# WAVE PROPAGATION PROCEDURE ON THE INFINITE BOUNDARY FOR DYNAMIC FEM ANALYSIS

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### SUMMARY

A new dynamic FEM analysis for soil-structure interaction problem is presented. The earthquake response can be divided to components of a free field motion and a scattering wave. Furthermore we seperate an analytical area to an interior zone and an exterior zone. Each component in the exterior zone is calculated individually by different approaches. In the interior zone a joined component(sum of free field motion and scattering wave) is calculated by FEM. These components are combined with themselves on the interface between two zones. This procedure is taken advantage of every seismic wave and every arbitrary model.

#### INTRODUCTION

Many numerical methods to analyze a dynamic soil-structure interaction problem of earthquake response have been proposed. Those methods are classified by their techniques; ①multi digree-of-freedom system of lamped mass and spring (MDFS), ②finite defference method(FDM), ③finite element method(FEM), ④boundary integral equation method(BIEM). MDFS and FDM are simpler than either FEM or BIEM, but not adaptable to complicated models. FEM is the most applicable method to several shape models, but it cannot correctly estimate nature of soil as infinite medium. BIEM is the only method that can evaluate soil nature as infinite medium exactly. It, however, restricts strata and model shapes. If the problem of estimating soil nature as infinite half space was resolved, FEM would become the most suitable method for dynamic interaction analysis of earthquake response.

This problem for dynamic FEM analysis consists of two independent problems. The first problem is that scattering waves, which are generated by and radiating from an inhomogeneous area, cannot be absorbed on outer boundaries. Here, this boundary which separate the analytical region and the surrounding soil is defined as an 'infinite boundary'. The second problem is that conventional procedure of arbitrary seismic wave propagating from outside of the infinite boundaries have not yet been established.

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For the first problem, Lysmer and Kuhlemeyer<sup>1)</sup>(1969) proposed a 'viscous boundary', which absorbs scattering waves effectively. Although their method is very convenient for calculation algorism, it is not crrect for waves which incident obliquely through the infinite boundaries. Lysmer<sup>2)</sup>(1970) proposed a 'energy transmitting boundary', which is intended to absorb body waves and surface waves on the lateral infinite boundaries. It cannot, however, be used on the bottom infinite boundaries. Smith<sup>3)</sup>(1974) proposed a 'non-reflecting boundary', which averages two results calculated by Neumann boundary condition and Dirichlet boundary condition. This operation needs one-time excution per one-time wave reflection on infinite boundary. This method entirely eliminates the reflection of every kind of scattering waves, but increase of calculation-time involves increase of additional results calculated by different boundary conditions because scattering waves are reflected several times on the infinite boundaries.

Kunar et. al<sup>4</sup>(1980) and Udaka<sup>5</sup>(1980) resolve the secound problem by means of supplying incident component at the bottom boundaries. It is very suitable method for seismic body waves propagating vertically, but it cannot deal with surface waves or body waves propagating obliquely for input seismic waves. We<sup>6</sup>(1982, 1983)had found a new method that would resolve these problems as for vibration to anti-plane direction. Its method have been developed and the present method is got, which can deal with not only body waves propagating to any direction but also surface waves vibrating within plane direction.

## BASIC THEORY

If soil is a homogeneous half space as shown in Fig.1, dynamic response of soil and structures induced by seismic body waves can be divided to three components of propagating wave: incident wave, reflected wave, and scattering wave. Incident wave is propagating to upward. Reflected wave, which is generated by reflection of incident wave on the free surface of half space medium, is retreating to downward. Scattering wave, which is caused by artificially or naturally irregular topography in a half space, is radiating to outward. In usual soil-structure models scattering waves are induced by three major causes; ①reflection of incident waves on the sides or the buttom face of the structure, ②lack of reflected waves due to existance of the structure, ③vibration of the structure. By the way, the sum of incident waves and

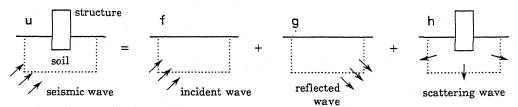


Figure 1. Earthquake response of soil and structure consists of three different components, i.e. f: incidence, g: reflection, h: scatter(u = f + g + h). u: joined component. f+g corresponds to free field motion( $u_f = f + g$ ).

reflected waves are equal to a free field motion, and seismic surface waves correspond to it. Generally in horizontally layered strata, it is difficult to separate the dynamic response induced by seismic waves into these three waves, but it is easy to separate its response into the free field motion and a scattering component. In this paper, dynamic response of soil-structure model is separated into these two components. The free field motion is computed exactly by one of several wave propagation theories and the scattering component is computed numerically.

Instead of separating the response in a whole analytical area, we divide its area to an interior zone and an exterior zone, which are partly overlapped each other as shown in Fig.2. In the interior zone, which is surrounded by free surfaces and restrained boundaries, dynamic response is not separated into each component. All inhomogeneous topographies and structures should exist only in this zone. In the exterior zone, which enclosed by inner boundaries, outer boundaries(i.e. infinite boundaries) and free surfaces, the response is separated into the free field motion and the scattering component. In this zone, free field motion and scattering component are computed individually.

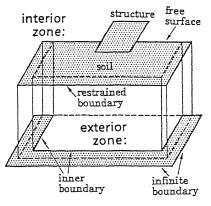


Figure 2. Seperation of analitical area into an interior zone and an exterior zone which partly overlap each other.

Dynamic response in the interior equals to the sum of the free field motion and the scattering component in the boundary. The free field motion can be calculated at any time step and any location point regardless of structure or dynamic interaction. Assuming that all components are known until a certain time-step, a joined component(i.e. dynamic response) at following time-step in the interior except on the restrained boundaries can be computed by step-by-step dynamic FEM analysis. The scattering component in the exterior propagates to only outerward and if its component was given on the inner boundaries, it could be calculated by FEM or FDM. Especially on the infinite boundaries of the exterior, any numerical technique that absorbs propagating waves can be used. Now two unknown components remained, but they will be found due to overlapping zone. A joined component on the restrained boundaries of the interior and a scattering component on the inner boundary of the exterior are unkwon at following time step. By the way, the joined component can be calculated by the sum of the free field motion and the scattering component at the same location in the interior region(eq. (1)). Scattering waves can be also calculated by subtracting free field motion at same location in the boundary from response component in the interior(eq. (2)). (in which uf: free field motion)

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\dot{u} = u_f + h (on the restarined boundary) (1)

h = u - u_f (on the inner boundary) (2)
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# CALCULATION AND RESULT

# Response of Half-Space Medium

At first completely half-spatial and homogeneous medium removed every irregularity is computed. Fig.3~5 show geometrical responses of SV-, P- and rayleigh waves, in which arrows indicate displacements of each point. Soil properties are listed in Table 1 and conditions of input waves are also shown in Table 2. Fig.6 shows seismograms of input waves and responses at a center point on free surface of the model as bold lines. Thin curves indicate responses at the same point calculated by wave propagation theory. In this figure, case I shows only horizontal displacement of SV-wave and caseII shows only vertical displacement of P-wave since these seismic waves, which have a half wavelength, are propagating vertically and perpendicular components are very small. With regard to body waves(i.e. P-wave and SV-wave), responses agree well with the theory although little reverbration, which is almost able to be neglect, remains behind a main schock. Its reverbration is probably referred to discreption of space and time and finite duration time of incident waves because initial edge and final edge of waves contain several



Figure 3. Response of half space medium due to SV-wave. It enters from bottom plane at 0sec and propagating vertically. Arrows indicate displace-ments at each nodal point. Time means second. Period of incident wave is 0.3sec and  $\Delta t$  is 0.01sec. Calculation is done step-by-step in time domain.

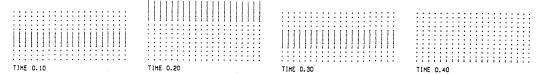


Figure 4. Response of P-wave. Its period is 0.15sec. Other conditions are the same as SV-wave.

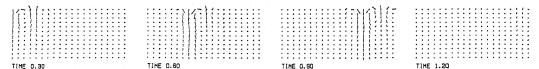


Figure 5. Response of Rayreigh wave(Period: 0.3sec). It enters from left side and propagating laterally. The duration time is equal to one period

high frequency components. It will be easily found if Fourier transform is accomplished as for input wave. Casell showes horizontal and vertical component of Rayreigh wave and its response. The wave lateraly propagates from left hand to right hand and have one wavelength. In this calculation vertical component delays n/2 against horizontal component. The response does not agree very well with the theory. It seems that phase velocity agrees well and group velocity does not agree well. Generally, phase velocity of surface waves depends on its frequency while phase velocity of body waves is independent of frequency, but Rayleigh waves in homogeneous half space medium have no dispersion. Therefore dispersion is not reason of disagreement. The reason, we thought, is time lag between horizontal and vertical components of input wave. Its evidence lies on a figure of casell. It is the respons of Rayleigh wave, which is the same as casell but having two wavelength. Its initial part and final part look like the seismograms of casell, and middle part well agrees well with the theory. This matter explain the fact that horizontal component and vertical component have to co-exist in Rayleigh waves at any time.

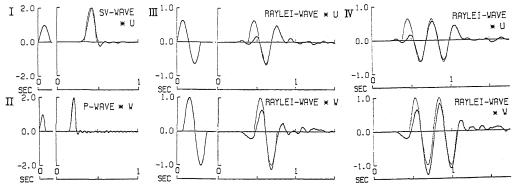


Figure 6. Incident waves and their responses. In each figure the left shows seismic wave just before entering a model and the right shows its response at a center on free surface of the model. Bold curves are calculated by the present method and thin curves are calculated by a wave propagation theory.

Seismogram I shows horizontal response of SV-wave.

Seismogram II shows vertical response of P-wave.

Seismograms III and IV show response of Rayreigh wave having one or two wavelength, respectively.

# Response of Soil-Structure model

Fig.7 shows a geometry of next analytical model. Continous lines indicate FEM mesh which corresponds to the interior zone. The exterior zone, in which FDM analysis is applied to compute scattering waves, is illustrated as assemblage of little open circuls. The interior region except an irregular structure is  $104m\times42m$ . The structure has 30m width, 30m height and 12m depth embedded under ground and therefore total height from its bottom to its top is 42m. Propaties of soil and structure are listed in Table 1. Both soil and structure are completely homogenous.

The exterior is calculated with two different sizes. First model is 152m width ×66m depth and second model is 120m width × 48m depth. They correspond to 20 imes 12 nodal points and 16 imes9 nodal points respectively since mesh size of FDM is 8m × 6m. On the infinite boundaries of the exterior viscous boundary is applied. Although it is not suitable to absorb scattering waves, it is easily managed for calculation. Fig. 9~11 show results of simulation computed with large exterior model and Fig. 8 shows seismograms of their responses at certain points on free surface of the model. Thin curves indicate responses at same location points of half-space model computed by wave propagation theory.

Table 1. Properties of materials.

	Poisson's ratio	S-wave velocity	P-wave velocity
soil	1/3	200	400
structure	1/3	500	1000

(Unit: m/s)

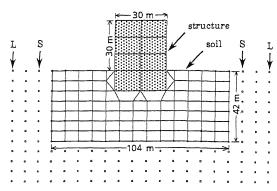


Figure 7. Geometry of analytical model. Area drawn dot mesh indicates structure. The other area indicates soil. Open circles indicate nodal points of FDM. L and S indicate infinite boundaries of large exterior model and small exterior model, respectively.

Table 2. Conditions of input waves.

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	velocity (m/s)	period (sec)	number of waves	
P-wave	400	0.15	1/2	
S-wave	200	0.3	1/2	
Rayleigh	186.5	0.3	1	

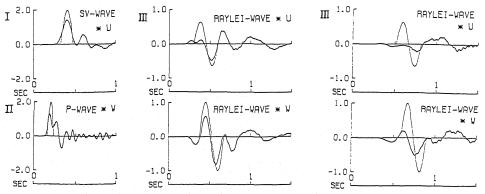


Figure 8. The responses of soil-structure model at certain points as the same hight that free surface of soil.

Seismogram I shows horizontal response of SV-wave at a center of the structure. Seismogram II shows vertical response of P-wave at a center of the structure. Seismograms III show response of Rayreigh wave in the left and the right of the structure.

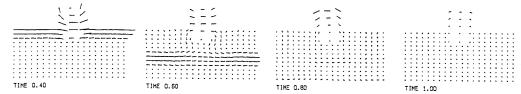


Figure 9. Response of soil-structure model due to SV-wave. Input conditions are the same with Fig.4.

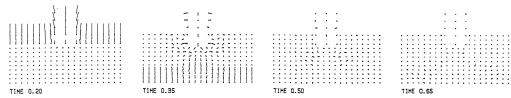


Figure 10. Response of soil-structure model due to P-wave. Input conditions are the same with Figure 5.

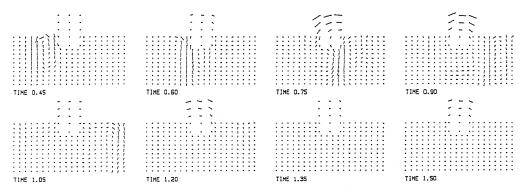


Figure 11. Response of soil-structure model due to Rayleigh wave. Input conditions are the same with Fig.6.

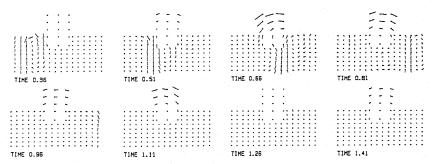


Figure 12. Response of soil-structure model due to Rayleigh wave using a small exterior.

Case I , II show body wave responses at a center of model as same height as free surface. CaseIII shows surface wave response of surface points at about 1/3 and 2/3 of the interior width from its left side. Therefore these points locate in the left and right hands of the structure. It is found from these seismograms that large structures disturb prgression of surface waves. Fig.12 illustrates the same result as Fig.11 except the exterior zone. It is computed with a small exterior model seen in Fig.7. Two results are almost same though the latter exterior zone is less than 60% of the former. This means that analytical zone can be modeled rather small to save calculation time and core size of CPU

### CONCLUSION

From these results, validity of the present method is proved for dynamic analysis, and following conclusions are derived.

- 1. The proposed method can be easily applied to analyze problems of seismic body wave and seismic surface wave identically, and gives very good results.
- 2. Surrounding soil is allowed to form horizontally homogeneous layers since free field motion is solved by wave propagation theory. The model of structure is able to have free geometry.
- 3. Time-integration of dynamic analysis should be calculated only by step-by-step method in time domain. It is not disadvantage since analytical model could be very small.

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# Reference

- 1) Lysmer, J. and R.kuhlemeyer (1969), "Finite Dynamic Model for Infinite media", ASCE, Vol. 95, EM 4, pp. 859-877
- Lysmer, J. (1970), "Lumped Mass Method for Rayleigh Waves", BSSA, Vol.60, pp.89-104
- 3) Smith, W.D.(1974), "Nonreflecting Plane Boundary for Wave Propagation Problems", J. comp. Phys., Vol.15, pp.492-503
- 4) Kunar,R.R. and L.rodriguez-ovejero(1980), "A Model with Non-Reflecting Boundaries for Use in Explicit Soil-Structure Interaction Analysis", Int. J. Earthq. Eng. and Struc. Dynam., Vol.8, pp.361-374
- 5) Udaka, T. (1980), "Super FLUSH User's Manual"
- 6) Akao, Y. and M.Hakuno (1982), "Wave Propagation Procedure on the Infinite Boundary In Dynamic Analysis of Anti-Plane Direction", Proc. of 6th. Jap. Earthq. Eng. Symp. pp.1625-1632
  - Akao, Y. and M. Hakuno (1983), "Dynamic Analysis of Wave Propagation Procedure on the Infinite Boundary", Proc. of JSCE, No. 338, pp. 21-29