-independent stiffness and damping coefficients by averaging the interaction forces over a frequency range of interest. Bycroft's solution is used and the following coefficients are obtained:

$$\hat{K}_T = 4.4\rho V_s^2 r \quad ; \quad \hat{C}_T = 2.7\rho V_s r^2 \tag{5a}$$

$$\hat{K}_R = 2.3\rho V_s^2 r^3 \quad ; \quad \hat{C}_R = 0.31\rho V_s^2 r^4 \tag{5b}$$

In this study, the frequency independent coefficients are obtained by comparing two systems; one with frequency-dependent interaction forces (denoted as Systems I) and the other with frequency-independent interaction forces (denoted as System II). The frequency-independent coefficients are determined by equating the average work done and average rate of energy dissipated by the interaction forces of both systems. In Refs. 6 and 7, the said coefficients are determined through transient analysis, whereas steady-state analysis is being adopted in the present study.

The structure-foundation system considered is a rigid circular plate of radius r and mass m_o situated on a surface of a homogeneous, isotropic and linearly elastic half-space. The interaction forces at the soil-structure interface are produced by the horizontal translation and rocking of the plate. If P(t) and Q(t) are the time-dependent applied force and moment acting on the plate, then

$$P(t) = K_T u_b + C_T \dot{u}_b + m_o \ddot{u}_b \tag{6a}$$

$$Q(t) = K_R \phi + C_R \dot{\phi} + I_o \ddot{\phi} \tag{6b}$$

where $I_o = \frac{1}{4} m_o r^2$ is the mass moment of inertia of the plate. If $P(t) = P_o e^{i\omega t}$ and $Q(t) = Q_o e^{i\omega t}$ then $u_b = u_o e^{i\omega t}$ and $\phi(t) = \phi_o e^{i\omega t}$ where P_o, Q_o are the amplitudes of the respective loading, and u_o, ϕ_o the amplitudes of the resulting displacements. Since the relationships given in Eqs. 6(a) and 6(b) are similar in expression, the following general relationships is used for brevity:

$$N(t) = N_o e^{i\omega t} = K_o x + C_o \dot{x} + Y\ddot{x}\ , \quad N^*(t) = K_o x + C_o \dot{x} \tag{7}$$

where N(t) corresponds to applied loading P(t) or Q(t), $N^*(t)$ the corresponding interaction force, $x(t) = x_o e^{i\omega t}$ corresponds to the resulting displacements $u_b(t)$ or $\phi(t)$, and Y corresponds to m_o or I_o. Furthermore K_o and C_o are the frequency-dependent stiffness and damping coefficients respectively for each mode of vibration. Let

$$x_o = x_R + i x_I \tag{8}$$

where x_R and x_I are the real and imaginary components of x_o respectively. Substituting Eq. 8 in Eq. 7 and equating the real and imaginary components yield

$$x_R = \frac{N_o\,(K_o - \omega^2 Y)}{(K_o - \omega^2 Y)^2 + (\omega C_o)^2}\ ;\ x_I = \frac{N_o\,(\omega C_o)}{(K_o - \omega^2 Y)^2 + (\omega C_o)^2} \tag{9}$$

Hence $$|x_o| = \sqrt{x_R^{\,2} + x_I^{\,2}} = \frac{N_o}{\left((K_o - \omega^2 Y)^2 + (\omega C_o)^2\right)^{1/2}} \tag{10}$$

Let $W(\omega)$ be the Work done per cycle by the interaction force $N^*(t)$. Then, for System I,

$$W_I(\omega) = \int_o^T N^*(t)\ x dt = K_o \int_o^T x^2 dt + C_o \int_o^T \dot{x}\ x dt \tag{11}$$

where $T = \frac{2\pi}{\omega}$ is the period of excitation. Using the real or the imaginary component of the displacement $x(t)$ depending on whether $N(t) = N_o \cos\omega t$ or $N_o \sin\omega t$, Eq. 11 yields

$$W_I(\omega) = K_o(\frac{\pi}{\omega})\ |x_o|^2 \tag{12}$$

Next, let $E(\omega)$ be the rate of energy dissipated per cycle by the interaction forces. Then, for system I,

$$E_I(\omega) = \int_o^T N_I^*\ (t)\ \dot{x}\ dt = K_o \int_o^T x\ \dot{x} dt + C_o \int_o^T \dot{x}^2 dt = C_o\ \pi\omega\ |x_o|^2 \tag{13}$$

For System II, the work done and the rate of energy dissipated per cycle take the form

$$W_{II}(\omega) = \hat{K}_o\ (\frac{\pi}{\omega})\ |x_o|^2;\ E_{II}(\omega) = \hat{C}_o(\pi\omega)\ |x_o|^2 \tag{14}$$

where $\hat{K}_o$ and $\hat{C}_o$ are the frequency-independent stiffness and damping coefficients. Equating the average work done and average rate of energy dissipated by both systems over a specified frequency range $0 \leq \omega \leq \omega_o$ yields

$$\frac{1}{\omega_o} \int_o^{\omega_o} W_I(\omega)\ d\omega = \frac{1}{\omega_o} \int_o^{\omega_o} W_{II}\ (\omega)\ d\omega \tag{15a}$$

$$\frac{1}{\omega_o} \int_o^{\omega_o} E_I(\omega)\ d\omega = \frac{1}{\omega_o} \int_o^{\omega_o} E_{II}\ (\omega)\ d\omega \tag{15b}$$

In view of Eqs. 12, 13 and 14, Eq. 15 yields the frequency independent stiffness and damping coefficients as

$$\hat{K}_o = \frac{\int_o^{\omega_o} \frac{K_o}{\omega\left((K_o - \omega^2 Y)^2 + (\omega C_o)^2\right)} d\omega}{\int_o^{\omega_o} \frac{1}{\omega\left((K_o - \omega^2 Y)^2 + (\omega C_o)^2\right)} d\omega} \tag{16a}$$

$$\hat{C}_o = \frac{\int_o^{\omega_o} \frac{\omega\, C_o}{\left((K_o - \omega^2 Y)^2 + (\omega C_o)^2\right)} d\omega}{\int_o^{\omega_o} \frac{\omega}{\left((K_o - \omega^2 Y)^2 + (\omega C_o)^2\right)} d\omega} \tag{16b}$$

The frequency independent coefficients evaluated using the frequency dependent K_o and C_o are made dimensionless constants K_o^* and C_o^* for translational and rocking modes of vibration as

$$K_T^* = \frac{\hat{K}_T}{\rho\, V_s^2\, r} \; ; \; C_T^* = \frac{\hat{C}_T}{\rho\, V_s\, r^2} \tag{17a}$$

$$K_R^* = \frac{\hat{K}_R}{\rho\, V_s^2\, r^3} \; ; \; C_R^* = \frac{\hat{C}_R}{\rho\, V_s\, r^4} \tag{17b}$$

RESULTS AND DISCUSSIONS

The frequency independent stiffness and damping coefficients obtained by the energy approach using Bycroft's solution are compared with the mean values given by Parmelee, et. al. (Ref. 5) in Table 1. It is seen that except C_R^*, the coefficients obtained by energy approach compare very well with the mean values. Over the frequency range used ($0 \leq a \leq 1.5$), the function of C_R in Eq. 3(b) varys widely with the frequency of excitation, which attributes to the large difference between the value of C_R^* obtained through energy approach and the mean value.

The frequency independent coefficients obtained through energy approach using the solution of Veletsos and Wei are compared with the mean value in Table 2, for the frequency range of $0 \leq a \leq 1.5$. Again except C_R^*, the agreement between the coefficients obtained through energy approach and the mean values is quite good.

In Table 3, the frequency independent coefficients K_T^*, C_T^* and K_R^* presented in Table 2 through energy approach are compared with the frequency independent coefficients given in Ref. 10. It is seen that the coefficients through energy approach are in very good agreement with the results given in Ref. 10 since the damping coefficients given in Ref. 10 for rotatioanl mode is a function of the mass moment of inertia of the foundation, it was not possible to compare C_R^* obtained through energy approach with that given in Ref. 10.

CONCLUSIONS

The frequency-independent stiffness coefficients for both translational and rotational modes as well as the damping coefficient for translational mode, determined by the energy approach, using the expressions for interaction forces given by Bycroft, and Veletsos & Wei are in good agreement with the corresponding mean values for the selected frequency range. However, the frequency-independent damping coefficient for the rotational mode, determined by the energy approach, differs somewhat from the mean value which is due to the fact that the corresponding frequency-dependent coeffcient varies widely with the frequency and hence, taking a mean value would introduce some error. With the proposed method, it is possible to develop frequency independent stiffness and damping coefficients for different types of foundation resulting in simplified models for dynamic analyses of structure-foundation system.

REFERENCES

1. Chopra, A.K. and Gutierrez, J.A., "Earthquake Response Analysis of Multi-storey Buildings Including Foundation Interaction", Earthquake Engineering and Structural Dynamics, Vol. 2, 1974, pp. 65-77.

2. Liu, S.C. and Fasel, L.W., "Earthquake Interaction by Fast Fourier Transform", Journal of the Engineering Mechanics Division, ASCE, Vol. 97, 1971, pp. 1223-1237.

3. Bielek, J., "Dynamic Behaviour of Structures with Embedded Foundations", Earthquake Engineering and Structural Dynamics, Vol. 3, 1975, pp. 259-274.

4. Jennings, P.C. and J. Bielak, "Dynamics of Building - Soil Interaction", Bulletin of Seismological Society of America, Vol. 63, 1973, pp. 9-48.

5. Parmelee, R.A., Perelman, D.S., and Lee, S.L., "Seismic Response of Multi-storey Structures on Flexible Foundations", Bulletin of the Seismological Society of America, Vol. 59, No. 3, 1969, pp. 1061-1070.

6. Balendra, T., Tan, Y.P. and Lee, S.L., "Simplified Dynamic Analysis of Buildings with Basements" Journal of the Engineering Mechanics Division, ASCE, Vol. 108, No. EM5, Oct, 1982, pp. 895-914.

7. Balendra, T., Tan, Y.P. and Lee, S.L. "Frequency Independent Stiffness and Damping Coefficients for Structure-Foundation System", International Journal of Mechanical Science, Vol. 33, No. 9, 1981, pp. 531-546.

8. Bycroft, G.N., "Forced Vibration of a Rigid Circular Plate on a Semi-infinite Elastic Space and on an Elastic Stratum", Philosophical

Transaction Royal Society of London, Series A, Vol. 248, No. 948, 1956, pp. 327-368.

9. Veletsos, A.S. and Wei, Y.T., "Lateral and Rocking Vibration of Footings", Journal of the Soil mechanics and Foundation Division, ASCE, Vol. 97, SM9, 1971, pp. 1227-1248.

10. Richart, F.E., Hall, J.R. and Woods, R.B., "Vibration of Soils and Foundations", Prentice Hall, New Jersey (1970).

Table 1 Comparison with Parmelee's results using Bycroft's solution ($\nu = 0$)

Method	K_T^*	C_T^*	K_R^*	C_R^*
(i) Mean (Ref. 5)	4.4	2.7	2.3	0.31
(ii) Energy approach	4.53	2.70	2.52	0.44

Table 2. Comparison with mean values using solution of Velotsos & Wei for $0 \leq \nu \leq 0.5$ and $0 \leq a \leq 1.5$

Poisson's ratio ν	K_T^*			C_T^*		
	Energy Approach	Mean	% Diff	Energy Approach	Mean	% Diff
0.0	3.97	3.82	3.63	2.76	2.73	1.09
0.33	4.77	4.66	2.27	2.87	2.84	0.99
0.45	5.14	5.05	1.83	2.99	2.96	0.98
0.50	5.31	5.21	1.77	3.07	3.04	1.00
ν	K_R^*			C_R^*		
0.0	2.52	2.29	10.31	0.44	0.31	44.05
0.33	3.80	3.48	9.21	0.55	0.38	45.50
0.45	4.62	4.23	9.40	0.65	0.45	45.91
0.50	5.08	4.63	9.69	0.72	0.49	46.10

Table 3. Comparison with the frequency independent coefficients given in Ref. 10

Poisson ratio ν	K_T^*		C_T^*		K_R^*	
	Energy approach	Ref. 10	Energy approach	Ref. 10	Energy approach	Ref. 10
0.0	3.97	4.57	2.76	2.63	2.53	2.67
0.33	4.77	4.92	2.87	2.83	3.80	3.98
0.45	5.14	5.18	2.99	2.98	4.63	4.85
0.5	5.31	5.33	3.07	3.07	5.08	5.33

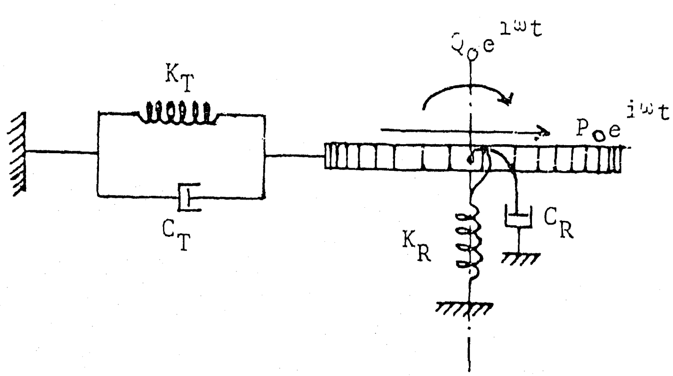

Figure 1 Simplified Model for Interaction Forces in Translational and Rotational Modes of Vibration