# SHEAR CAPACITY OF R/C SHORT COLUMN

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## SUMMARY

This paper presents a method to estimate the shear capacity of R/C column. The shear capacity is defined at the yielding of shear reinforcements crossing to the diagonal crack. An analytical procedure for the premature shear failure due to bond splitting of longitudinal reinforcement is also presented. By comparing with the past experimental data, it is shown that the proposed methods can be applicable to evaluate the shear capacities of R/C columns.

## INTRODUCTION

Under the earthquake load condition, the R/C columns in a framed structure are subjected to combined shear and flexure in addition to axial force. Therefore the column may fail in shear unless the adequate amount of shear reinforcement is arranged at the critical portion, and at worst results in the collapse of whole structure. In order to prevent such shear failure the column has to be designed according to the rational design formulae which is based on the shear resisting mechanisms. The object of this study is to make clear the shear resisting mechanisms of R/C column through the detailed investigations on the experimental data. On the basis of proved mechanisms, a evaluation method for shear capacity of R/C short column is proposed.

## DIAGONAL TENSION CRACKING DUE TO FLEXURAL BOND

The flexural bond seems to be necessary to explain the mechanisms of diagonal tension cracking in the light of Kani's theory (Ref.1). It is intended to derive an equation to estimate the shear force at diagonal tension cracking on the basis of the flexural bond action.

### Flexural bond stress

Flexural bond stress is given by the following well known equation for cracked beam section.

$$\tau_b = Q / jd\cdot\Sigma\psi \qquad (1)$$

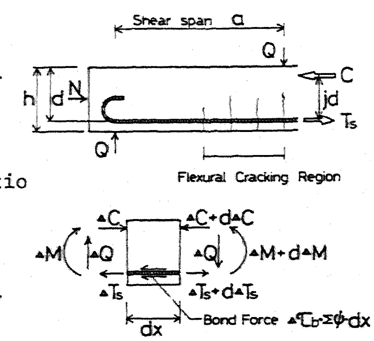
Q:shear force, d:effective depth, j:lever arm ratio
$\Sigma\psi$:sum of perimeter of tensile reinforcement

On the other hand in case of column Eq.1 is not applicable due to the existence of axial force. The moment at certain cracked section is given by

$$M = T\cdot jd + (h/2-d+jd)\cdot N \qquad (2)$$

T:tension force of longitudinal reinforcement
h:over all depth of section, N:axial force

From Eq.2 moment increment $\Delta M$ is given by $\Delta M=\Delta Tjd$ provided that the change of lever arm jd corre-




$M = T_s\cdot jd + N(\frac{h}{2}-d+jd)$ $\quad \Delta M = \Delta T_s jd$

$\Delta\tau_b = \frac{d\Delta T_s}{dx\cdot\Sigma\psi} = \frac{d\Delta M}{dx}\cdot\frac{1}{\Sigma\psi jd} = \frac{\Delta Q}{\Sigma\psi\cdot jd}$

Fig.1 Flexural bond in R/C column

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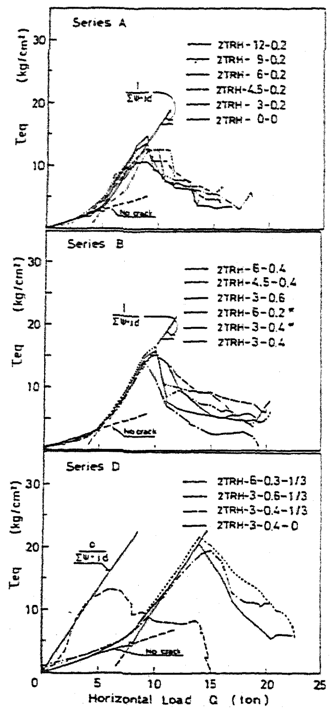


Fig.2 Nominal flexural bond stresses

sponding to ΔM is ignored. For the short length dx of reinforcement, flexural bond stress for cracked section can be expressed by

$$\Delta\tau_b = \Delta Q \;/\; jd \qquad \text{———} \quad (3).$$

This calculation procedure is shown in Fig.1. For uncracked section, the tensile force of longitudinal reinforcement is given by

$$T = M\cdot(d-h/2)\cdot n\cdot A_{st} \;/\; I_{eq} \qquad \text{———} \quad (4).$$

n:modular ratio, $A_{st}$:area of tension reinforcement, $I_{eq}$:moment of inertia for uncracked section transformed to concrete

By differenciating Eq.4 with respect to x,

$$dT/dx = Q\cdot(d-h/2)\cdot n\cdot A_{st} \;/\; I_{eq} \qquad \text{———} \quad (5).$$

Nominal bond stress is obtained as follows.

$$\tau_b = Q\cdot(d-h/2)\cdot n\cdot A_{st} \;/(I_{eq}\;\Sigma\psi) \quad \text{——} \quad (6)$$

To investigate the applicability of Eq.3 and Eq.6 with experimental data, the results of author's column shear tests (details are described in next chapter) are reffered. In Fig.2 nominal flexural bond stresses of tension reinforcement between sections A and B of tested columns (see Fig.6) are plotted against applied shear forces. In the derivation of flexural bond stresses $\tau_{eq}$ Eq.7 is used , where the contribution of shear reinforcement on the bond stress are took away by cosidering the fan-shaped internal truss mechanism as shown in Fig.3.

$$\tau_{eq} = [(T_a - T_b) - \Sigma(T_{wi}\cdot l_i \;/\; d)] \;/\; (\tilde{l}\cdot\Sigma\psi) \;\text{—}\; (7)$$

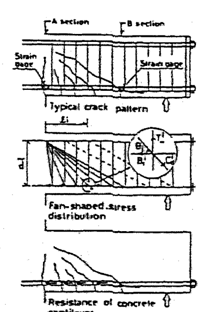


Fig.3 Internal mechanisms

$T_a$,$T_b$:tensile forces of longitudinal reinforcement at sections A and B, $T_{wi}$:tensile force of ith shear reinforcement, $\tilde{l}$:length of longitudinal reinf. from section A to B

In Fig.2, two straight lines are also drawn and each of them corresponds to Eq.2 and Eq.6. From the inspection of experimental curves, shear force $Q_o$ at the intersection of two lines is determined as the shear corresponding to a certain moment at which the stress of tension reinforcement becomes zero at critical section. From the comparison between experimental curves and two straight lines, it is recognized that Eq.3 and Eq.6 can be applicable to estimate the flexural bond stress with sufficient accuracy.

Diagonal tension cracking

Flexural bond stress in Fig.2 must be carried by the concrete cantilevers before diagonal tension cracking. In Fig.2, sharp peaks can be seen at certain values of $\tau_{eq}$ and from experimental observations it is confirmed that these peaks correspond to the diagonal tension cracking. Based on such fundamental considerations, an equation to estimate the shear force $Q_s$

is derived by following procedure. Flexural bond stress at diagonal tension cracking can be given by Eq.8.

$$\tau_b=Q_o\cdot(d-h/2)\cdot n\cdot A_{st}\ /\ (I_{eq}\cdot\Sigma\psi)+(Q_s-Q_o)\ /\ jd\cdot\Sigma\psi \quad (8)$$

From the failure condition of concrete cantilever a following equation is obtained (see Fig.4).

$$\tau_b\cdot C_w\cdot\Sigma\psi = b\cdot s^2\cdot f_t\ /\ (6\cdot s) \quad (9)$$

By substituting Eq.8 into Eq.9, we obtain

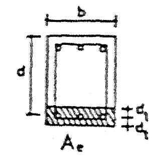

Fig.4

$$Q_s=\frac{7}{48}f_t\cdot b\cdot d\cdot(C_w/s)+[1-\frac{7}{8}d\cdot(d-h/2)\cdot n\cdot A_{st}/I_{eq}]\cdot Q_o \quad (10)$$

$Q_o=\sigma_o\cdot I_{eq}/(d-h/2)/a$, $j=7/8$, $\sigma_o$:compressive stress at centroidal axis due to axial force, s:length of concrete cantilever, $C_w$:crack spacing, $f_t$:tensile strength of concrete, b:width of section, a:shear span length

In Eq.10, it is assumed that $C_w$ value can be expressed in a form of $C_w=\alpha+\beta(A_e/m)$ according to Morita's proposal (Ref.2). From this assumption the following equation is given.

$$(C_w/s/d)= k_1 + k_2\cdot(A_e/m) \quad (11)$$

In Eq.11, two coefficients $k_1$ and $k_2$ are determined by applying a regression analysis in order that the values from Eq.10 become as possible as close to the diagonal tension cracking loads of 175 column shear tests which have been performed in Japan (Ref.3). Finally a following equation is derived.

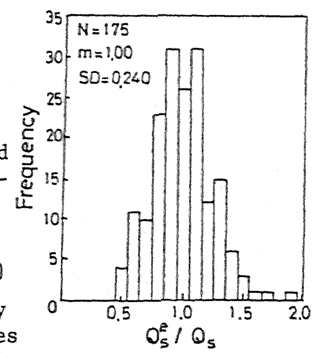

Fig.5 Hystogram for $Q_s^e$ / $Q_s$ values

$$Q_s= [\ 9.11+0.0716\cdot(A_e/m)]\cdot f_t\cdot b + [\ 1-\frac{7}{8}d\cdot(d-h/2)\cdot n\cdot A_{st}/I_{eq}\ ]\cdot Q_o \quad (12)$$

$A_e$:effective embedment area, m:number of re-bar (see Fig.4) (unit:kg,cm)

The comparison between calculated values of $Q_s$ and experimental values $Q_s^e$ are shown in Fig.5 as the histogram of the ratios of them.

## EFFECTIVENESS OF TRANSVERSE REINFORCEMENT

To elucidate the reinforcing efficiency of shear reinforcement, the column shear tests were carried on 15 specimens. Each specimen having 25x25 cm squre section was reinforced longitudinally by eight 16 mm in dia. deformed bars with the yield strength of 4990 kg/cm$^2$. As the shear reinforcement, high strength steel bars were arranged passing through the ducts as shown in Fig.6. Tensile forces of these bars $F_b$ were adjusted to satisfy the given conditions ,that is, in elastic range $F_b=p_w\ b\ E_s\ w_e\ l_d/l_h$ and after yielding of shear reinforcement $F_b=p_w\ b\ l_d\ {}_wf_y$ where $p_w$:shear reinforcement ratio, $w_e$:shear crack width measured in transverse direction, $l_d$:spacing of shear reinforcement, $l_h$:distance between tension and compression reinforcement. Details of each specimen are summerized on table 1. Obtained Q (horizontal shear force) $-\delta_t$ (deflection at column top) curves are shown in Fig.7. In Fig.8, the relations between Q and $Q_R$ (carried shear force by shear reinforcements crossing to the diagonal crack) are drawn for each test series. From these figures,

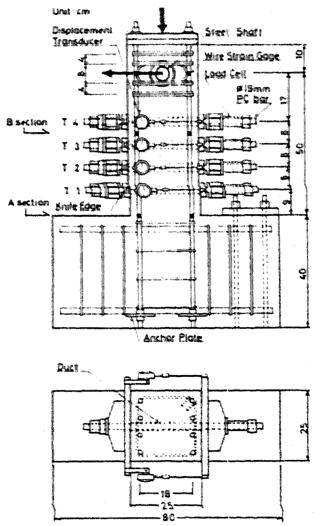


Fig.6 Details of specimen

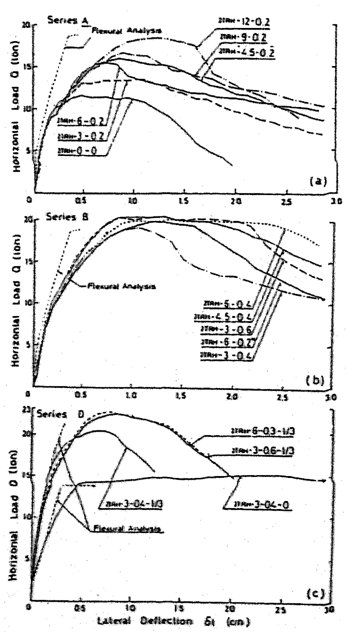


Fig.7 $Q-\delta_t$ curves

Table 1 List of test specimens

| SERIES | SPECIMEN | SHEAR REINFORCEMENT $p_w$(%) | SHEAR REINFORCEMENT $_wf_y$(kgf/cm²) | STRENGTH OF CONCRETE $F_c$(kgf/cm²) | STRENGTH OF CONCRETE $F_t$(kgf/cm²) | AXIAL LOAD N(tonf) |
|---|---|---|---|---|---|---|
| A | 2TRH-0-0 | - | - | 238 | 26.4 | 27 |
| | 2TRH-3-0.2 | 0.2 | 3000 | " | " | " |
| | 2TRH-4.5-0.2 | 0.2 | 4500 | " | " | " |
| | 2TRH-6-0.2 | 0.2 | 6000 | " | " | " |
| | 2TRH-9-0.2 | 0.2 | 9000 | " | " | " |
| | 2TRH-12-0.2 | 0.2 | 12000 | " | " | " |
| B | 2TRH-3-0.4 | 0.4 | 3000 | 234 | 23.2 | 27 |
| | 2TRH-4.5-0.4 | 0.4 | 4500 | " | " | " |
| | 2TRH-6-0.4 | 0.4 | 6000 | " | " | " |
| | 2TRH-3-0.6 | 0.6 | 3000 | " | " | " |
| | 2TRH-6-0.2* | 0.2 | 6000 | " | " | " |
| D | 2TRH-3-0.4-1/3 | 0.4 | 3000 | 242 | 25.6 | 54 |
| | 2TRH-3-0.6-1/3 | 0.6 | 3000 | " | " | " |
| | 2TRH-6-0.3-1/3 | 0.3 | 6000 | " | " | " |
| | 2TRH-3-0.4-0 | 0.4 | 3000 | " | " | 0 |

it is recognized that the shape of $Q-Q_R$ curves are quite similar to each other after the diagonal cracking regardless of different values of $p_w$, $_wf_y$, $p_w \cdot {}_wf_y$ and axial load level. From these curves, a relation between $p_w \cdot {}_wf_s$ and $v_R$ (increment of nominal shear stress after diagonal tension cracking) can be derived by following procedure. From $Q_R = p_w \cdot b \cdot \alpha \cdot jd \cdot {}_wf_s$, we obtain

$$p_w \cdot {}_wf_s = Q_R/(b \cdot \alpha \cdot jd), \quad v_R = (Q - Q_s^e)/(b \cdot jd) \quad \text{—} \quad (13)$$

$Q_s^e$:shear force at diagonal cracking, $\alpha$:coefficient to represent the diagonal cracking region

As shown in Fig.8, some columns showed more increase of load carrying capacity after the yielding of shear reinforcements. However this additional capacity is not so reliable and should not be expected to avoid the excessive opening of diagonal crack. On such point of view, if the available limit of reinforcing efficiency of shear reinforcement can be defined at the yielding of that, a capacity equation corresponding to the shear reinforcement is obtained by replacing $_wf_s$ with $_wf_y$ in $v_R - p_w \cdot {}_wf_s$ (product of the ratio and the average stress of shear reinforcement) relations. Results obtained are shown in Fig.9 and a best fitting curve is also determined by using a least square method.

$$v_R = 3 \cdot (p_w \cdot {}_wf_y)^{5/8} \quad \text{———} \quad (14)$$

(unit:kg/cm²)

PREMATURE FAILURE IN BOND SPLITTING

There are two types of premature failure before the yielding of shear reinforcements. One is a failure due to the crushing of web concrete. Another one is a progressive failure in bond splitting of longitudinal reinforcement. Here it is intended to introduce an analytical method to calculate the shear strength of column failing in

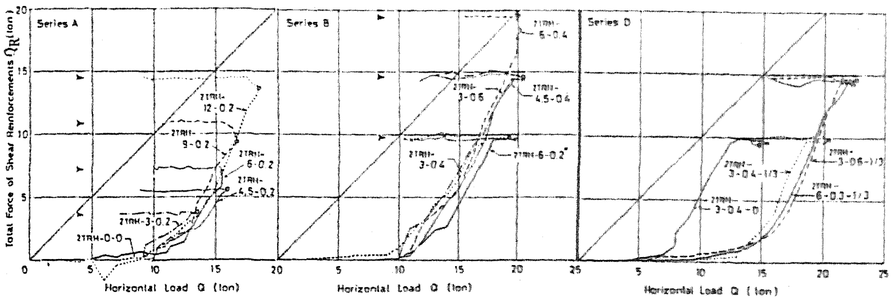


Fig.8 $Q$-$Q_R$ relations for each test series

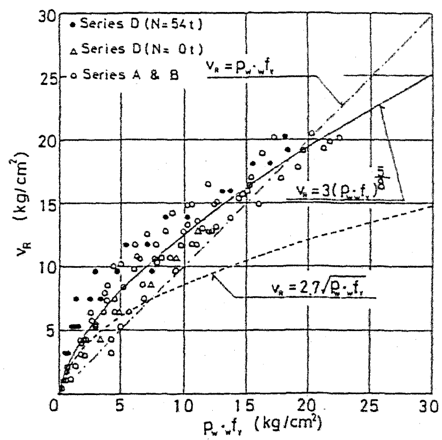


Fig.9 $v_R$-$p_w \cdot {}_wf_y$ relations

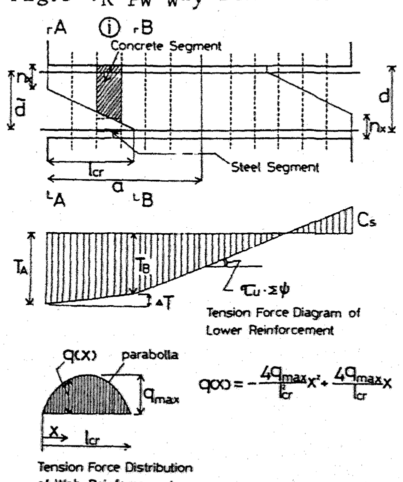


Fig.10 Basic assumptions

bond splitting. This method can be applicable to columns subjected to monotonically increasing load and the basic assumptions are summerized in Fig.10. In the analysis , a concrete free body formed by two diagonal cracks is devided into finite concrete segments along column axis. Out of this free body, fan-shaped compression stress field is assumed as shown in Fig.11. In Fig.10, section B is determined as the intersection of diagonal crack and longitudinal reinforcement, and the distance from critical section A is represented by $l_{cr}$.

$$l_{cr} = (1 - Q_o / Q_s) \cdot a \qquad (15)$$

$Q_s$:diagonal tension cracking load (Eq.12)
$Q_o$:shear at which the stress of tension reinforcement becomes zero at section A
a:shear span (half of column length)

According to Eq.14, the shear force carried by shear reinforcements can be obtained by

$$Q_R = v_R \cdot b \cdot jd = 3 \cdot (p_w \, {}_wf_s)^{5/8} \cdot b \cdot jd \qquad (16).$$

where ${}_wf_s$ is an average stress of shear reinforcements. From the assumption of parabollic distribution for the stress of shear reinforcements as shown in Fig.10,

$$2 \cdot q_{max} \cdot l_{cr}/3 = p_w \cdot {}_wf_s \cdot b \cdot l_{cr} \qquad (17).$$

By eliminating $p_w \, {}_wf_s$ from Eq.16 and Eq.17, we obtain

$$q_{max} = 1.5 \cdot (Q_R/2.63/b/d)^{1.6} \, b \qquad (18).$$

Tension force of longitudinal reinforcement at certain section being $x_i$ apart from the critical section is given by Eq.19 according to the fan-shaped internal truss.

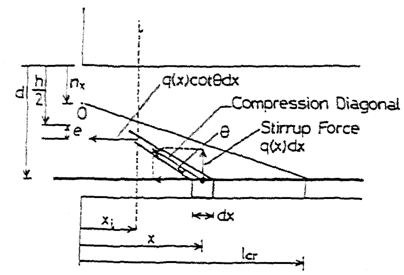


Fig.11 Fan-shaped truss mechanism

$$T_{xi}=T_a-\left[\frac{q_{max}}{d-n_x}\right]\left[-\frac{4x_i^4}{4l_{cr}^2}+\frac{4x_i^3}{3l_{cr}}\right] \quad \text{—— (19)}$$

$T_a$: tension force of longitudinal reinforcement at critical section, $n_x$: newtral axis depth, $l_{cr}$: length of shear cracking region

Tension force of top or bottom reinforcement is given by Eq.20 or Eq.21 (see Fig.12) where $\tau_u$ is the bond splitting strength of longitudinal reinforcement.

For top bar,

$$T^u_{xi}=C_s+x_i\cdot\tau_u\cdot\Sigma\psi=T_b-(2\cdot a-l_{cr}-x_i)\tau_u\cdot\Sigma\psi \quad (20)$$

For bottom bar,

$$T^l_{xi}=\begin{cases} T_a-\dfrac{q_{max}}{d-n_x}\left[\dfrac{-4x_i^4}{4l_{cr}^2}+\dfrac{4x_i^3}{3l_{cr}}\right] & 0<x_i\leq l_{cr} \\ T_b-(x_i-l_{cr})\cdot\tau_u\cdot\Sigma\psi & l_{cr}<x_i\leq a \end{cases} \quad (21)$$

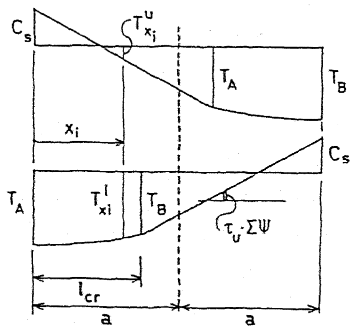


Fig.12 Force distribution of longit. reinf.

Resultant forces and internal moments at any section having a distance of $x_i$ from critical section A can be expressed by following equations for each resisting mechanism.

For fan-shaped truss mechanism,

$$N_{fan}(x_i)=\int_{x_i}^{l_{cr}}q(x)\cot\theta dx=\frac{1}{d-n_x}\int_{x_i}^{l_{cr}}q(x)xdx \quad -(22)$$

$$M_{fan}(x_i)=\left(\frac{h}{2(d-n_x)}-\frac{d}{d-n_x}+1\right)\int_{x_i}^{l_{cr}}q(x)xdx-x_i\int_{x_i}^{l_{cr}}q(x)dx \quad -(23)$$

$q(x)$: tension force of shear reinforcement per unit column length

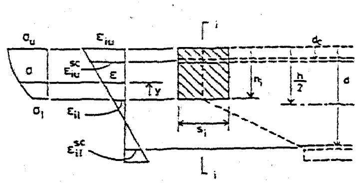

Fig.13 Stress strain distributio of concrete segment

For concrete segments (see Fig.13),

$$N_c(x_i)=\frac{b\,n_i}{\varepsilon_{iu}-\varepsilon_{il}}\int_{\varepsilon_{il}}^{\varepsilon_{iu}}\sigma\,d\varepsilon \quad \text{—— (24)}$$

$$M_c(x_i)=\left[\frac{b\,n_i(0.5h-n_i)}{\varepsilon_{iu}-\varepsilon_{il}}-\left(\frac{n_i}{\varepsilon_{iu}-\varepsilon_{il}}\right)^2\cdot\varepsilon_{il}\cdot b\right]\int_{\varepsilon_{il}}^{\varepsilon_{iu}}\sigma d\varepsilon+b\left(\frac{n_i}{\varepsilon_{iu}-\varepsilon_{il}}\right)^2\int_{\varepsilon_{il}}^{\varepsilon_{iu}}\sigma\varepsilon d\varepsilon \quad -(25)$$

$n_i$ height of concrete segment, $\varepsilon_{iu}$, $\varepsilon_{il}$: top and bottom fiber strains of concrete segment, $\sigma$: function for stress strain curve of concrete, $\varepsilon$: strain of concrete

For longitudinal reinforcement,

$$N_s(x_i)=-(T^u_{xi}+T^l_{xi}) \quad \text{— (26)} \qquad M_s(x_i)=(d-h/2)(T^l_{xi}-T^u_{xi}) \quad \text{— (27)}$$

From Eq.22 to Eq.27, we obtain the resultant force $N_i$ and internal moment $M_i$.

$$N_i = \begin{cases} N_{fan}(x_i)+N_c(x_i)+N_s(x_i) & 0 < x_i \le l_{cr} \\ N_c(x_i)+N_s(x_i) & l_{cr} < x_i \le a \end{cases} \qquad (28)$$

$$M_i = \begin{cases} M_{fan}(x_i)+M_c(x_i)+N_s(x_i) & 0 < x_i \le l_{cr} \\ M_c(x_i)+M_s(x_i) & l_{cr} < x_i \le a \end{cases} \qquad (29)$$

Failure condition is given by the crushing of concrete at critical section ,that is, extreme fiber strain of concrete attains the available limit strain in flexural compression (Ref.4).

In the numerical calculation, a special compatibility condition is used ,that is, the elongation of longitudinal reinforcement along the column length should be identical to the virtual elongation of concrete at the layer of that. This condition has been used for the analysis of unbonded prestressed concrete. The detailed procedure is shown in Fig.11 in a form of computer flow chart, where $\varepsilon_{iu}^{sc}$ and $\varepsilon_{il}^{sc}$ are the virtual concrete strains at the layer of longitudinal reinforcement, $\varepsilon_{iu}^{ss}$ and $\varepsilon_{il}^{ss}$ are the strains of upper and lower reinforcements. Compatibility condition is given by following equations.

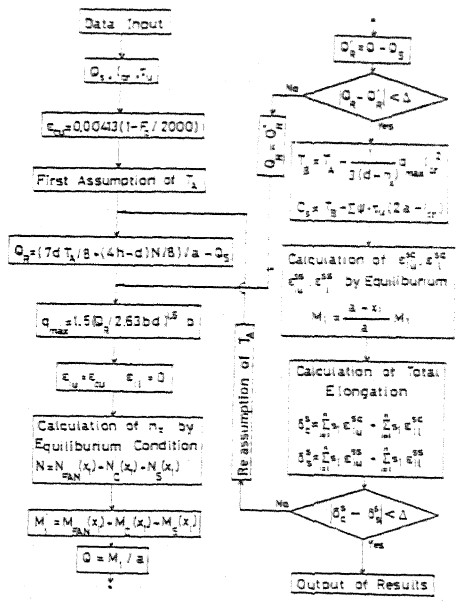

Fig.14 Flow chart for numerical caluculation

$\delta_c^s = \delta_s^s$ (30), where $\delta_c^s = \Sigma s_i\ \varepsilon_{iu}^{sc} + \Sigma s_i\ \varepsilon_{il}^{sc}$ , $\delta_s^s = \Sigma s_i\ \varepsilon_{iu}^{ss} + \Sigma s_i\ \varepsilon_{il}^{ss}$

$s_i$:length of each concrete segment

EVALUATION OF SHEAR CAPACITY OF R/C COLUMN

Based on a fundamental concept that the shear capacity of reinforced concrete column should be determined at the yielding of shear reinforcements , a following strength equation is proposed.

$Q_u = Q_s + v_R \cdot b \cdot jd$ — (31) $\qquad$ $Q_s$:given by Eq.12, $v_R$:given by Eq.14

Eq. 31 was checked by comparing with the results obtained from the shear tests on 266 column specimens carried in the past (Ref.3) as shown in Fig.15. In Fig.15, $Q_u$ denotes the capacity obtained from Eq.31, $Q_f$ the shear force at which the theoretical flexural strength is developed at critical section and $Q_u^e$ the maximum shear capacity obtained experimentally. From Fig.15, it is recognized that the almost columns showen the larger values of the capacity than the values from Eq.31. However, some columns showed the considerably smaller values of $Q_u^e$ (indicated by triangles). For such columns the shear strength $Q_{u,sp}$ due to bond splitting of longitudinal reinforcement were calculated by the proposed analytical method mentioned in a previous chapter and the obtained results are shown in Fig.15 by solid circles. In the calculation of $Q_{u,sp}$ , the splitting bond strength of reinforcement was given by an experimental equation proposed by Fujii and Morita (Ref.5). From the figure it is

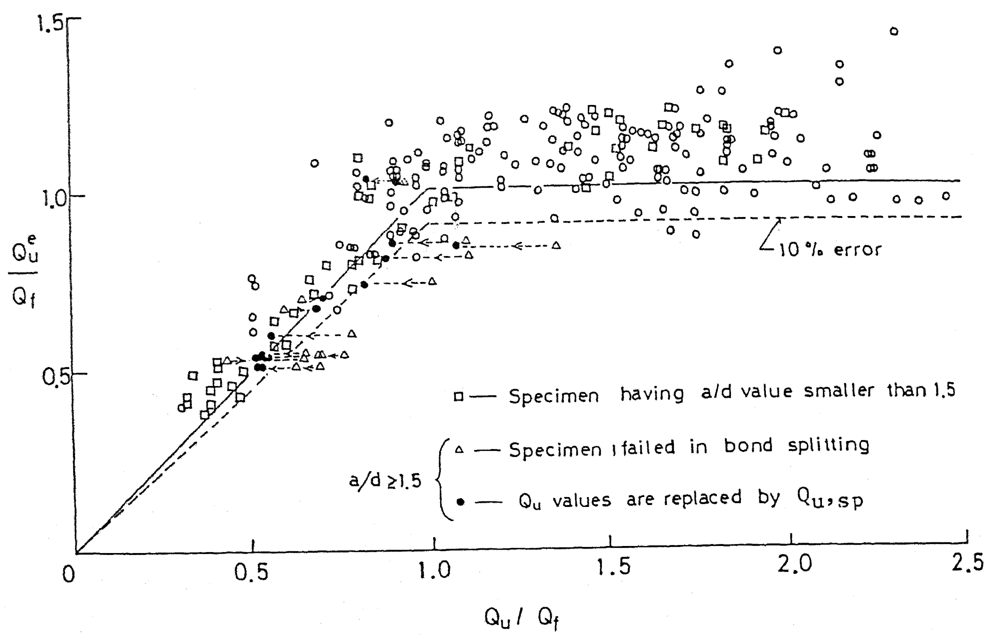


Fig.15 Evaluation of shear strength of R/C column

known that the shear strength of R/C column can be evaluated by considering the two failure modes ,that is, the yielding of shear reinforcements and the premature failure due to bond splitting.

## CONCLUSIONS

From this study the following conclusions can be derived.

(1) Diagonal tension cracking load can be estimated by Eq.12 based on the flexural bond action and the resistance of concrete cantilever like a tooth.

(2) Contribution of shear reinforcement is expressed by an empirical formula where the efficiency of shear reinforcement is defined at yieldings of it.

(3) Shear strength of R/C column can be evaluated by Eq.31. However the check for the premature shear failure due to bond splitting is required.

## ACKNOWLEDGEMENT

The author would like to acknowledge the continuiting guidance and encouragement of Prof. Dr. H.Muguruma.

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