

A THREE-DIMENSIONAL NONLINEAR SOIL-STRUCTURE INTERACTION  
ANALYSIS CONSIDERING THE ELASTO-PLASTIC SOIL BEHAVIOR

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SUMMARY

A time-domain analysis using the nonlinear constitutive equations for soil, valid for multi-dimensional states of stresses, has been conducted to determine the approximate nonlinear seismic response of a three-dimensional soil-structure interaction system. The elasto-plastic soil behavior is represented by the Prevost-Iwan multiple-yield-surface theory of plasticity based on assumed strain distributions. The nonlinear response, which is both time and path dependent, is treated as a perturbation from a linear reference solution. Numerical examples are illustrated by comparing the accelerations and in-structure response spectra of a reactor containment building.

INTRODUCTION

In the linear analyses of soil-structure interaction (SSI), the energy dissipative mechanism resulting from the inelastic soil behavior has been approximately simulated by representing the soil medium by a viscoelastic halfspace. A true nonlinear representation of the inelastic soil behavior requires that the analysis be performed on a time domain using direct integration. Roesset and Tassoulas (Ref. 1) have emphasized the mathematical and physical consistency of using the nonlinear constitutive equations of soil in a time-domain analysis based on the theory of plasticity. Recently, nonlinear finite element models considering elasto-plastic soil representation have been developed (Refs. 2-3). Analytical solutions using a nonlinear halfspace do not appear feasible at the present time. However, approximate solutions from an analytical standpoint as herein demonstrated can be obtained by systematically reducing the problem to a manageable level with the use of simplifying assumptions.

THEORETICAL DEVELOPMENT

Mathematical idealizations have been adopted in two areas to (i) separate the radiation (geometric) damping from the soil material nonlinearity and to (ii) use prescribed functions for the unknown distributions of the strain field at the basemat-soil interface. The nonlinear responses are treated as perturbations from a linear reference solution which is taken to be a frequency-independent analysis. It is assumed that the radiation damping coefficients are insensitive to the localized nonlinear excursions in the vicinity of the basemat. Therefore, the elasto-plastic soil behavior can be handled as a separate entity from the radiation damping matrix which is assumed unaltered throughout the duration of the dynamic solutions.

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### Modified Equations of Motion

Consider the seismic model of a surface-founded nuclear power plant as depicted in Figure 1a. The consideration of path-dependent nonlinear response necessitates some modifications in the governing equations. The conventional formulation excluding the structure static weight from the dynamic solution does not give a correct starting point of the stress path in the soil. In the present analysis, the soil deformation due to structure weight prior to the seismic event is included in the dynamic solution as an initial condition. This is done by using the absolute displacement coordinates in vertical direction so that the gravity effects will appear in the equations of motion. The modified governing equations may be written as (Ref. 4):

$$\{\ddot{q}\} + [m^{SB}]\{\ddot{U}\} + [2\zeta\omega]\{\dot{q}\} + [\omega^2]\{q\} = \{0\} \quad (1)$$

$$[m^{BB}]\{\ddot{U}\} + [m^{BS}]\{\ddot{q}\} + [C^R]\{\dot{U}\} - \{\dot{U}^0\} + g\{m^{SW}\} + g[m^{IP}]\{U\} = \{Q(t)\} \quad (2)$$

with initial condition  $\{U\} = \{u_0\}$  at  $t = 0$ . Equation (1) is written in terms of the generalized coordinates of superstructure modes. The  $U$  and  $U^0$  in Eq. (2) are the vectors of basemat displacement and foundation input motion. The symbol  $g$  is the gravitational constant,  $C^R$  is the frequency-independent radiation damping matrix and  $Q(t)$  is the vector of contact forces and moments at the base-soil interface. The components of the force and moment in  $Q(t)$  are defined as

$$P_i = - \iint_S \sigma_{i3} dS \quad \text{and} \quad M_i = - \iint_S e_{ijk} x_j \sigma_{k3} dS. \quad (3)$$

( $i, j, k = 1, 2, 3$ )

Equation (3) and other expressions carrying subscripts are written in tensor notation for conciseness.  $\sigma_{ij}$  is the stress tensor,  $e_{ijk}$  is the permutation symbol, directions 1,2,3 correspond to x,y,z and  $S$  represents the contact area. The last two terms on the left hand side of Eq. (2) account for the structure weight and the inverted-pendulum effect (Fig. 1b), the latter becoming important for relatively taller buildings.

### Stress And Strain

Let the problem be confined to the case subjected to vertically propagating seismic waves so that

$$\{U^0\} = (u_1^g, u_2^g, u_3^g, 0, 0, 0)^T; \quad \{U\} = (u_1^B, u_2^B, u_3^B, \psi_1, \psi_2, \psi_3)^T, \quad (4)$$

where  $U_i^g$  are components of the free-field motion,  $u_i^B$  and  $\psi_i$  designate the translational and rotational components of basemat displacement. On the surface ( $z=0$ ) the total displacement field of free-field motion and the scattered field must satisfy the mixed boundary conditions, therefore the scattered field  $u_i^S$  in the contact region on  $z=0$  may be written as

$$u_i^S = u_i^B - U_i^g + A_{ij} \psi_j, \quad (i, j=1, 2, 3) \quad (5)$$

where  $A_{ij}$  is a transformation matrix whose coefficients involve spatial coordinates only. Based on the strain-displacement relations and Eq.(5), the major strain components on the surface at the contact region may be expressed in terms of the approximate distribution functions associated with each basemat degree of freedom as

$$\begin{aligned}\epsilon_{13} &= (u_1^B - u_1^G)h_1(\bar{x}) + A_{13}\psi_3\bar{h}_3(\bar{x}) ; \quad \epsilon_{23} = (u_2^B - u_2^G)h_2(\bar{x}) + A_{23}\psi_3\bar{h}_3(\bar{x}) ; \\ \epsilon_{33} &= (u_3^B - u_3^G)h_3(\bar{x}) + A_{31}\psi_1\bar{h}_1(\bar{x}) + A_{32}\psi_2\bar{h}_2(\bar{x}).\end{aligned}\quad (6)$$

where  $h_i(\bar{x})$  and  $\bar{h}_i(\bar{x})$  are distribution functions. Stresses in the elastic states are governed by the linear stress-strain relations and the nonlinear stresses are determined by using the constitutive equations for soil.

#### Linear Reference Solution

The selection of distribution shapes is based on the contact stress distribution on the elastic halfspace. For example, the functions  $h_2(\bar{x})$  and  $\bar{h}_1(\bar{x})$  for the vertical and rocking modes may be prescribed to correspond to the stress distributions (Figure 2) as presented in Reference 5. Furthermore, they must also satisfy the conditions

$$D_{33} \int_S h_3(\bar{x}) \, dS = \frac{4Gr_o}{1-\nu} , \quad D_{33} \int_S y^2 \bar{h}_1(\bar{x}) \, dS = \frac{8Gr_o^3}{3(1-\nu)}, \quad (7)$$

where  $D_{33}$  involves the Lamé constants and  $r_o, G$  and  $\nu$  are, respectively, the base radius, soil shear modulus and Poisson ratio. These conditions ensure that in the absence of plastic deformation, the analysis reduces to the linear solution using Richart soil parameters (Ref.6). This linear solution may be chosen to be some other appropriate reference analysis (Ref.7).

#### Nonlinear Constitutive Equations

The surface integrals in Eqs.(3) and (7) were quantized through contact area discretization (Fig.5). The multiple-yield-surface theory of plasticity is used to model the elasto-plastic soil medium. The general expression used for the  $m$ -th yield surface with isotropic and kinematic hardening and normal stress dependency, according to the Prevost-Iwan theory (Refs.8-9), may be written as

$$f^{(m)} = \frac{3}{2} \left[ s_{ij} - \alpha_{ij}^{(m)} \right] \left[ s_{ij} - \alpha_{ij}^{(m)} \right] + \bar{\alpha}^2 \left[ p - \beta^{(m)} \right]^2 - \left[ \kappa^{(m)} \right]^2 = 0 \quad (8)$$

where  $s$ ,  $\kappa^{(m)}$ ,  $p$ ,  $\alpha^{(m)}$  and  $\beta^{(m)}$  are, respectively, the deviatoric stress, radius of surface, effective mean normal stress and centers of surfaces. The flow rule associated with each surface, in terms of plastic strain increment, is given by

$$(d\epsilon_{ij}^p)^{(m)} = \frac{\partial f^{(m)}}{\partial \sigma_{ij}} d\lambda^{(m)} \quad (9)$$

where  $d\lambda$  is obtained through the consistency condition of Eq.(8) as

$$d\lambda^{(m)} = - \frac{\frac{\partial f^{(m)}}{\partial \sigma_{kn}} d\sigma_{kn}}{\left[ -c \frac{\partial f^{(m)}}{\partial \alpha_{ij}^{(m)}} + \frac{\partial f^{(m)}}{\partial \kappa^{(m)}} \frac{\partial \kappa^{(m)}}{\partial (\epsilon_{ij}^p)^{(m)}} \right] \frac{\partial f^{(m)}}{\partial \sigma_{ij}}} \quad (10)$$

when the second term in Eq.(8) is neglected. The constant  $c$  relates the center coordinates linearly to the plastic strain. The elasto-plastic solution was obtained by using the established numerical procedures with minor modifications. Figure 3 illustrates the behavior of a field of yield surfaces. A stress-strain relation predicted by using a two-surface model is displayed in Figure 4 showing the desired Bauschinger effect.

### Results and Discussions

The computer program developed from the present formulation (hereafter referred to as NONSSI) is extremely efficient that it is economically feasible to carry out extensive parametric studies on the nonlinear SSI. Table 1 presents a set of relevant data obtained from a comparison study on the seismic response of a typical nuclear power plant. The NONSSI model consists of two eccentrically located superstructures supported by a rigid basemat with a soil shear wave velocity of 1800 FPS. The system is subjected to a single-component artificial earthquake input of 0.15g with a 10-second duration. Accelerations at three critical elevations were compared with its linear reference response obtained by using the same model but with the plastic deformation omitted. The linear reference solution of NONSSI was also checked out by comparing its responses against those given by the linear analysis of MODSAP (Ref.10). As confirmed by other studies (Refs.11-12), the small differences observed here between the two linear analyses are attributed to the use of composite modal damping in MODSAP.

Table 1. Maximum Acceleration (g) Obtained From Linear and Nonlinear Analyses

		Foundation Mat	Core Midheight	Containment Shell (EL.104 Ft.)
Linear Analyses	MODSAP	0.165	0.230	0.259
	NONSSI (Linear Reference)	0.153	0.242	0.356
Nonlinear Analyses	NONSSI (A)	0.133	0.179	0.211
	NONSSI (B)	0.133	0.191	0.228
Notes: Soil strain distributions used are: (A) nonuniform as shown in Fig.2; (B) proportional to displacement				

Since the nonlinear excursions are treated as perturbations from a linear reference state based on assumed soil strain distributions, the non-linear

stress distributions are assumed to satisfy the equilibrium conditions approximately. The justification for this assumption is that the contact stresses are averaged out by the integrals of Eq.(3). Therefore the dynamic response of the overall system is not sensitive to any small deviation from the true stress distributions in the contact region. To substantiate this conclusion, the nonlinear response of NONSSI was determined by using two different distribution shapes and their numerical values are given in the last two rows of Table 1. The comparison of in-structure response spectra is presented in Figure 6. The general reduction exhibited by the nonlinear response over a wide range of frequency indicates that the effects of the elasto-plastic soil behavior resemble those provided by the aseismic isolators. Typical CPU times required to complete a 10-second dynamic solution will be about 30 to 60 minutes for a model using 52 subregions for the contact area.

### Conclusions

A method has been presented for analyzing the three-dimensional nonlinear SSI considering the elasto-plastic soil behavior. The purpose of adopting the present approach is twofold: (i) to develop an economic computing tool suitable for performing repeated nonlinear SSI analysis as required in a statistical approach or parametric study, and (ii) to provide an independent verification for the results obtained by other nonlinear SSI analyses such as those using finite-element methods. The method provides an efficient means of incorporating nonlinear effects in three-dimensional SSI analyses without the costs associated with nonlinear finite element methods. It is more rigorous than trying to estimate an equivalent reduced shear modulus for the soil under the structure since this reduction compared to the free-field shear modulus is not accurately known and must be assumed to remain constant rather than vary with soil strain resulting from structure response in the time domain as with the method presented here. The technique can easily be adapted to treat other nonlinearities such as sliding and uplift in presence of elasto-plastic soil behavior and strain softening.

The uncertainty involved in prescribing the assumed strain distributions can be handled by parameter-variation studies in which the assigned distributions are varied over all possible shapes. In view of the path-dependency of the nonlinear response, earthquake traces having different temporal characteristics should be used as variations in input motions.

The comparison in response spectra as displayed in Figure 6 indicates that the overall structural system tends to move as a rigid body in the presence of plastic deformation of soil, thereby suppressing the modal response of the fixed-base modes of superstructures. The fact that similar behavior of nonlinear response was not observed in Reference 2 warrants additional research work in this area. Comparison with Reference 3 unfortunately could not be made since no response spectra are presented in this reference.

Inclusion of the nonlinear soil effects has been shown to result in reduced loads throughout the structure especially above the base slab. These reductions are also reflected in reduced seismic input to equipment which is particularly apparent in the amplified portions of the in-structure response spectra. The conservatism which results from using a linear soil-structure interaction model may be acceptable or even desirable in a design analysis. However, for the analysis of many soil-structure interaction systems such as

those which occur during the evaluation of existing structures or in the determination of median centered capacities for probabilistic risk assessment, it is important to determine realistic seismic response characteristics without the introduction of significant but often unknown levels of conservatism. The method described in this paper provides a cost effective means of assessing the effects of nonlinear soil behavior which can be used to quantify this conservatism.

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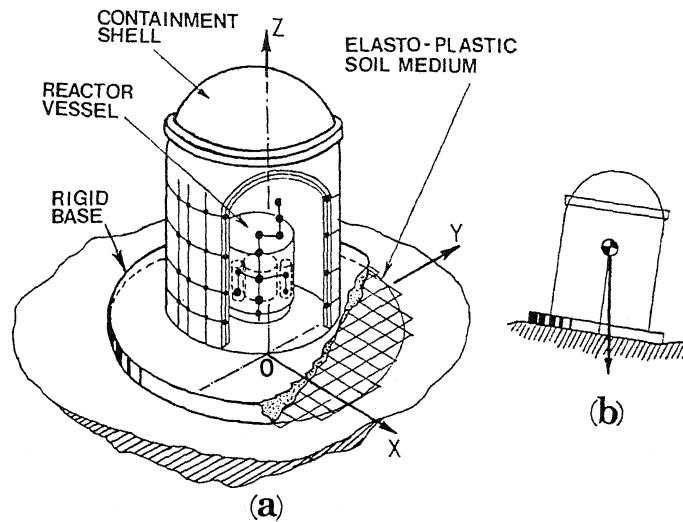


Figure 1. Nonlinear Soil-Structure Interaction Model: (a) Reactor Containment Building; (b) Inverted-Pendulum Effect

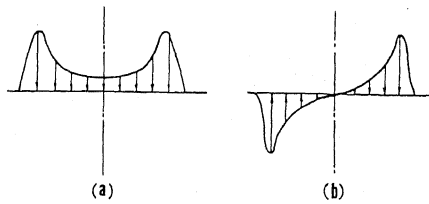


Figure 2. Distribution Functions Used For Strain Field: (a) Vertical Motion; (b) Rocking

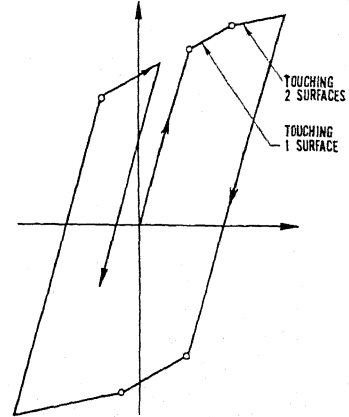


Figure 4. One-Dimensional Stress-Strain Relation Simulated By A Two-Surface Model

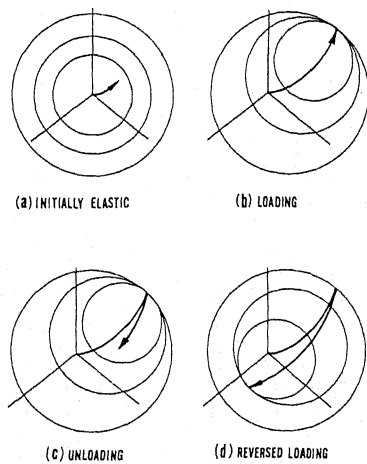


Figure 3. Behavior of A Field of Yield Surfaces

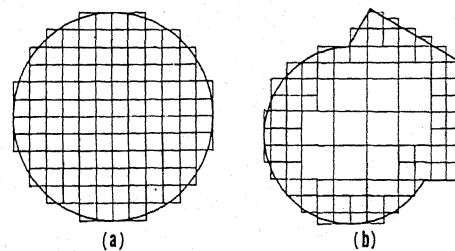


Figure 5. Discretization of Contact Area: (a) Circular Area With Uniform Mesh Size; (b) Arbitrarily-Shaped Area With Nonuniform Mesh Size

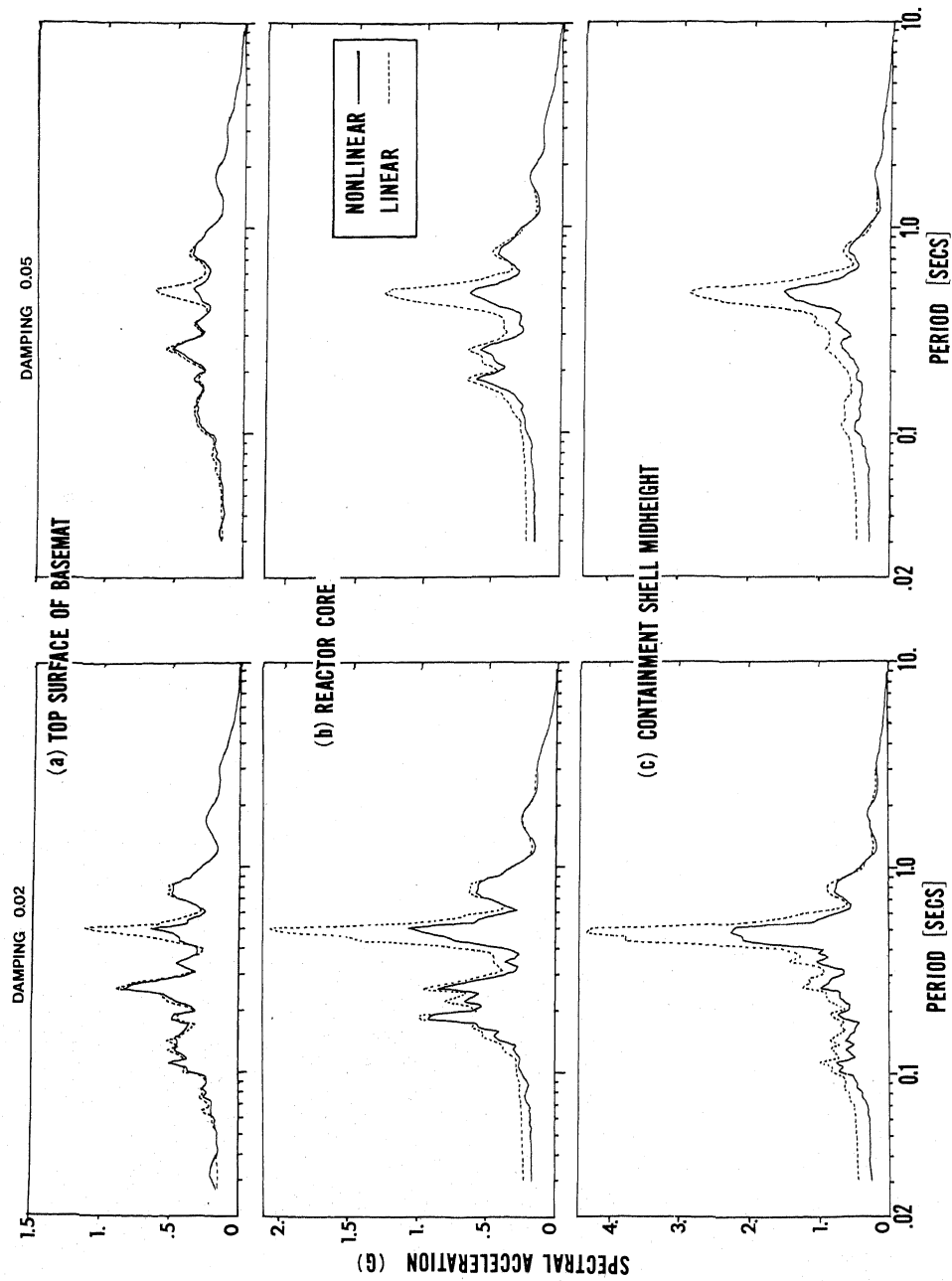


Figure 6. Comparison Of In-Structure Response Spectra