

## WHY THE "ELEPHANT'S FOOT" PHENOMENON OF LIQUID STORAGE TANK HAPPENED

Guanqing Chen (I)

### SUMMARY

This paper presents a method for analyzing the seismic response of a flat bottomed cylindrical liquid storage tank to vertical earthquake excitation. Taking into account the vertical and horizontal earthquake loads, hydrostatic pressure, and considering restrictive moment and shear forces at shell-bottom welded joint, the author has calculated circumferential and longitudinal stresses. The calculated result closely conforms to the actual damage, termed "elephant's foot," observed in the fuel storage tanks damaged in the Tangshan earthquake. The effect due to vertical earthquake load is more than the effect from the horizontal load.

### INTRODUCTION

In the Tangshan earthquake of July 1976, four 1000 m<sup>3</sup> fuel storage tanks belonging to Tangshan Steel Factory and Tianjin Chemical Industrial Plant at Hangu developed one (partial filled) and two (full filled) outward bulge deformations at the base of the shells respectively. These bulges are generally called "elephant's foot." The welded shell-bottom joint failed at various places. Similar "elephant's foot" damage has also occurred in major earthquakes in the U.S.A., Japan and other countries, indicating that "elephant's foot" damage is not an occasional phenomenon.

Through analysis and observation of fuel tank damage caused by the Tangshan earthquake, the author considers that "elephant's foot" phenomenon in storage tanks are mainly caused by vertical earthquake loads and hydrostatic pressure. Horizontal earthquake loads are secondary, with the moment and shear force due to the boundary effect at the shell-bottom joint being taken into account. Under actions from all of these loads, the combined stresses, consisting of circumferential stresses and longitudinal bending stresses, exceeded the yield strength of the shell material. Therefore, it is not a stability problem of the tank shell, but a strength problem of the shell material.

### THE METHOD FOR CALCULATING VERTICAL EARTHQUAKE LOAD

#### Vertical Seismic Response Spectra

Ground motion during an earthquake is three-dimensional. The ratio of the vertical peak of earthquake acceleration to the horizontal peak is not a constant during an earthquake. The vertical component of the peak ground motion is commonly considered to be about 1/2 or 2/3 that of the horizontal. But in the epicenter or near it, the peak of vertical accelerations were equal to or even greater than the horizontal. The Engineering Mechanics

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(I) Mechanical Engineer, Pipeline Design and Research Institute of Petroleum Ministry, Langfang, Hebei Province, P.R.C.

Research Institute of the Academy of Sciences of China has made a preliminary study of the vertical response spectra of earthquakes and has concluded that there are no particular differences in character as to the peak, period of peak, curve and so on between the vertical and horizontal response spectra in the same earthquake and same area. Therefore, the vertical response spectra can be considered the same as the horizontal (Ref. 1). Therefore, we can use the typical design response spectra (horizontal) presented in "The Aseismic Design Code for Industrial and Civil Building" TJ11-78 to calculate the vertical earthquake loads (Ref. 2).

#### Damping Ratio of Tank-Liquid System

Typical response spectra correspond to elastic systems with one degree of freedom and an average damping ratio equal to 5%. In earthquake excitation, the maximum ratio value of acceleration response  $a_H$  to gravity acceleration  $g$ , i. e. the coefficient of earthquake effect  $\alpha = a_H/g$  is shown in Figure 1.

When the natural vibrational period  $T$  and soil conditions are known, the coefficient of earthquake effect can be obtained from Figure 1.

Accurate data about the damping ratio of radial vibration in a liquid system has not yet been obtained. The ranges of about 0.5 - 2% is represented by some references. When the damping ratio does not equal 5%, correction should be made for the response spectra of acceleration by multiplying by the coefficient  $\psi = 1/\sqrt{20\xi}$  (Ref. 3). Substituting  $\xi = 0.5 - 2\%$  into the equation, we obtain  $\psi = 1.6 - 3.2$ . In reference (1), it is stated that the value of response spectra is inversely proportional to  $n$  times the damping ratio. So the correct coefficient is  $\psi = (0.05/\xi)^n$  and substituting  $\xi = 0.5 - 2\%$  into the equation, we obtain  $\psi = 1.3 - 3.2$ . Therefore we can closely calculate the response spectra of the tank-liquid system by doubling the value of typical response spectra.

#### The Radial Vibrational Period of Storage Tanks

The storage tank is taken as a system with one degree of freedom. Under vertical earthquake excitation, courses of tank shell undergo radial expansive vibration around the vertical central axis of the tank without distortion of the circular cross section. (Ref. 4).

It is well known that circumferential stress in the shell due to hydrostatic pressure is

$$\sigma_{st} = \frac{\gamma HR}{t} \quad (\text{kg/cm}^2) \quad (1)$$

where

- $\gamma$  = density of liquid ( $\text{kg/cm}^3$ )
- $H$  = height of liquid column above the point in consideration (cm.)
- $R$  = radius of tank (cm.)
- $t$  = thickness of the shell (cm.)

The resultant force in the unit height shell is

$$F_{st} = \sigma_{st} \cdot t \cdot 1 = \gamma HR \quad (2)$$

Liquid mass force in the unit height shell is

$$M = \frac{F_{st}}{g} = \frac{\gamma HR}{g} \quad (\text{kg-sec}^2 / \text{cm}) \quad (3)$$

Where

$g$  = acceleration of gravity ( $\text{cm/sec}^2$ )

Radial displacement due to hydrostatic pressure is

$$\Delta R_{st} = \epsilon R = \frac{\sigma_{st} R}{E} = \frac{\gamma HR^2}{Et} \quad (\text{cm}) \quad (4)$$

$E$  = modulus of material elasticity of the shell ( $\text{kg/cm}^2$ )

Under the action of hydrostatic pressure, the circumferential elongation of the unit height shell is

$$\delta_{st} = 2\pi \Delta R_{st} = 2\pi \cdot \frac{\gamma HR^2}{Et} \quad (\text{cm}) \quad (5)$$

From dynamics, we know the radial vibrational period of a system with one degree of freedom is

$$T = 2\pi \sqrt{\frac{\delta_{st}}{g}} = 2\pi \sqrt{\frac{2\pi R^2 \gamma H}{Etg}} = 2\pi \sqrt{\frac{2\pi R \sigma_{st}}{Eg}} \quad (\text{sec}) \quad (6)$$

$$E = 2.1 \times 10^4 \quad (\text{kg/cm}^2)$$

$$g = 981 \quad (\text{cm/sec}^2)$$

Substituting these values into Equation (6), we obtain

$$T = 0.000356 \sqrt{R \sigma_{st}} \quad (\text{sec}) \quad (7)$$

#### Vibrational Load in Tank Shell Due to Vertical Earthquake Load

The resultant force in the unit height shell due to vertical earthquake load is

$$F_d = M a_v = \frac{\gamma HR}{g} a_v \quad (\text{kg}) \quad (8)$$

Where  $a_v$  = Maximum of vertical acceleration response of liquid subjected to vertical earthquake load. Because the damping ratio of the tank-liquid system should be corrected as above,  $a_v = 2a_H$ .

#### Dynamic Stress in the Shell

Dynamic stress in the shell under the action of a vertical earthquake load is

$$\sigma_d = \frac{F_d}{t \cdot 1} = \frac{\gamma HR}{t} \cdot \frac{a_v}{g} = \frac{2a_H}{g} \cdot \frac{\gamma HR}{t} = 2\alpha \sigma_{st} \quad (\text{kg/cm}^2) \quad (9)$$

$$\text{Here } \alpha = \frac{a_H}{g}$$

From the above Equation (9), it is seen that the dynamic stress due to vertical earthquake load  $\sigma_d$  is  $2\alpha$  times the stress due to hydrostatic pressure.

#### Equivalent Density of Liquid in the Tank Under Vertical Earthquake Load

From Equation (9) it is seen that under vertical earthquake loads the storage liquid density in the tank increases  $2\alpha$  times. Then the liquid density in the tank becomes

$$\gamma_d = \gamma \left(1 + \frac{\sigma_d}{\sigma_{st}}\right) = \gamma (1 + 2\alpha) \text{ (kg/cm}^3\text{)} \quad (10)$$

#### CALCULATION OF STRESSES IN THE BASE SHELL COURSE

##### Computational Model

Analysis of bending and circumferential stresses at the shell-bottom joint is rather complicated. Due to the space limitations, the computational model is briefly introduced without any detail developments.

The shell at the bottom joint is subjected to restraining action at the bottom of radial force  $Q_0$  and moment  $M_0$ , so the radial displacement of base shell at the bottom joint equals zero. When the tank is filled with liquid, the deformation and forces on the shell and bottom are shown on Figure 2.

Because the ratio  $L/R$  is very small, we can consider the length of bottom  $L$  as a beam supported by moment  $M_0$ , vertical force  $T_w$ , hydrostatic pressure and reactions  $R_1$  and  $R_2$  from the foundation, and calculate bending stress  $\sigma_b$  and circumferential stress  $\sigma_c$  in the base shell course.

#### INSPECTION OF STORAGE TANKS IN TANGSHAN EARTHQUAKE

##### Circumstance of Damage

At Tianjin Chemical Industrial Plant in Hangu (intensity IX zone in the Tangshan Earthquake), there were two fuel tanks which were made of steel and constructed in 1972. Each tank had a 1000 m<sup>3</sup> capacity and consisted of 7 courses each approximately 1400 mm high. The base shell courses have 6 mm thickness (full filled) and 5 mm thickness (partial filled) respectively, the second and third courses 5 mm thick, the remaining courses and the top were 4.5 mm thick, and the bottom was 5 mm thick. One tank was filled with fuel, the other was 50% filled during the Tangshan Earthquake. Two tanks used A3 steel, with a yield strength of 2300 kg/cm<sup>2</sup> and ultimate strength of 3800 kg/cm<sup>2</sup> (approximately equal to the grade of A 285). The deformation of the tank shell after the earthquake are as shown in Figures 3, 4 and 5.

##### Tank Data

Inside Diameter  $D = 1204.8$  cm

Total height of tank shell  $H_t = 960$  cm

Thickness of the base shell course  $t = 0.55$  cm (tank A, 100% filled)  
 $t = 0.45$  cm (tank B, 50% filled), (excluding corrosion allowance 0.05cm)  
 Thickness of bottom annular ring  $t = 0.5$  cm  
 Height of liquid in tank  $H = 0.9 H_t = 864$  cm (tank A)  
 $H = 0.45 H_t = 432$  cm (tank B)  
 Density of liquid  $\gamma = 0.9 \times 10^{-3}$  kg/cm<sup>3</sup>  
 Total weight of tank  $W_s = 35160$  kg (including insulation)  
 Total weight of storage liquid  $W = 886,500$  kg (tank A)  
 $W = 443,250$  kg (tank B)  
 Length of bottom outside tank shell  $b = 50$  cm  
 Earthquake Intensity at tank location : Intensity IX on the MM Scale

### Combined Stresses

There were bending stresses  $\sigma_b$  and circumferential stresses  $\sigma_c$  due to hydrostatic pressure, and horizontal bending stresses  $F$  and vertical bending stresses  $\sigma'_c$  and circumferential stresses  $\sigma'_b$  due to earthquake. The bending stresses in base shell course  $\sigma_b$  and  $\sigma'_b$  are the bending stresses in the range of shell thickness due to meridional bending moment, so they are positive and negative in a vertical section. Circumferential stresses  $\sigma_c$  and  $\sigma'_c$  are tensile stresses. The bending stresses  $F$  (calculated in accordance with Appendix E of the API Standard 650) due to the horizontal earthquake load are the bending stresses in the annular area on the whole tank shell, thus of the whole tank section. Part of the shell is in tension and part is in compression. We take the unfavorable condition for the computation of the combined stress, i.e.  $F$  as compressive stress. It should be noticed as well that earthquake loads are vibrational ones and stresses caused by them are alternating.

A check according to maximum shear stress theory was also made. The condition in which plastic deformation of the tank shell does not occur is

$$\sigma_1 - \sigma_3 < \sigma_s \quad (\text{kg/cm}^2) \quad (11)$$

where  $\sigma_1$  = total circumferential stress in the shell (kg/cm<sup>2</sup>)

$$\sigma_1 = \sigma_c + \sigma'_c \quad (12)$$

$\sigma_3$  = total longitudinal stress in the shell (kg/cm<sup>2</sup>)

$$\sigma_3 = \sigma_b + \sigma'_b - F \quad (13)$$

$\sigma_s$  = minimum specified yield strength of the shell material (kg/cm<sup>2</sup>)

$$\sigma_s = 2300 - 2400 \quad (\text{steel A3}) \quad (14)$$

### Combined Stresses Table

From the following table 1, it is seen that the combined stresses of

tank shell ( $\sigma_1 - \sigma_2$ ) exceeded the yield strength of the tank shell at the range  $x = 20 - 70$  cm (tank A) and  $x = 0 - 30$  cm (tank B) of vertical distance from the bottom of shell section. Therefore, under the action of earthquake loads, the "elephant's foot" phenomenon occurred in these parts of the tank shell. This calculated results conform completely to observed data.

### CONCLUSION

The "elephant's foot" phenomenon that may appear on a storage tank in an earthquake is caused by resultant stress at the base shell course exceeding the yield strength of the material over a large area. Therefore, it is not a stability problem of the tank shell, but a strength problem of the material.

The action of vertical earthquake acceleration upon the storage tank corresponds to increasing the density of storage liquid. When the earthquake intensities are VII, VIII, IX, it corresponds to increasing the density of storage liquid by 0.46, 0.90, 1.8 times respectively. Thus, the bending stresses and circumferential stresses in the tank shell due to hydrostatic pressure are increased by 0.46, 0.9, 1.8 times (only for soft soil).

The effect of vertical earthquake acceleration on the storage tank is more than the effect of horizontal earthquake acceleration at higher intensities near the epicenter of the earthquake. The circumferential stress caused by vertical earthquake acceleration is 10 times more than the bending stress caused by horizontal earthquake acceleration.

The tanks previously mentioned were on soft soil and the period of radial vibration of the tanks is so short ( $T = 0.2 - 0.3$ ) that the earthquake acceleration corresponds to the maximum of the response spectra. Otherwise, the calculation of earthquake load will relate to the soil condition under storage tank.

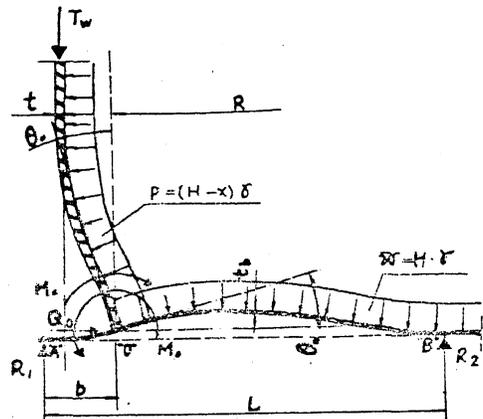
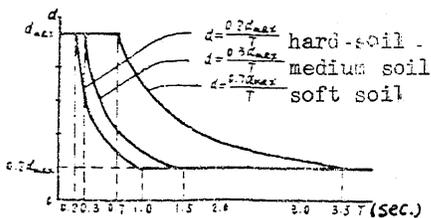


Figure 1. The coefficient of earthquake effect. Figure 2. Shell-bottom free body diagram

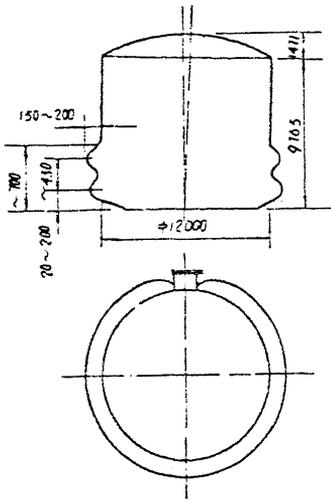


Figure 3. The deforeamation of Tank A after the Tangshan earthquake

Figure 4. The photograph of Tank A with two outward bulges elephant' s foot deformation

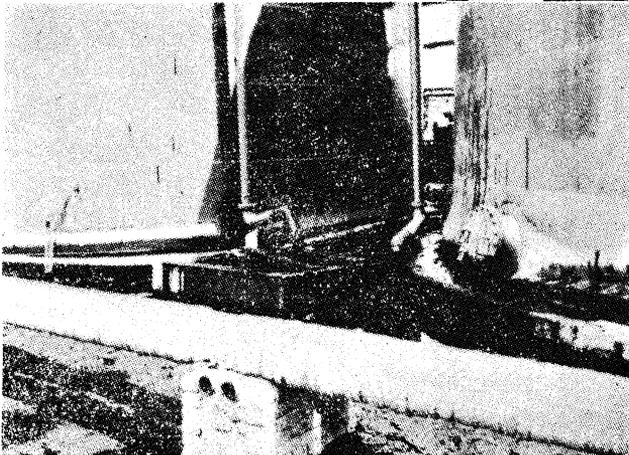
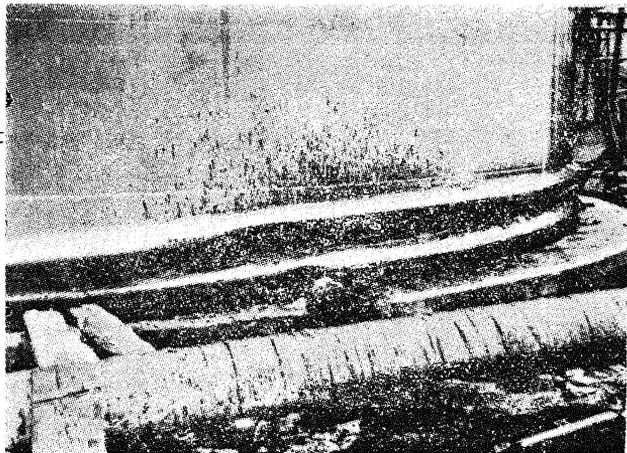


Figure 5. The photograph of Tank A (left) and Tank B (right) with "elephant's foot

Table 1 Combined Stresses along Tank Height

Vertical distance from bottom to shell section being conesidered x (cm)	Total circumferen- tial stresses in the shell <sup>2</sup> $\sigma_1$ (kg/cm <sup>2</sup> )		Total longitudinal stresses in the shell $\sigma_3$ (kg/cm <sup>2</sup> )		Combined stresses $\sigma_1 - \sigma_3$ (kg/cm <sup>2</sup> )	
	tank A <sup>1</sup>	tank B	tank A <sup>3</sup>	tank B	tank A	tank B
0	0.3	0	+2956 -3496	-6955 +4787	0 3496	6955 0
10	837	2632	- 281 - 177	-3421 +3203	1118 1014	6053 0
20	1770	2744	-1024 + 628	- 972 + 758	2794 1770	3716 0
30	2251	2128	- 784 + 432	+ 70 - 278	3035 2251	2058 2406
40	2372	1588	- 414 + 96	+ 266 - 468	2786 2372	1322 2056
50	2332	1299	- 190 - 100	+ 151 - 347	2522 2432	1148 1646
60	2260	1193	- 103 - 165	+ 8 - 200	2363 2425	1185 1393
70	2201	1170	- 89 - 157	- 73 - 113	2290 2358	1243 1283
80	2162	1162	- 100 - 134	- 104 - 76	2262 2296	1266 1238
90	2131	1148	- 103 - 115	- 101 - 73	2234 2246	1249 1221
100	2106	1120	- 100 - 92	- 93 - 77	2206 2144	1213 1197
120	2052	1056	- 90	- 79	2142	1135

Note: Take  $\sigma_3 = 0$ , when  $\sigma_3 > 0$   
 This part of the tank material underwent plastic deformation and appeared as "elephant's foot."

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