CONTINUOUS MODELS FOR FRAME AND SHEAR-WALL BUILDINGS WITH FLEXIBLE FLOORS

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SUMMARY

This paper presents some simple analytical methods for the dynamic analysis of long, narrow buildings that have significant in-plane floor deformations. One- and two-story buildings with stiff end walls have been analyzed by treating their floors and walls as bending and shear beams, respectively. The resulting equations of motion and boundary conditions are solved exactly to obtain the natural frequencies and the mode shapes. Floors of multistory buildings with end walls are modeled as a uniformly-distributed beam system. Buildings with uniformly-distributed frames or walls are modeled as vertically-oriented anisotropic plates.

INTRODUCTION

In the dynamic analyses of buildings, it is normally assumed that the floors are rigid in their own plane. This assumption, although acceptable for many structures, is not realistic for buildings with certain configurations. Forced vibration tests conducted on some buildings (e.g., Ref. 1 and 2) and performance of many others during past earthquakes (some such examples are described in Ref. 3) clearly show that the in-plane floor flexibility of long, narrow buildings and buildings with stiff end walls should be taken into consideration in their earthquake analysis. This paper presents new analytical methods for the treatment of some important classes of buildings for which the floor diaphragm deformations are significant.

EFFECTS OF IN-PLANE FLOOR FLEXIBILITY

Significant in-plane floor flexiblity affects the dynamic behavior of buildings in many ways. First of all, the dynamic properties (e.g., frequencies and mode shapes) are different from those obtained with assumptions of rigid diaphragms. This influences the determinations of total dynamic forces acting on the structure. Also, the distribution of total lateral forces among the various vertical members (frames or walls) is governed by the in-plane stiffness of the floors. Finally, large in-plane floor deformations lead to twisting of vertical members. As a result, the joints between these and the floors, or the vertical member itself can suffer damage during an earthquake if not designed for such deformations.

PROPOSED MODELS AND ASSUMPTIONS

It is assumed that the structure is linearly elastic and that frequencies and mode shapes are not significantly affected by damping. Thus, in the

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analysis presented in this paper, damping is neglected. However, it can be introduced later in the modal equations. The buildings are assumed to be long and narrow, and therefore, in-plane flexibility of the floors is significant only in the transverse direction of the structure. In the longitudinal direction, the building can be analyzed by conventional methods that treat the floor as rigid in its own plane. It is assumed that the lateral load resisting elements in the longitudinal direction (frames and/or walls) do not contribute any stiffness in the transverse direction. However, it is possible to include their influence in these models in an approximate manner (Ref. 3).

The proposed method analyzes single- and double-story buildings with end walls by treating the floors as bending (Bernoulli-Euler) and shear beams, respectively. The dynamic equations of motion and the boundary conditions for the floors and the walls are solved exactly to obtain a transcendental characteristic equation. This equation can be solved numerically to obtain the natural frequencies of the structure, which can be substituted in the appropriate expressions to obtain mode shapes. Once the frequencies and the mode shapes are known for a structure, complete dynamic response can be obtained either by time-history analysis or by response-spectrum techniques.

The floors of multistory buildings with end walls are idealized as equivalent, distributed bending beams while the end walls (or frames) are treated as bending or shear beams. Again, the resulting equations are solved for appropriate boundary conditions. Finally, buildings that consist of a uniform distribution of identical transverse frames (or walls) along the length of the building and of identical floors along the height of the building are modeled as vertically-oriented anisotropic plates, described later in the paper. This last model enables one to draw some useful qualitative conclusions about such structures.

SINGLE-STORY BUILDINGS WITH END WALLS

Long and narrow single-story buildings whose lateral load resistance in the transverse direction is provided by only two walls placed at the two ends, have been of considerable interest in the past (e.g., Ref. 4). Such buildings are commonly used for schools, hospitals or offices where movable partition walls are preferred for functional flexibility.

Consider one such building that has two identical end walls of height h and a long roof of length 2L (Fig. 1). Any intermediate columns that may exist are assumed to be sharing only the vertical loads and provide no lateral support. For vibrations in the z'-direction, the roof can be treated as a bending beam due to its large length-to-width ratio, and the end walls as shear beams (low height-to-width ratio). The equations of motion for free vibrations for the roof and the wall can be written in terms of nondimensional coordinates as:

$$E_1 I_1 u_{,xxxx}(x,t) + m_1 L^4 u_{,tt}(x,t) = 0 \qquad (1a)$$

$$k_2 v_{1,yy}(y,t) - m_2 h^2 v_{1,tt}(y,t) = 0 \qquad (1b)$$

$$k_2 v_{2,yy}(y,t) - m_2 h^2 v_{2,tt}(y,t) = 0 \qquad (1c)$$

in which E_1I_1 is the flexural stiffness of the roof; $k_2(=k'A_2G_2)$ is the shear stiffness of the walls; k' is the shape factor; A_2 is the area of cross-section of the wall; G_2 is the shear modulus for the wall; m_1 and m_2 are mass per unit length or height of the roof and the wall, respectively; and (x,y) are defined as:

$$x = \frac{x'}{L} \quad , \quad y = \frac{y'}{h} \tag{2}$$

Using the symmetry of the structure, only the right half of the structure needs to be analyzed. The boundary conditions for the symmetric (translational) modes of vibration of the structure are:

(1) $u_{,x}(x=0,t) = 0$ (slope is zero at mid-span)

(2) $u_{,xxx}(x=0,t) = 0$ (shear is zero at mid-span)

(3) $v_1(y=0,t) = 0$ (zero displacement at the bottom end of walls)

(4) $u(x=1,t) = v_1(y=1,t)$ (displacement compatability)

(5) $u_{,xxx}(x=1,t) = q_1 v_{1,y}(y=1,t)$ (force balance at the junction)

$$\text{where} \quad q_1 = \frac{k_2 L^3}{E_1 I_1 h} \tag{3}$$

(6) $u_{,xx}(x=1,t) = -q_2 u_{,x}(x=1,t)$ (moment balance at the junction)

$$\text{where} \quad q_2 = \frac{C_2 L}{E_1 I_1} \tag{4}$$

and C_2 is the torsional rigidity of the end walls.

Equations (la,b,c) can be solved using the method of separation of variables for the above boundary conditions (Ref. 3) to obtain the following characteristic equations:

$$(\alpha \cosh \alpha + q_2 \sinh \alpha)(\alpha^3 \sin \alpha \sin \beta - q_1 \beta \cos \alpha \cos \beta) + \\ + (\alpha \cos \alpha + q_2 \sin \alpha)(\alpha^3 \sinh \alpha \sin \beta - q_1 \beta \cosh \alpha \cos \beta) = 0 \tag{5}$$

$$\text{where} \quad \alpha^4 = \frac{m_1 L^4}{E_1 I_1} \omega^2 \quad , \quad \beta^2 = \frac{m_2 h^2}{k_2} \omega^2 \tag{6}$$

and ω is the natural frequency of the structure. Equations (5) and (6) can be solved numerically to obtain the natural frequencies of the system. These can then be substituted into the following expressions to obtain the corresponding mode shapes.

$$U(x) = A[(\alpha \cosh \alpha + q_2 \sinh \alpha) \cos \alpha x + (\alpha \cos \alpha + q_2 \sin \alpha) \cosh \alpha x] \qquad -1 \le x \le 1 \qquad (7a)$$

$$V_1(y) = V_2(y) = A[\cos \alpha \, (\alpha \cosh \alpha + q_2 \sinh \alpha) + \cosh \alpha \, (\alpha \cos \alpha + q_2 \sin \alpha)] \cdot \frac{\sin \beta y}{\sin \beta} \qquad 0 \le y \le 1 \qquad (7b)$$

Similar expressions for antisymmetric (torsional) modes of vibration have been derived in Ref. 3. In general the torsional stiffness of the end walls (C_2) is quite insignificant and can be neglected. Hence, expressions (5) and (7) can be further simplified by taking the limit as $q_2 \to 0$. Ref. 3 also illustrates the use of perturbation techniques to obtain the fundamental frequency of the structure in an approximate, simpler manner.

The method described herein can also be applied for more complex single-story buildings and similar two-story structures. Some of these cases are solved in Ref. 3.

MULTISTORY BUILDINGS WITH END WALLS

The method described in the previous section becomes less attractive for buildings with several stories due to the increased complexity of the algebra. Therefore, for multistory buildings with two end walls, another approach is proposed. This requires an additional approximation wherein the floors are idealized as a continuous beam system in which the mass and the stiffness properties of the floors are distributed uniformly over the height of the structure. These distributed beams are such that the adjacent beams of infinitesimal depth have no contact with each other. In this approach, one needs to write only one differential equation for the floor system and one differential equation for each end wall.

Consider one such structure as shown in Fig. (2). The floors, being long and narrow, can be treated as bending beams. The end walls are assumed to have large height-to-width ratio and are treated as bending beams. They can, however, be modeled by shear beams or even by Timoshenko beams depending upon the building configuration. The equations of motion for the floor system and the right-side wall can be written in the nondimensional coordinates x and y as

$$E_1 I_1{}^* u_{,xxxx}(x,y,t) + m_1{}^* L^4 u_{,tt}(x,y,t) = 0 \qquad (8a)$$

$$E_2 I_2 v_{1,yyyy}(y,t) + m_2 h^4 v_{1,tt}(y,t) = \frac{E_1 I_1{}^* h^4}{L^3} u_{,xxx} \qquad (x=1,y,t) \qquad (8b)$$

where E_1 and E_2 are the modulus of elasticity for the floors and the walls, respectively; $I_1{}^*$ is the moment of inertia of floor-system cross-section per unit height; $m_1{}^*$ is the mass per unit area (in x'-y' plane) of the floors; I_2 is the moment of inertia of the end wall cross-section; and m_2 is the mass per unit height of walls.

The last term in equation (8b) is introduced by end shears in the distributed floor system. These equations can be solved for appropriate boundary conditions (Ref. 3) to obtain the following characteristic equation and mode shape expressions for the symmetric modes of vibration of the structure:

$$\cos\beta\cosh\beta + 1 = 0 \tag{9a}$$

where

$$\beta^4 = \frac{m_2 h^4}{E_2 I_2}\omega^2 + \frac{E_1 I_1{}^* h^4}{E_2 I_2 L^3}\frac{\alpha^3}{2}(\tan\alpha + \tanh\alpha) \tag{9b}$$

$$\alpha^4 = \frac{m_1{}^* L^4}{E_1 I_1{}^*}\omega^2 \tag{9c}$$

and

$$V_1(y) = V_2(y) = B\left[\frac{\sin\beta y - \sinh\beta y}{\sin\beta + \sinh\beta} - \frac{\cos\beta y - \cosh\beta y}{\cos\beta + \cosh\beta}\right] \qquad 0\le y\le 1 \tag{10a}$$

$$U(x,y) = \frac{B}{2}\left[\frac{\cos\alpha x}{\cos\alpha} + \frac{\cosh\alpha x}{\cosh\alpha}\right]V_1(y) \qquad \begin{array}{c}-1\le x\le 1\\ 0\le y\le 1\end{array} \tag{10b}$$

where x and y are defined by equations (2). The above expressions have been obtained for the case where the torsional stiffness of end walls is negligible. Expressions for antisymmetric (torsional) modes of vibration and for the case when the end walls (or frames) are more appropriately modeled as shear beams can be found in Ref. 3. Note that the equation (9a) is the same as the characteristic equation of a cantilever beam.

MULTISTORY BUILDINGS WITH UNIFORMLY DISTRIBUTED FRAMES (OR WALLS)

This important class of buildings consisting of several frames (or walls) that are uniformly spaced along the length of the building, is modeled as a vertically-oriented anisotropic plate. The plate model is the two-dimensional analog of the shear-beam models that are often used for studying the dynamics of buildings (e.g., Ref. 5). The plate is such that a horizontal strip of the plate has only bending flexibility while vertical strips have only shear flexibility (or bending flexibility in case of walls). The twisting stiffness of the floors and the frames (or walls), being small compared to the bending stiffness, is neglected. This leads to the following equation of motion for the plate (Ref. 3):

$$\overline{D}_1 \frac{\partial^4 w(x,y,t)}{\partial x^4} - \overline{K}_2 \frac{\partial^2 w(x,y,t)}{\partial y^2} = -m\frac{\partial^2 w(x,y,t)}{\partial t^2} \tag{11}$$

in which $\overline{D}_1$ is the flexural stiffness of a horizontal strip of the plate, of unit width; $\overline{K}_2$ is the shear stiffness of a vertical strip of the plate, of unit width; and m is the mass per unit area (in x-y plane) of the plate. The coordinate system is shown in Fig. (3).

Due to the absence of cross derivatives, this equation can be solved by the method of separation of variables for the boundary conditions: fixed at the bottom and free at the remaining edges. The resulting expressions for the frequency and mode shape are:

$$\omega_{ij} = \frac{(2j-1)\pi}{2h}\sqrt{\frac{\bar{K}_2}{m}} \qquad \begin{array}{l} i=1,2 \\ j=1,2,3,\ldots \end{array} \tag{12a}$$

$$\omega_{ij} \simeq \left[\frac{\left(i-\frac{3}{2}\right)^4 \pi^4 \bar{D}_1}{ml^4} + \frac{(2j-1)^2\pi^2\bar{K}_2}{4mh^2}\right]^{\frac{1}{2}} \qquad \begin{array}{l} i=3,4,5,\ldots \\ j=1,2,3,\ldots \end{array} \tag{12b}$$

$$W_{1j}(x,y) = A \sin\frac{(2j-1)\pi}{2h} y \qquad j=1,2,3,\ldots \tag{13a}$$

$$W_{2j}(x,y) = A\,(x-\tfrac{1}{2}) \sin\frac{(2j-1)\pi}{2h} y \qquad j=1,2,3,\ldots \tag{13b}$$

$$W_{ij}(x,y) = A\left[\frac{\sin\alpha_i x + \sinh\alpha_i x}{\sin\alpha_i l - \sinh\alpha_i l} - \frac{\cos\alpha_i x + \cosh\alpha_i x}{\cos\alpha_i l + \cosh\alpha_i l}\right]\cdot$$

$$\cdot \sin\frac{(2j-1)\pi y}{2h} \qquad \begin{array}{l} i=3,4,5,\ldots \\ j=1,2,3,\ldots \end{array}, \quad \cos\alpha l \cosh\alpha l = 1 \tag{13c}$$

This indicates that the frequencies of the structure can be obtained by simply taking the square root of the sum of the squares of the floor frequencies, when treated as free-free beams, and of the frame frequencies. Similarly, the mode shapes can be obtained by superposition of the floor modes and the frame modes. Due to the rigid body modes of the floors with free-free boundary conditions, the structure possesses all the modes that one obtains by analysis based on the assumption of rigid floors, plus some additional modes involving in-plane deformations. It can be shown (Ref. 3) that the modes with diaphragm deformations have zero participation factors for uniform ground motion. Thus, it is concluded that such buildings can be analyzed for uniform earthquake motion with the usual rigid-floor assumption without introducing an additional approximation. This result has also been shown for a discrete lumped-mass model of such structures (Ref. 6). A similar result holds when the building has a uniform distribution of walls instead of frames. It is interesting to note the results of a finite element, parametric study on a building with five cross-walls reported by Unemori, et al. (Ref. 7). They found that the modes involving floor diaphragm deformations had very small participation factors for uniform ground motion.

DISCUSSION AND CONCLUSIONS

Simple analytical procedures have been presented for the dynamic analysis of several classes of buildings that have the possiblity of significant inplane floor deformations. The proposed methods are exact for some of the buildings. For others, although approximate, they yield very useful information about the way such structures behave dynamically, without requiring significant numerical effort. Because of their simplicity, they can be employed as useful design tools, to evaluate whether or not the floor deformations are significant, and if so, their impact on the determination of the dynamic forces on the structure. Even though relatively simple structures have been dealt with in the present paper, the technique is general enough to be used for more complex structural systems. For example, the method has been applied to the earthquake response of the Imperial County Services Building in its transverse direction (Ref. 3). This six-story structure consisted of two rather stiff end walls in the upper stories and several walls in the ground story, thus requiring the second floor slab to transfer the lateral forces from the end walls above it to the first-story walls below.

An anisotropic plate has been proposed to model buildings with uniformly placed frames or walls. This model is also conveniently solved for mode shapes, frequencies and participation factors. From these results, it has been shown that although these structures possess modes involving significant floor deformations, these modes are not excited by the uniform ground motion. Thus, this last category of buildings can be analyzed by conventional methods, treating the floors as rigid in their own planes.

ACKNOWLEDGMENTS

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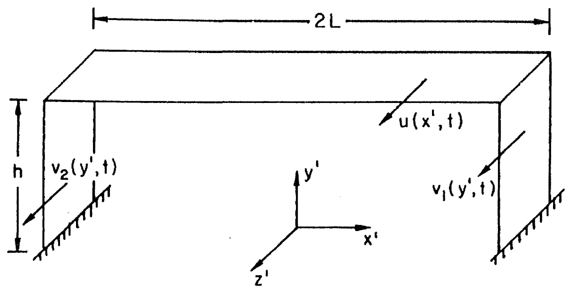

Fig. 1. Model of a One-Story Building with End Walls.

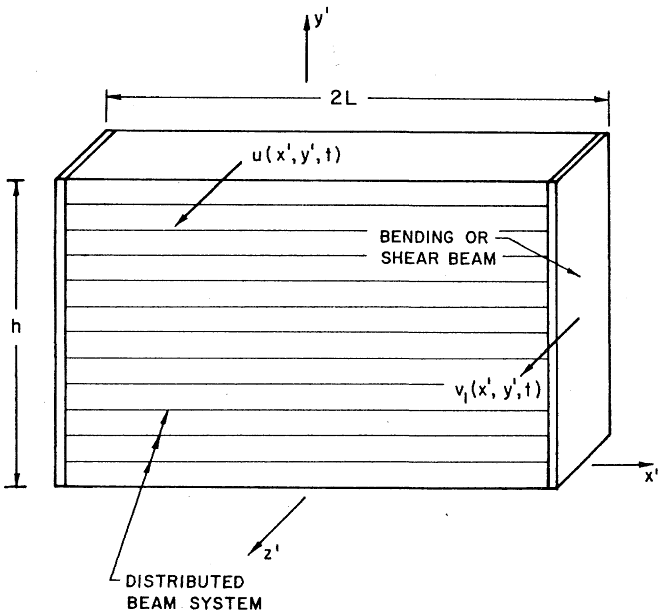

Fig. 2. Model of a Multistory Building with End Walls.

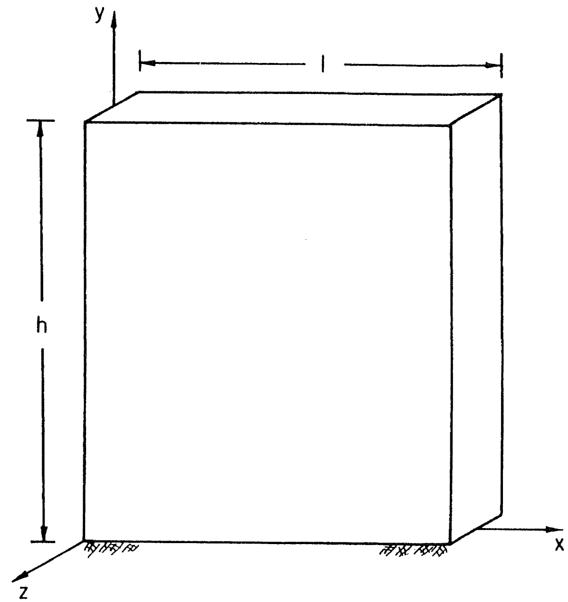

Fig. 3. Continuous Model for Buildings with Uniformly Distributed Frames or Walls.